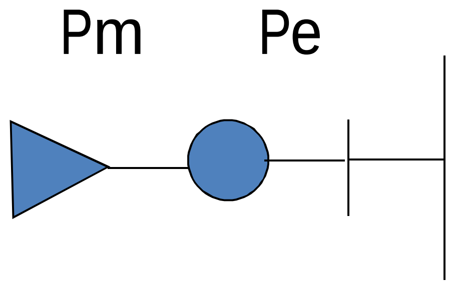
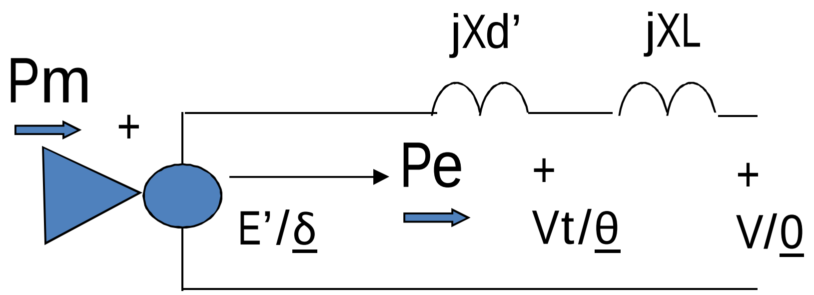
Transient Stability Solution Methods

- There are two methods for solving the transient stability problem

1. Numerical integration
   - this is by far the most common technique, particularly for large systems; during the fault and after the fault the power system differential equations are solved using numerical methods
2. Direct or energy methods; for a two bus system this method is known as the equal area criteria
   - mostly used to provide an intuitive insight into the transient stability problem

A generator connected to an infinite bus through a line. Initially Pm=Pe





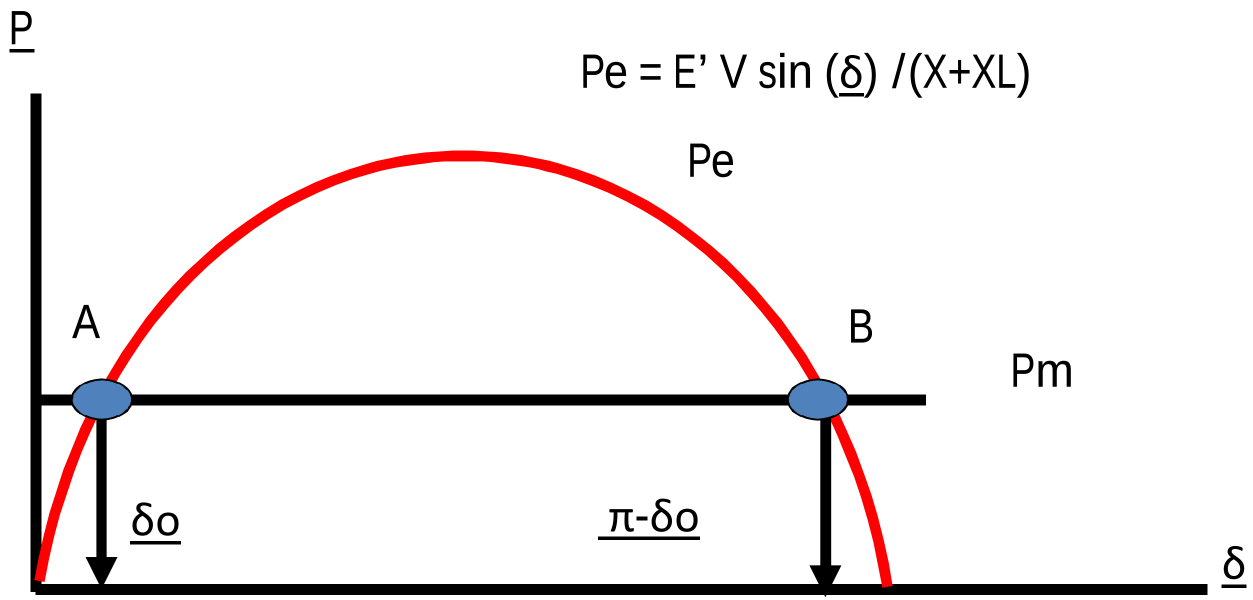
Also note, if Vt/θ is the terminal voltage of the generator

$$Pe = Vt\ V \sin(\theta) / (XL) \Rightarrow \theta = \operatorname{asin}[(Pe\ XL) / (Vt\ V)]$$

In Power Flow we simulate steady state (equilibrium)

