Power System Stability

- **The ability of the power system to remain in synchronism and maintain the state of equilibrium following a disturbing force**
  - Steady-state stability: analysis of small and slow disturbances
    - gradual power changes
  - Transient stability: analysis of large and sudden disturbances
    - faults, outage of a line, sudden application or removal of load

Generator Dynamic Model

- **Under normal conditions, the relative position of the rotor axis and the stator magnetic field axis is fixed**
  - the angle between the two is the power angle or torque angle, δ
  - during a disturbance, the rotor will accelerate or decelerate w.r.t. the rotating stator field
  - acceleration or deceleration causes a change in the power angle

$$T_e = \frac{P_e}{\omega_e} = \frac{P_e}{2\pi(60Hz)} \qquad \frac{P_m}{\omega_{rotor}} = T_m$$

$$T_{accelation} = \Delta T = T_m - T_e$$

$$J\frac{d^2\theta_m}{dt^2} = \Delta T = T_m - T_e \qquad \theta_m = \omega_{ms}t + \delta_m \qquad \frac{\omega_{rotor}}{\omega_{ms}} = \frac{poles}{2}$$

J	the total moment of inertia of the rotor masses, in kg-m^2
θ_m	the angular displacement of the rotor with respect to a stationary axis, in mechanical radians (rad)
t	time, in seconds (s)
T_m	the mechanical or shaft torque supplied by the prime mover less retarding torque due to rotational losses, in N-m
T_e	the net electrical or electromagnetic torque, in N-m
T_a	the net accelerating torque, in N-m

where ω_{sm} is the synchronous speed of the machine in mechanical radians per second and δ_m is the angular displacement of the rotor, in mechanical radians, from the synchronously rotating reference axis

Generator Dynamic Model

$$\omega_m = \frac{d\theta_m}{dt} = \omega_{ms} + \frac{d\delta_m}{dt} \qquad \alpha_m = \frac{d^2\theta_m}{dt^2} = \frac{d^2\delta_m}{dt^2}$$

$$J\frac{d^2\theta_m}{dt^2} = J\frac{d^2\delta_m}{dt^2} = T_m - T_e$$

$$J\omega_m\frac{d^2\delta_m}{dt^2} = \omega_m T_m - \omega_m T_e = P_m - P_e$$

$$W_{KE} = \tfrac{1}{2}J\omega_m^2 = \tfrac{1}{2}M\omega_m \qquad M = \frac{2\,W_{KE}}{\omega_m} = J\omega_m$$

$$\omega_m \approx \omega_{ms} \rightarrow M \approx \frac{2\,W_{KE}}{\omega_{ms}} = J\omega_{ms}$$

Generator Dynamic Model

$$M\frac{d^2\delta_m}{dt^2}=P_m-P_e$$

$$\delta=\delta_e=\frac{poles}{2}\delta_m \quad \rightarrow \quad \frac{p}{2}M\frac{d^2\delta}{dt^2}=P_m-P_e$$

$$\frac{p}{2}M\frac{d^2\delta}{dt^2} \quad = \quad \frac{p}{2}\frac{2W_{KE}}{\omega_{ms}}\frac{d^2\delta}{dt^2} \quad = \quad \frac{2W_{KE}}{\omega_s}\frac{d^2\delta}{dt^2}$$

$$\frac{2W_{KE}}{\omega_s}\frac{d^2\delta}{dt^2}=P_m-P_e \quad \rightarrow \quad \frac{2W_{KE}}{\omega_s S_B}\frac{d^2\delta}{dt^2}=\frac{P_m}{S_B}-\frac{P_e}{S_B}$$