Given: Vt, V and P   Find θ

The power angle curve allows us to visualize dynamics and equilibria

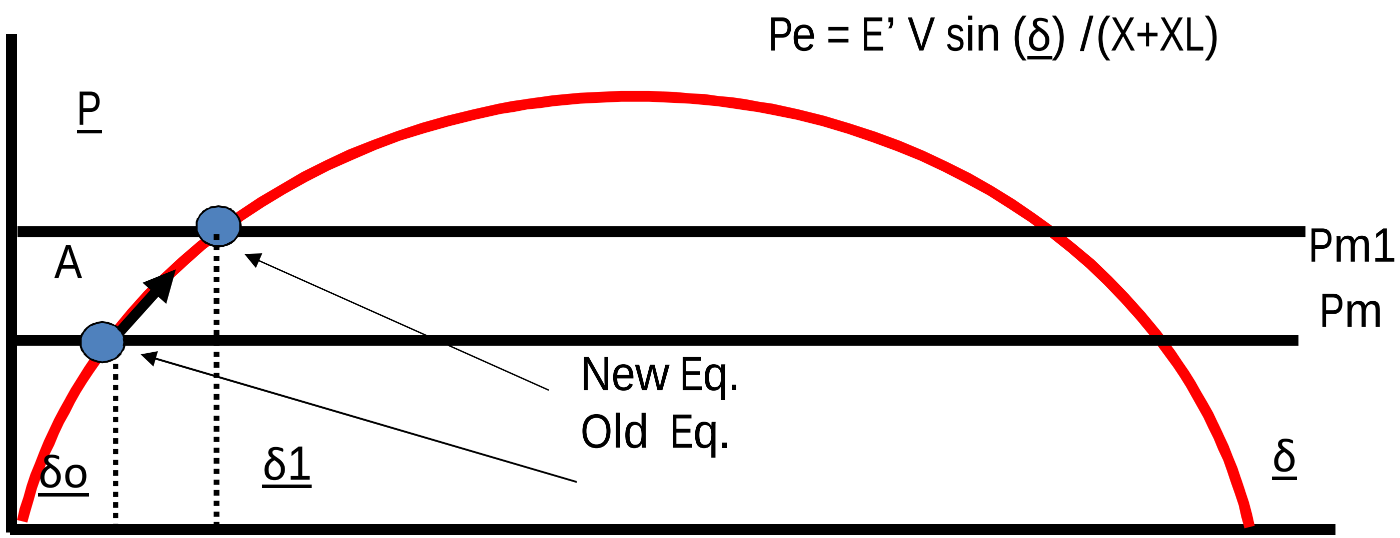


Intersection, A, is an equilibrium point where Pm=Pe.

The speed would remain constant at synchronous speed

B is also an equilibrium

Stable and unstable Equilibria – SMALL increase in Mechanical Power



Starting at point A. Increases Pm slowly/slightly to Pm1

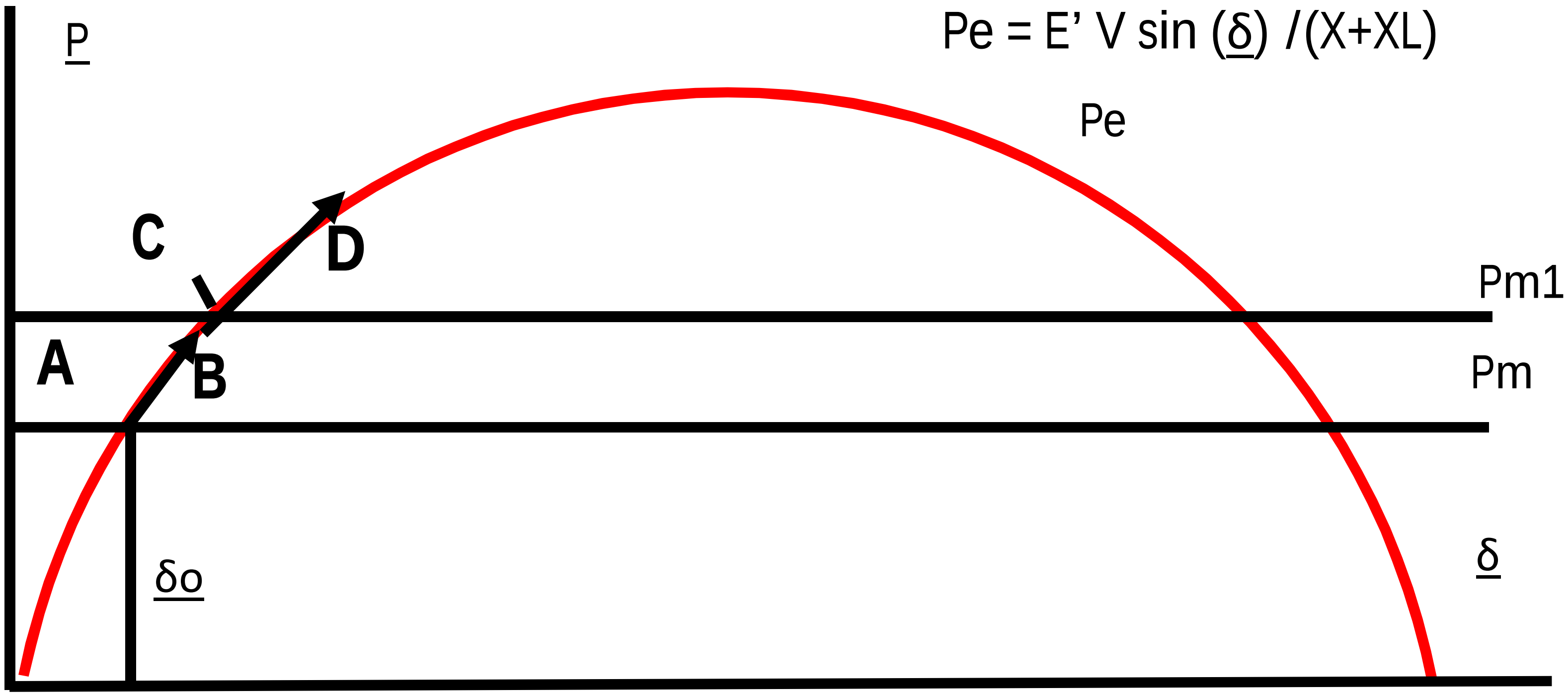
Pm1> Pe $d\omega/dt = (\pi f/H)(Pm-Pe)$ =>ω increases fromωsyn

Since ω> ωsyn and $d\delta/dt = \omega - \omega_{syn}$ => δ increases

For our system

Pe = E' V sin (δ) /(Xd'+XL)=>Pe increases

Stable Equilibrium



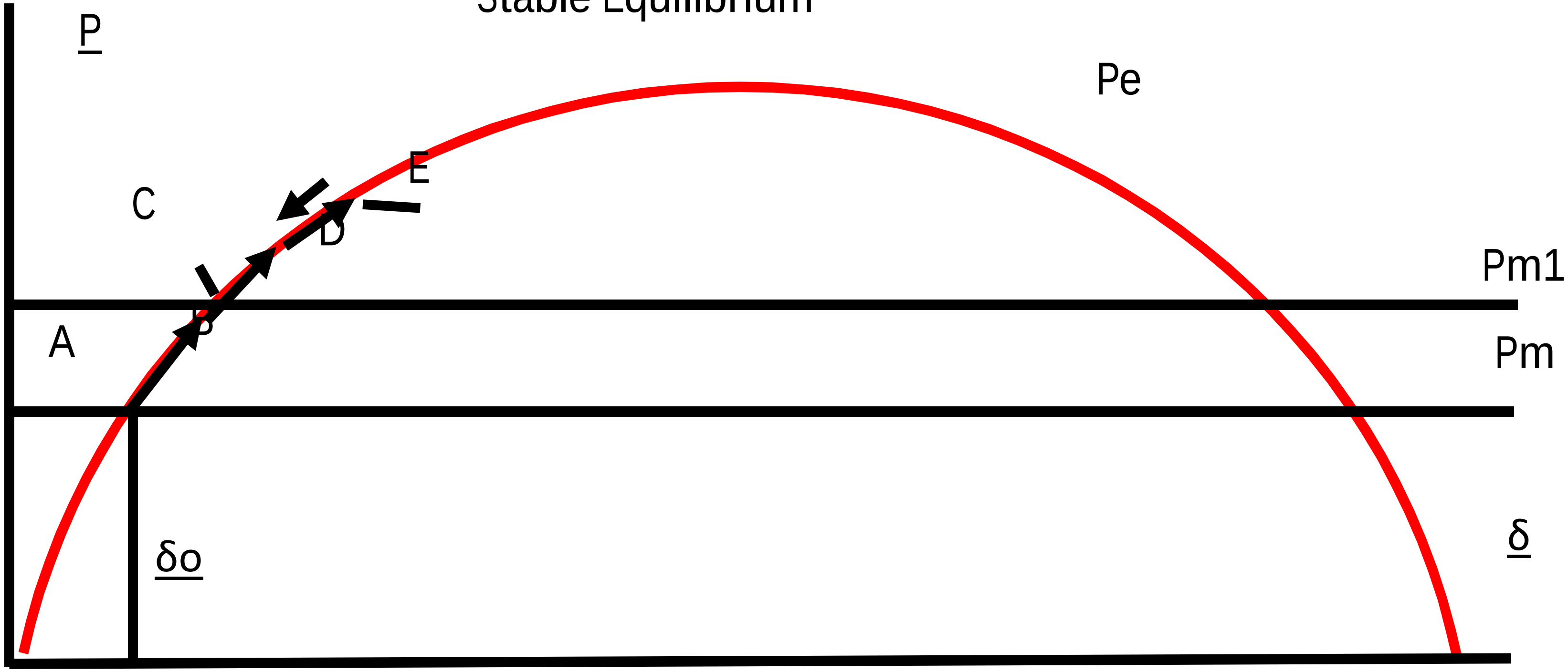
At point B ω> ωsyn => δ increases, say to C