Generator Dynamic Model

$$\frac{2W_{KE}}{\omega_s S_B}\frac{d^2\delta}{dt^2} = P_{m(pu)} - P_{e(pu)}$$

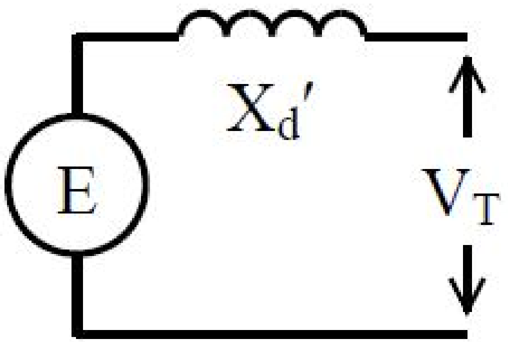
$$\frac{W_{KE}}{S_B} = \frac{\text{kinetic energy in MJ at rated speed}}{\text{machine power rating in MVA}} = H$$

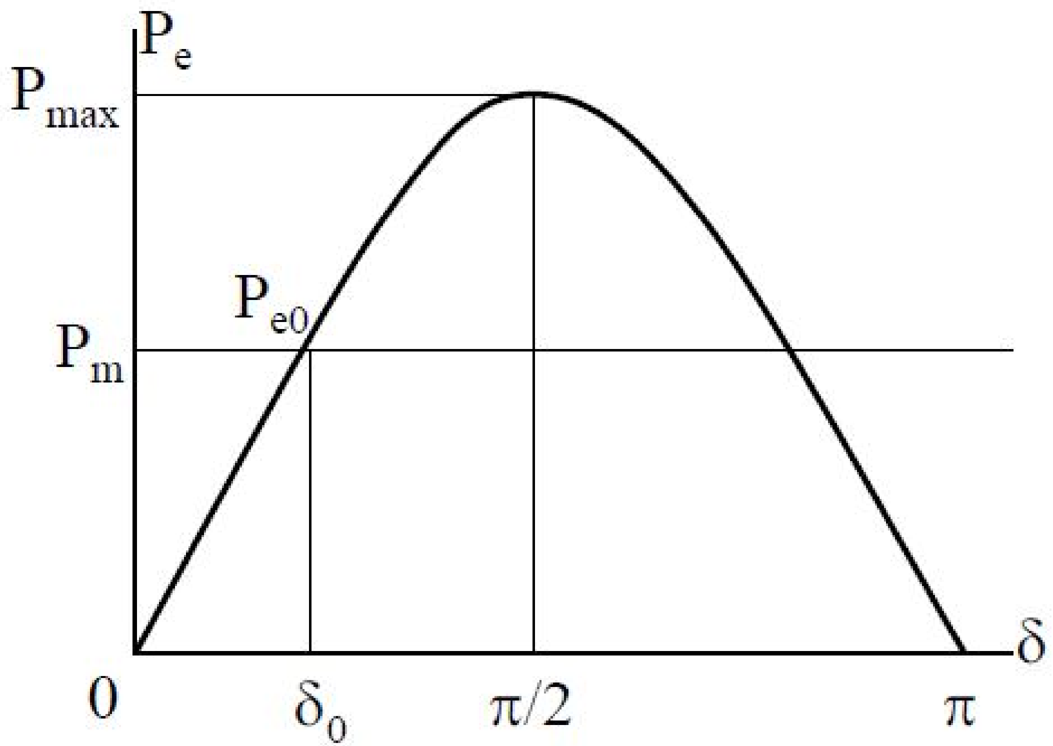
$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_{m(pu)} - P_{e(pu)}$$

$$\rightarrow \quad \frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_{m(pu)} - P_{e(pu)} \quad (radians)$$

$$\rightarrow \quad \frac{H}{180 f}\frac{d^2\delta}{dt^2} = P_{m(pu)} - P_{e(pu)} \quad (degrees)$$

Synchronous Machine Model





power angle curve

Round Rotor Machine Model

$$E' = |E'|\angle\delta$$

$$V_G = |V_G|\angle 0^\circ$$

$$B = 1/X_d'$$

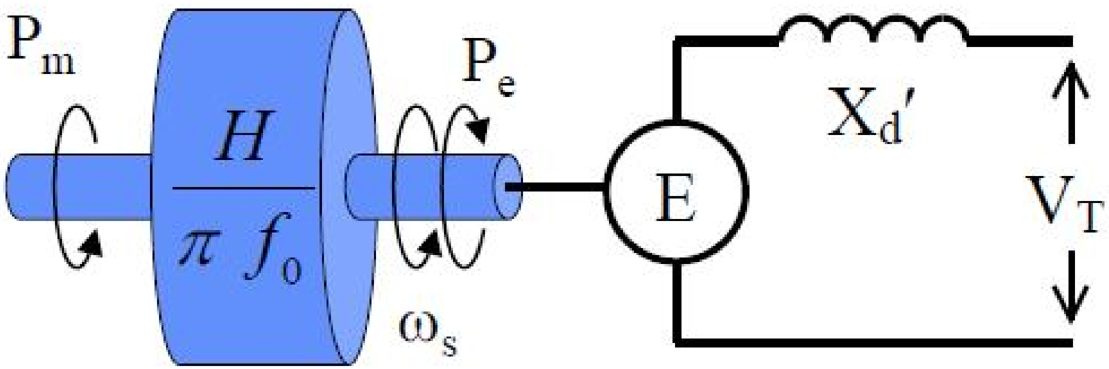
$$P_e = |E'||V_G||B|\cos(\delta - 90^\circ) = \frac{|E'||V_G|}{X_d'}\sin\delta = P_{max}\sin\delta$$

The Swing Equation

$$\frac{H}{\pi\, f_0}\frac{d^2\delta}{dt^2} = P_m - P_e$$ Dynamic Generator Model

$$P_e = P_{\max}\sin\delta$$ Synchronous Machine Model

$$\frac{H}{\pi\, f_0}\frac{d^2\delta}{dt^2} = P_m - P_{\max}\sin\delta$$ Forming the Swing Equation



Example 16.3. The single-line diagram of Fig. 16.4 shows a generator connected through parallel transmission lines to a large metropolitan system considered as an infinite bus. The machine is delivering 1.0 per-unit power and both the terminal voltage and the infinite-bus voltage are 1.0 per unit. Numbers on the diagram

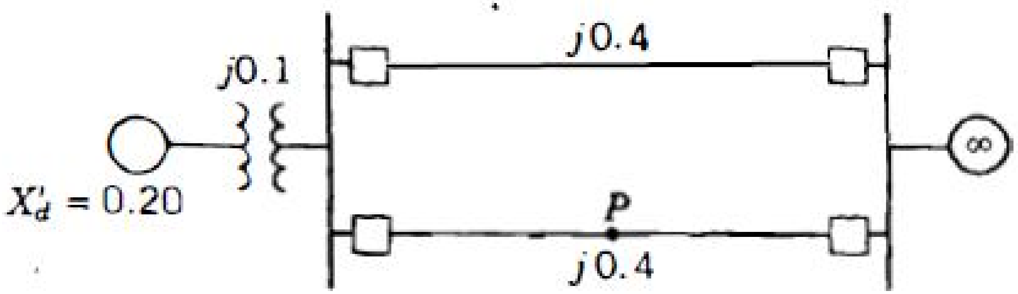


FIGURE 16.4
One-line diagram for Examples 16.3 and 16.4. Point P is at the center of the line.