Pm = Pe now so ω stops increasing

But ω> ωsyn so δ increases further

Now, at D Pm<Pe and ω begins to decrease

Stable Equilibrium



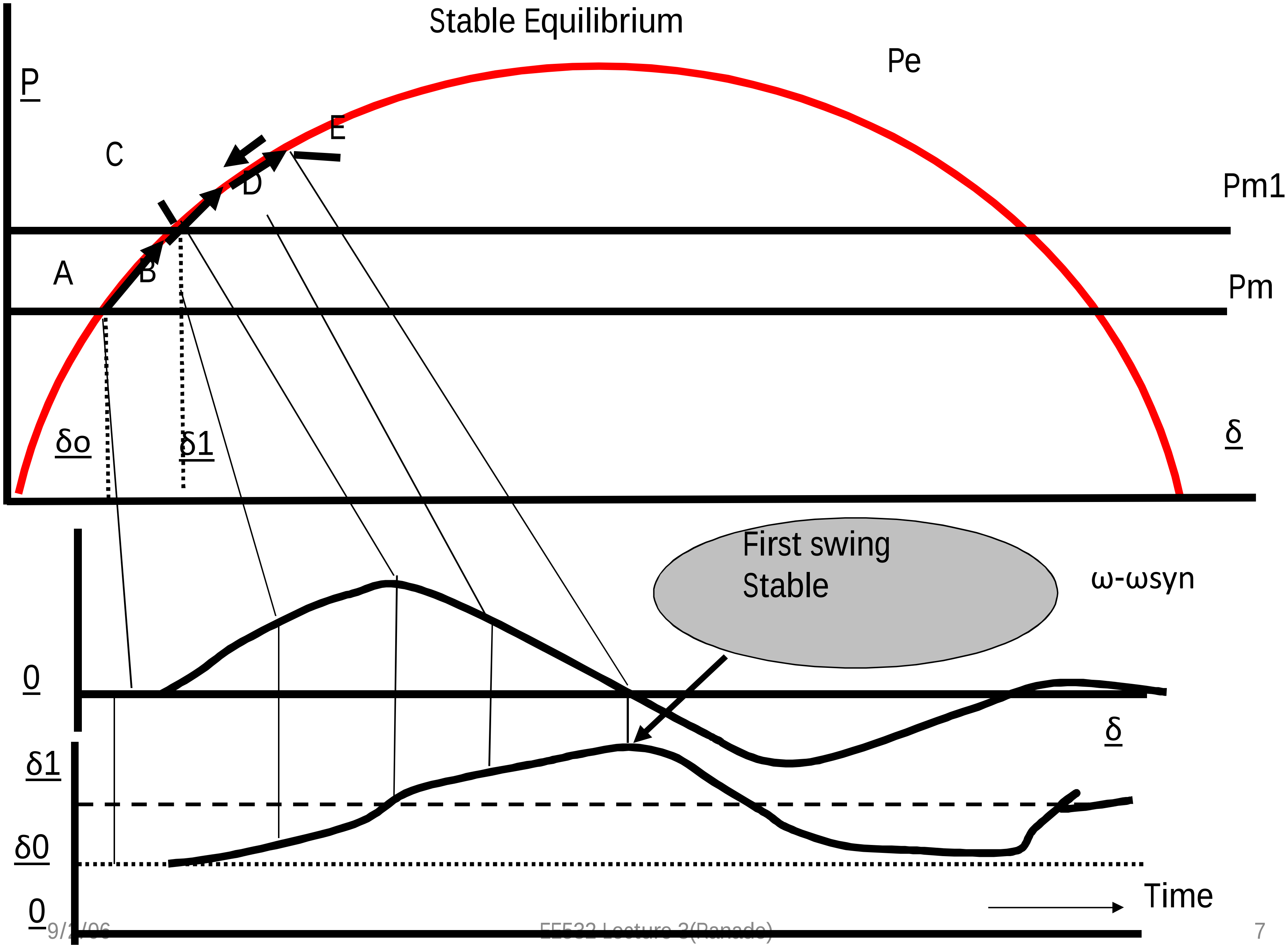
At D ω is decreasing but > ωsyn

δ increases further say to point E

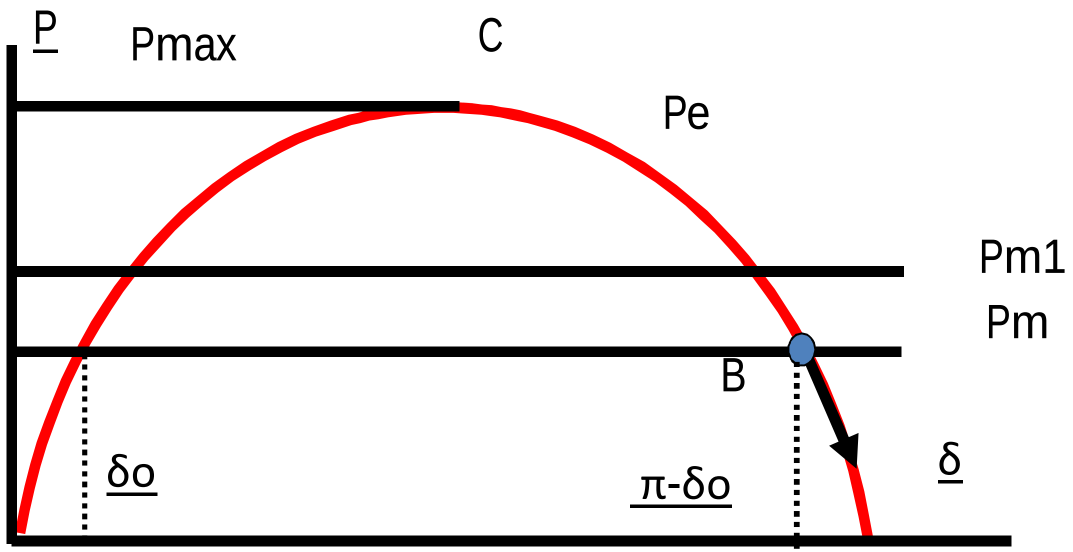
By now suppose ω is back to zero and decreasing

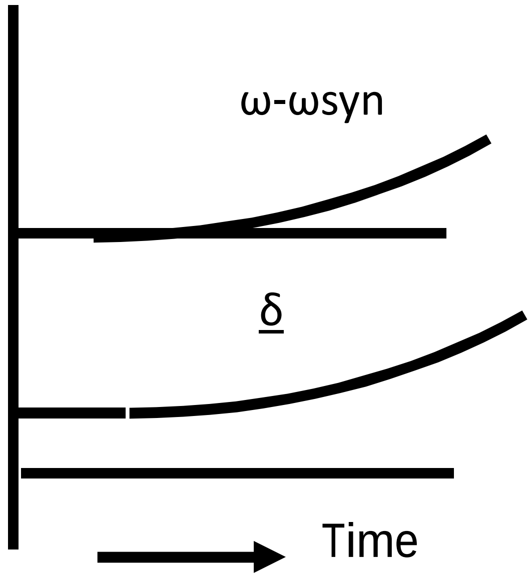
ω becomes < ωsyn as the generator continues to slow

Since ω< ωsyn δ decreases towards B First swing stable!


Stable Equilibrium
Pe
P
C
E
D
Pm1
A
B
Pm
δo
δ1
δ
First swing
Stable
ω-ωsyn
0
δ
δ1
δ0
0
Time

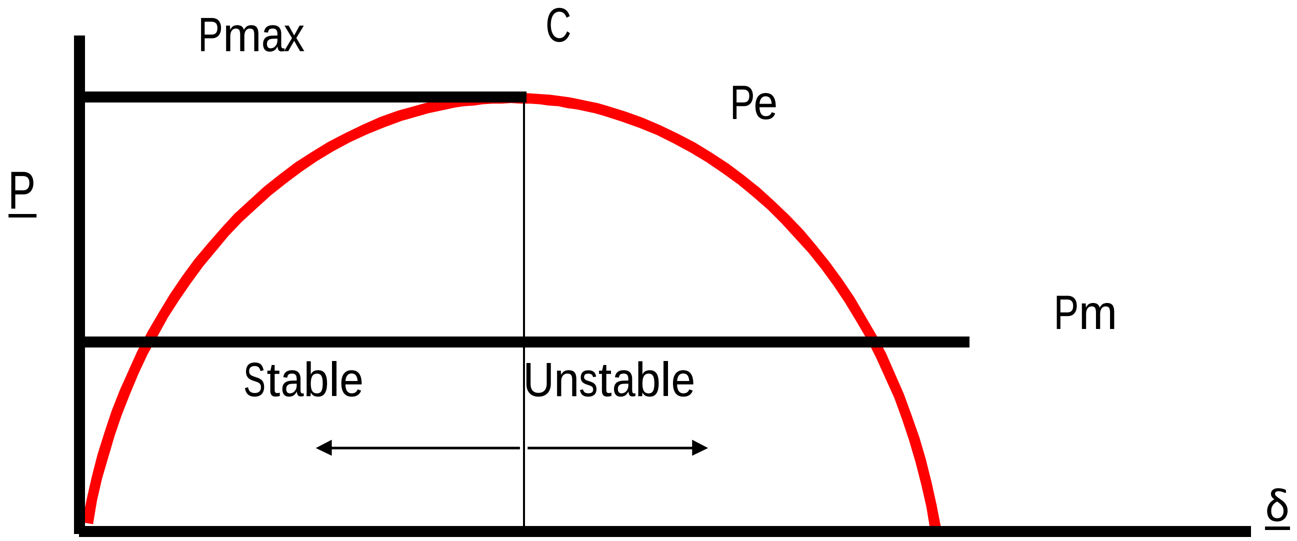
Unstable Equilibrium





Point B is unstable
Raising Pm increases speed which increases angle
But this time Pe decreases raising speed further…

Stable and unstable Equilibria-Steady State Stability Limit



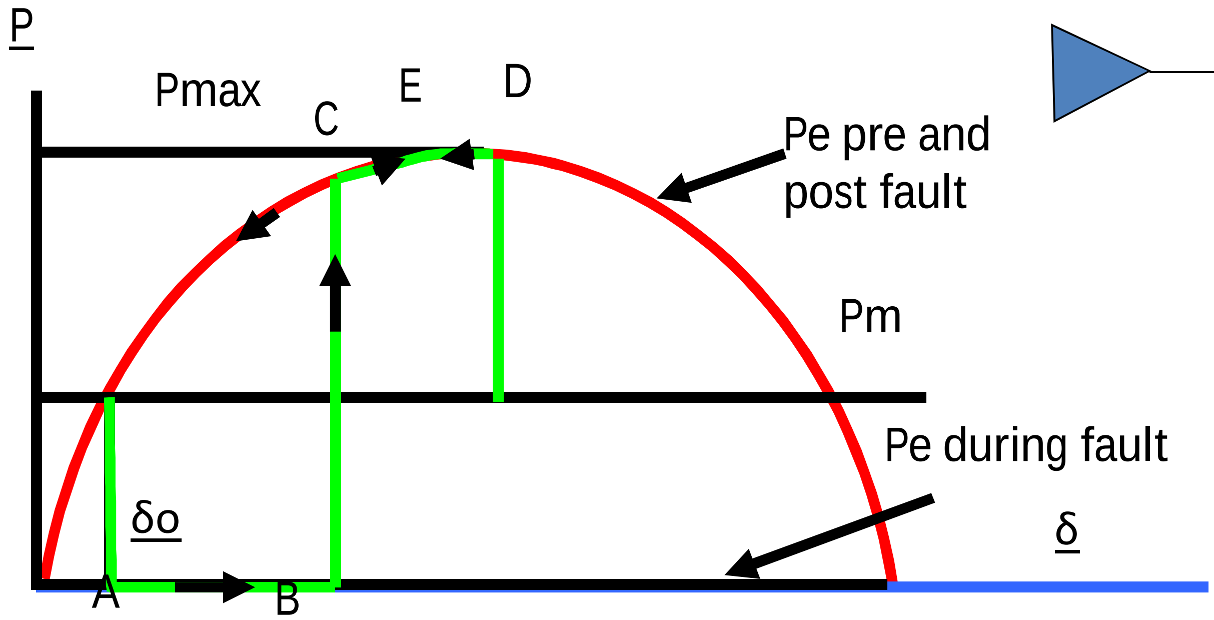
Point C represents the maximum power that can be generated and transmitted