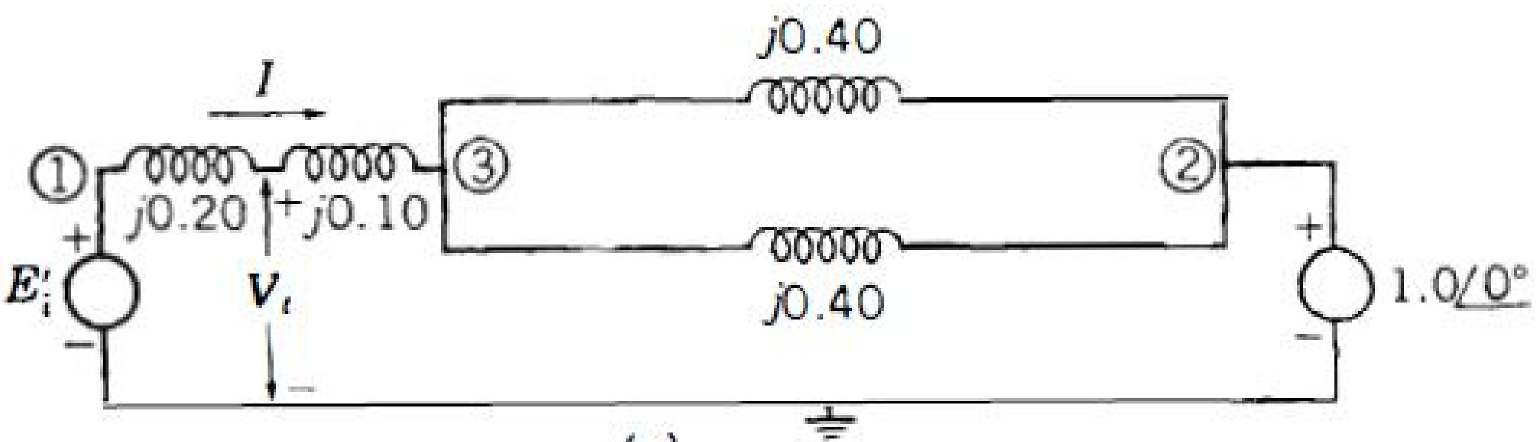
indicate the values of the reactances on a common system base. The transient reactance of the generator is 0.20 per unit as indicated. Determine the power-angle equation for the given system operating conditions.

Solution. The reactance diagram for the system is shown in Fig. 16.5(a). The series reactance between the terminal voltage and the infinite bus is

$$X = 0.10 + \frac{0.4}{2} = 0.3 \text{ per unit}$$

and therefore the 1.0 per-unit power output of the generator is determined by

$$\frac{|V_t||V|}{X} \sin \alpha = \frac{(1.0)(1.0)}{0.3} \sin \alpha = 1.0$$



(a)

where V is the voltage of the infinite bus and α is the angle of the terminal voltage relative to the infinite bus. Solving for α, we obtain

$$\alpha = \sin^{-1} 0.3 = 17.458°$$

so that the terminal voltage is

$$V_t = 1.0\angle 17.458° = 0.954 + j0.300 \text{ per unit}$$

The output current from the generator is now calculated as

$$I = \frac{1.0\angle \delta_2 - 1.0\angle 0°}{j0.3}$$

$$= 1.0 + j0.1535 = 1.012\angle 8.729° \text{ per unit}$$

and the transient internal voltage is then found to be

$$E_1' = (0.954 + j0.30) + j(0.2)(1.0 + j0.1535)$$

$$= 0.923 - j0.5 = 1.050\angle 28.44^\circ \text{ per unit}$$

The power-angle equation relating the transient internal voltage E_i' and the infinite-bus voltage V is determined by the *total* series reactance

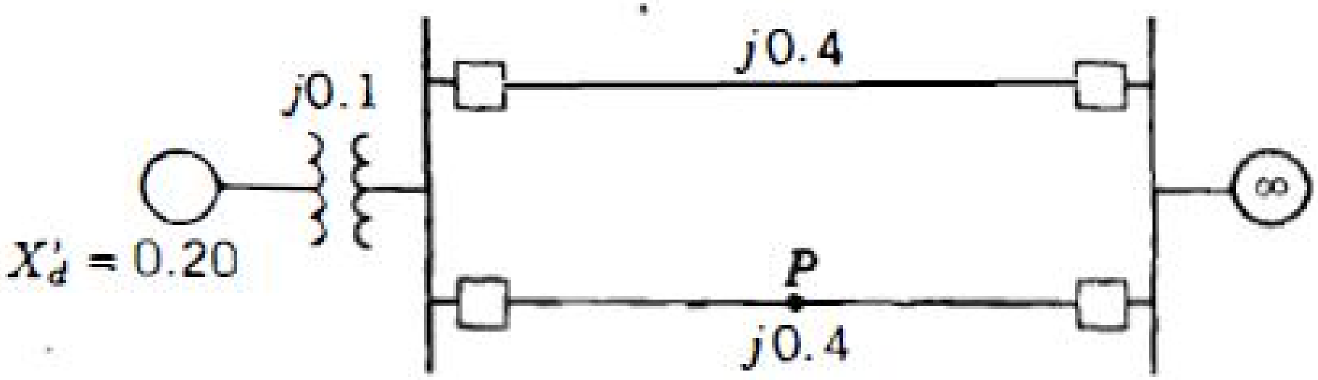
$$X = 0.2 + 0.1 + \frac{0.4}{2} = 0.5 \text{ per unit}$$