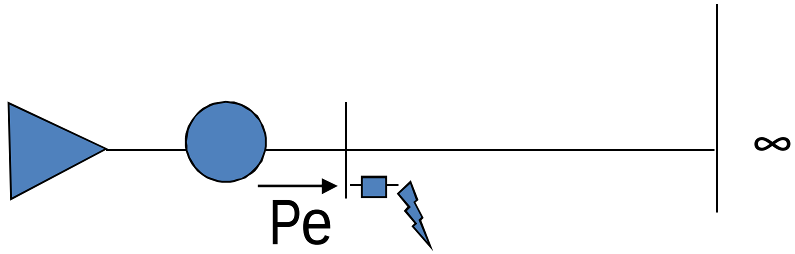
Pmax= E'V/(X+XL)

For loading > Pmax there are no equilibria

Pmax is the STEADY STATE STABILITY limit, i.e., the maximum operating power below which stability is guaranteed for sufficiently small changes

Stable and unstable Equilibria – Large Disturbances–
Fault at generator terminals – cleared quickly



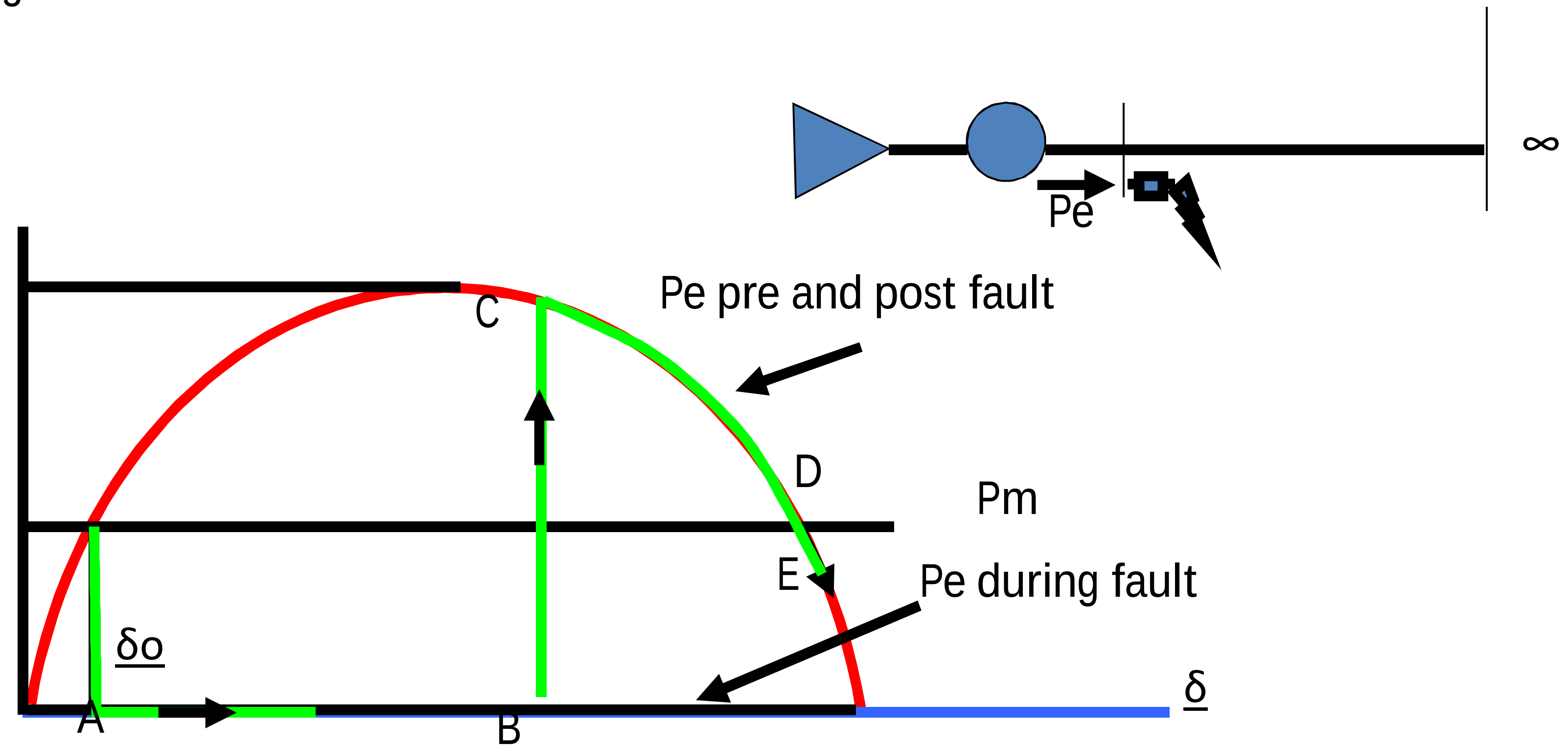


Stable Case

A, AB	Fault occurs Pm>Pe	ω ↑ δ↑	
B, CD	Fault Cleared Pm < Pe ω > ωsyn ↓		δ↑
D	Pm<Pe	ω =0 ↓ δ mom. constant	
DE	Pm<Pe	ω < ωsyn ↓ δ ↓ Swings back!	

First swing stability- Qualitative Behavior

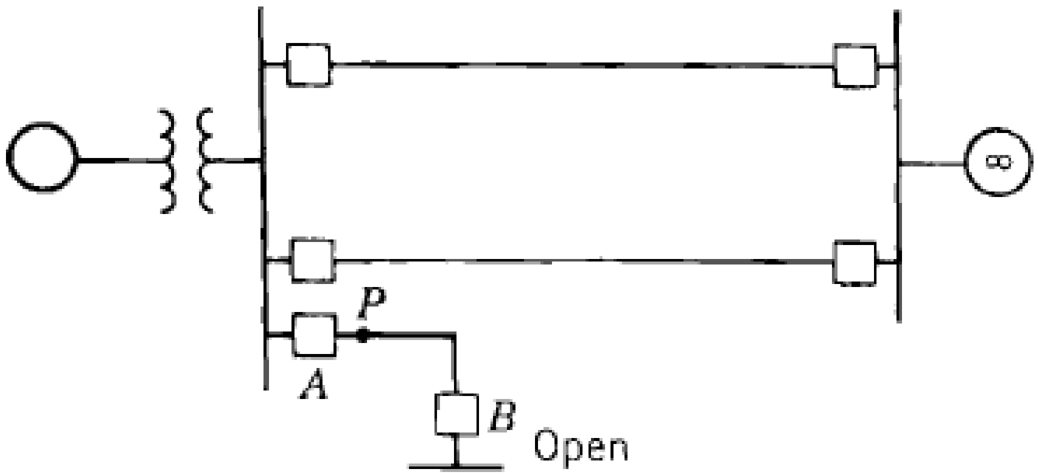
Stable and unstable Equilibria – Large Disturbances–
Fault at generator terminals – cleared much later



Unstable Case

A, AB Fault occurs Pm>Pe ω ↑ δ↑
B, CD Fault Cleared much later Pm > Pe ω > ωsyn ↓ δ↑
DE Pm>Pe ω > ωsyn ↑ δ↑

Equal Area Criterion of Stability



One line diagram of a generator connected to infinite power system

For analysis of the system

After the three phase fault Pe will be zero leaving the swing equation as follows

$$\frac{d^2\delta}{dt^2} = \frac{\omega_s}{2H} P_m$$

To obtain speed

$$\frac{d\delta}{dt} = \int_0^t \frac{\omega_s}{2H} P_m \, dt = \frac{\omega_s}{2H} P_m t$$