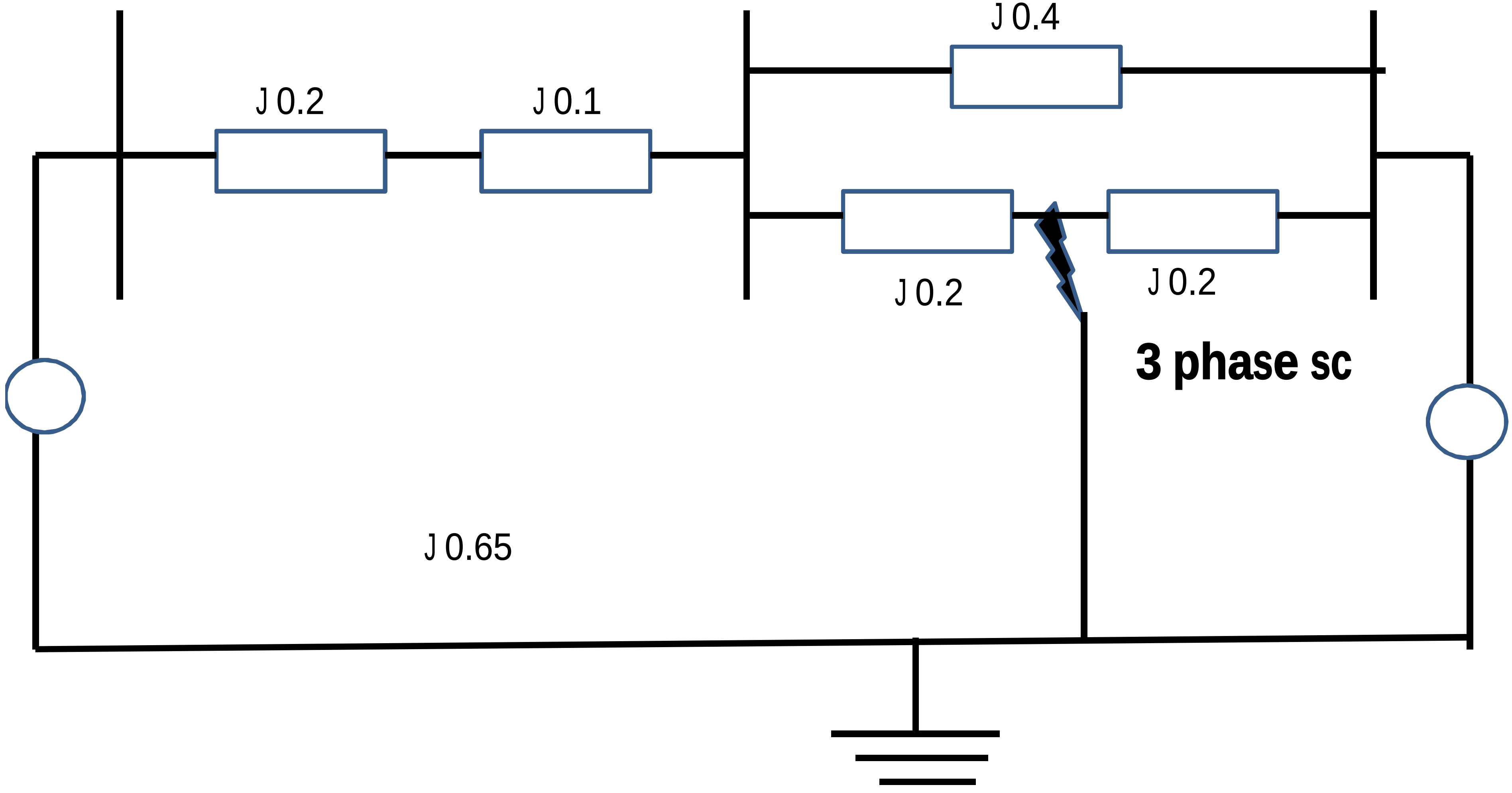
Hence, the desired equation is

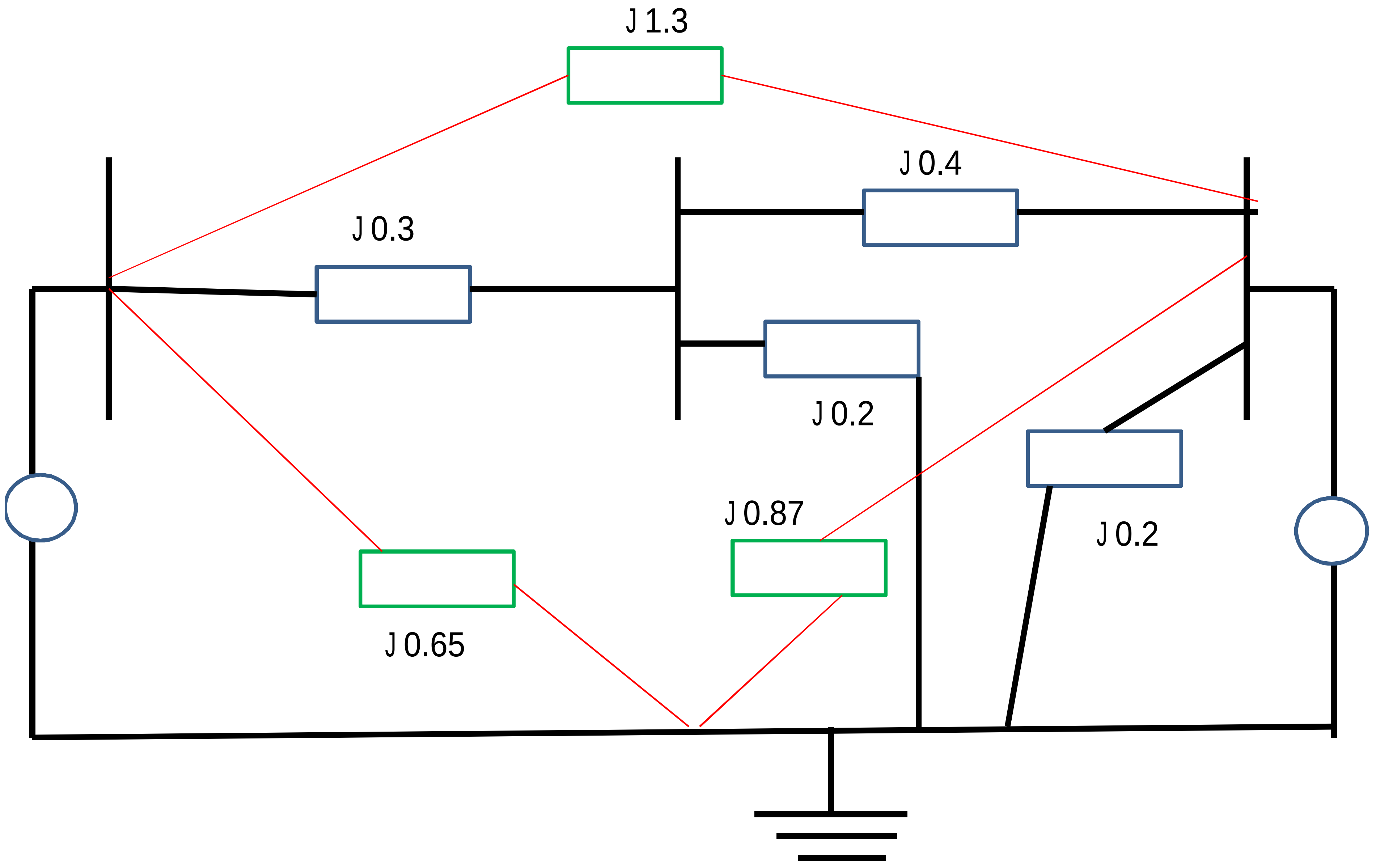
$$P_e = \frac{(1.050)(1.0)}{0.5} \sin\delta = 2.10 \sin\delta \text{ per unit}$$

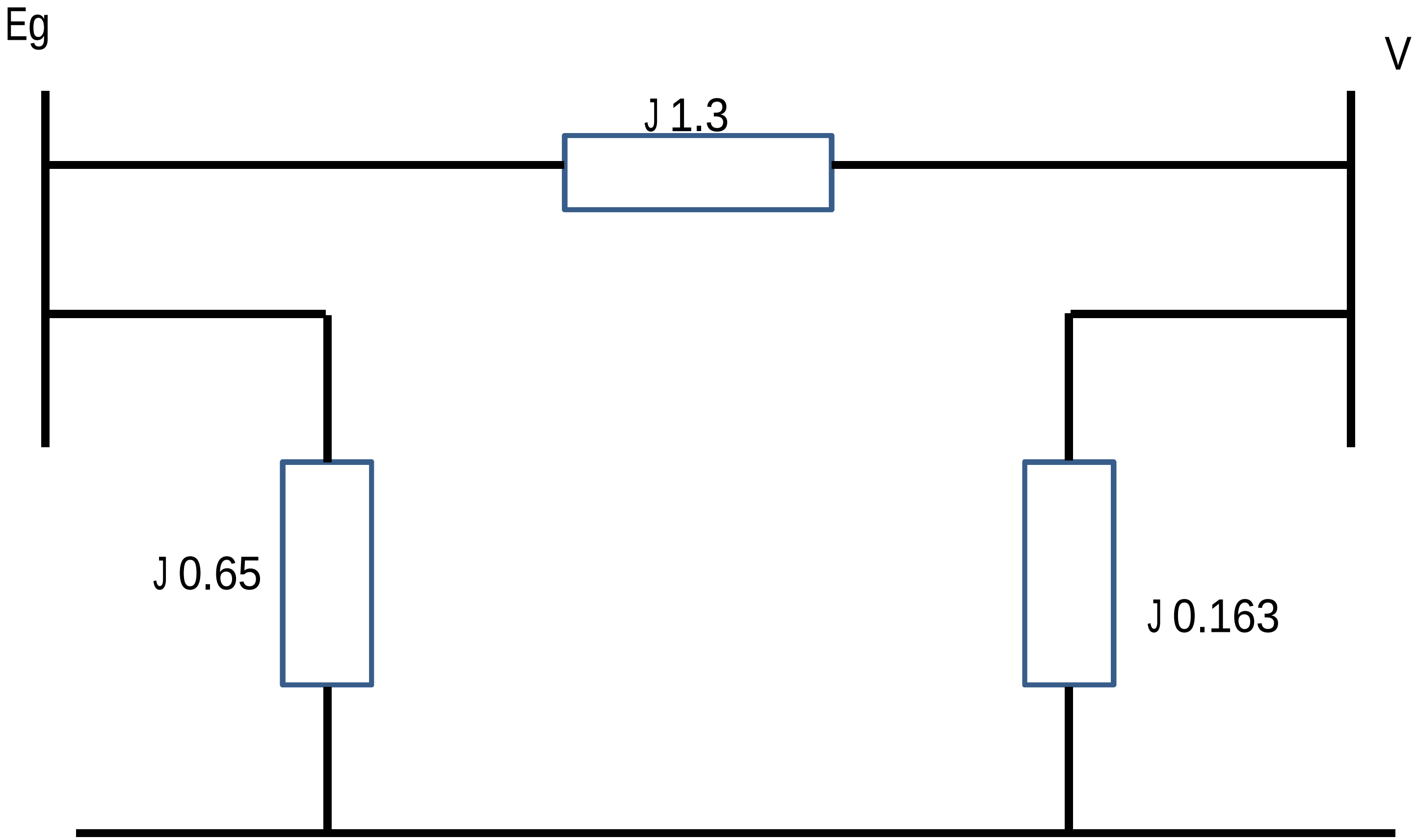
where δ is the machine rotor angle with respect to the infinite bus.

Example 16.4. The system of Example 16.3 is operating under the indicated conditions when a three-phase fault occurs at point P of Fig. 16.4. Determine the power-angle equation for the system with the fault on and the corresponding swing equation. Take $H = 5$ MJ/MVA.




J 0.4
J 0.2
J 0.1
J 0.2
J 0.2
3 phase sc
J 0.65


J 1.3
J 0.4
J 0.3
J 0.2
J 0.87
J 0.2
J 0.65


Eg
V
J 1.3
J 0.65
J 0.163

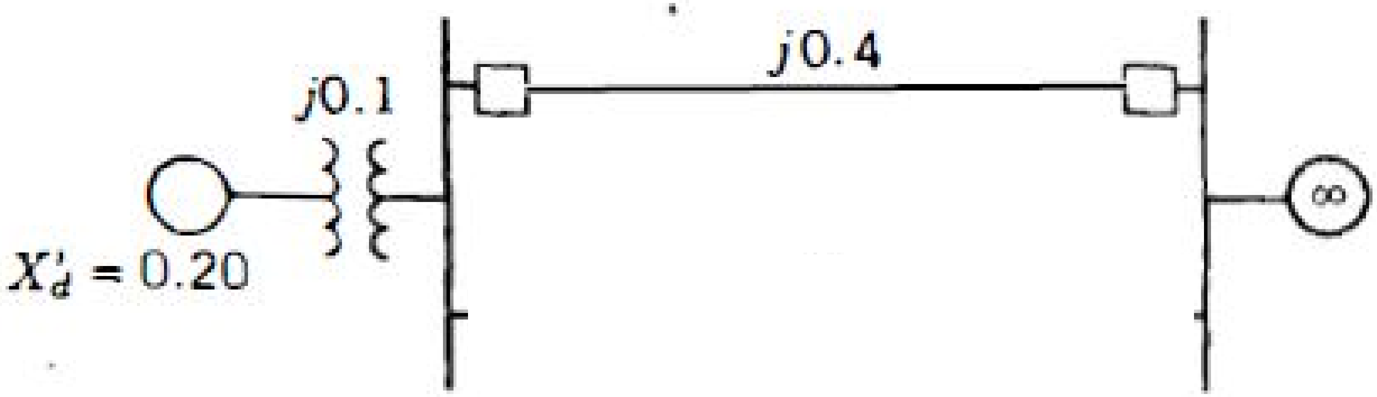
The power-angle equation with the fault on the system is therefore

$$P_e = 0.808 \sin \delta \text{ per unit}$$

and the corresponding swing equation is

$$\frac{5}{180f} \frac{d^2\delta}{dt^2} = 1.0 - 0.808 \sin \delta \text{ per unit}$$

Example 16.5. The fault on the system of Example 16.4 is cleared by simultaneous opening of the circuit breakers at each end of the affected line. Determine the power-angle equation and the swing equation for the postfault period.



Therefore, the postfault power-angle equation is

$$P_e = \frac{(1.05)(1.0)}{0.7} \sin \delta = 1.500 \sin \delta$$

and the corresponding swing equation is

$$\frac{5}{180f} \frac{d^2\delta}{dt^2} = 1.0 - 1.500 \sin \delta$$