By further integration we obtain the rotor angle

$$\delta = \frac{\omega_s P_m}{4H} t^2 + \delta_0$$

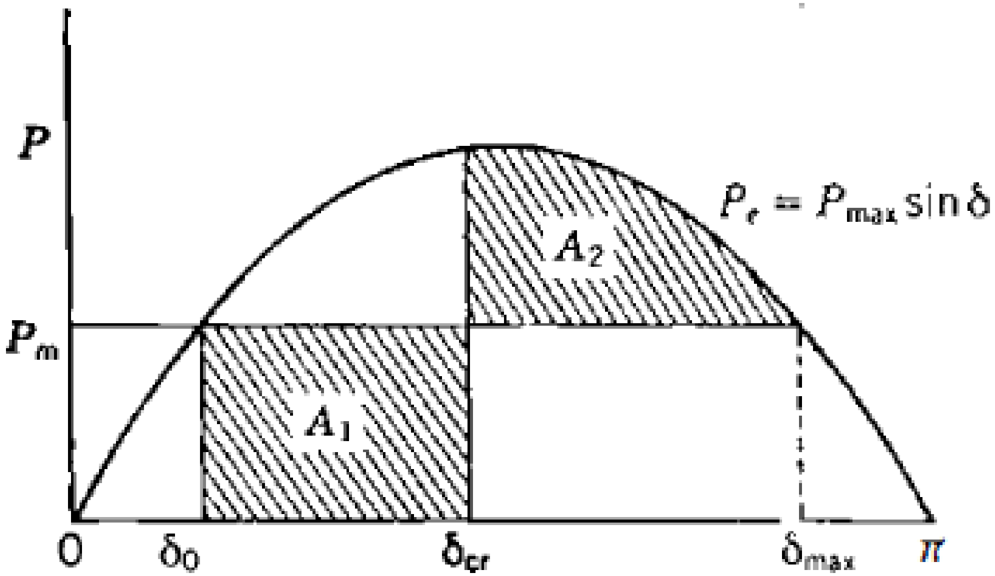
At instant of fault clearance

$$\left.\frac{d\delta}{dt}\right|_{t=t_c} = \frac{\omega_s P_m}{2H} t_c$$

$$\delta(t)\Big|_{t=t_c} = \frac{\omega_s P_m}{4H} t_c^2 + \delta_0$$

For the system to restore its stability, the area A1 should be equal to area A2

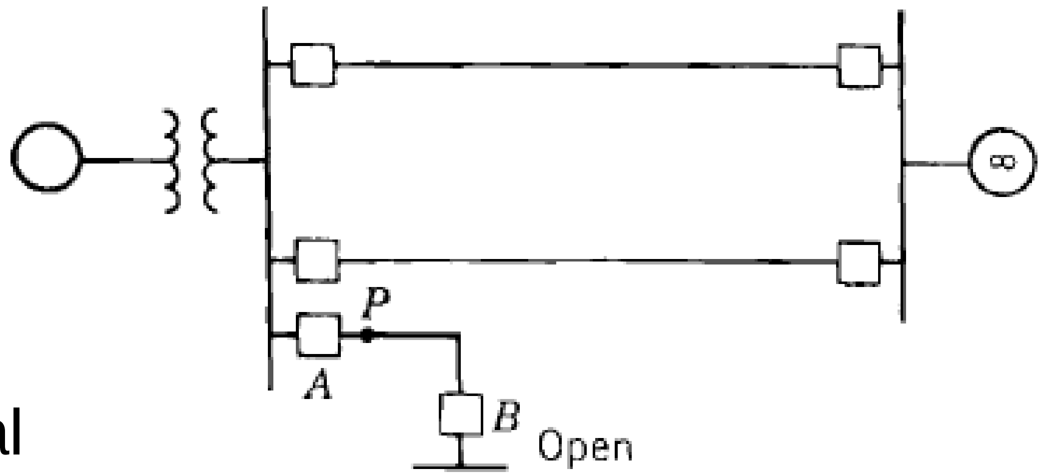
A1 represents the increase in kinetic energy of the rotor while it is accelerating
A2 represents the decrease of kinetic energy of the rotor while it is decelerating



For critical clearing angle

$$\delta_{cr} = \frac{\omega_s P_m}{4H} t_{cr}^2 + \delta_0$$

Example 16.7. Calculate the critical clearing angle and the critical clearing time for the system of Fig. 16.8 when the system is subjected to a three-phase fault at point P on the short transmission line. The initial conditions are the same as those in Example 16.3, and $H = 5$ MJ/MVA.



The given initial conditions are

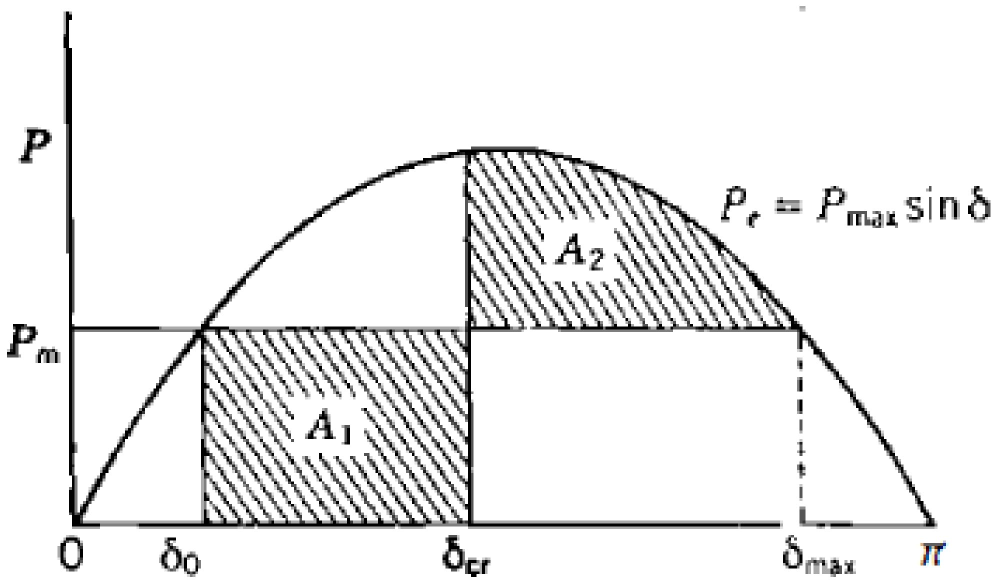
$$P_e = P_{max} \sin \delta = 2.10 \sin \delta$$

$$\delta_0 = 28.44° = 0.496 \text{ elec rad}$$

Mechanical input power P_m is 1.0 per unit

Considering the equal area criteria
where

A1=A2



$$A_1 = \int_{\delta_0}^{\delta_{cr}} P_m \, d\delta = P_m(\delta_{cr} - \delta_0)$$

$$A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_{max} \sin \delta - P_m) \, d\delta$$

$$= P_{max}(\cos \delta_{cr} - \cos \delta_{max}) - P_m(\delta_{max} - \delta_{cr})$$

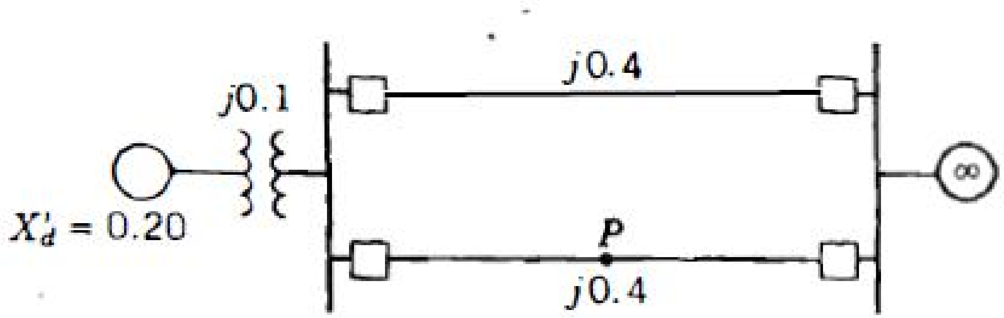
$$\delta_{cr} = \cos^{-1}[(\pi - 2 \times 0.496)\sin 28.44° - \cos 28.44°]$$

$$= 81.697° = 1.426 \text{ elec rad}$$

$$t_{cr} = \sqrt{\frac{4 \times 5(1.426 - 0.496)}{377 \times 1}} = 0.222 \text{ s}$$

Example:

Determine the critical clearing angle for the system shown in the following figure



Given the power angle equation during normal operation is

P=2.1 sin∂

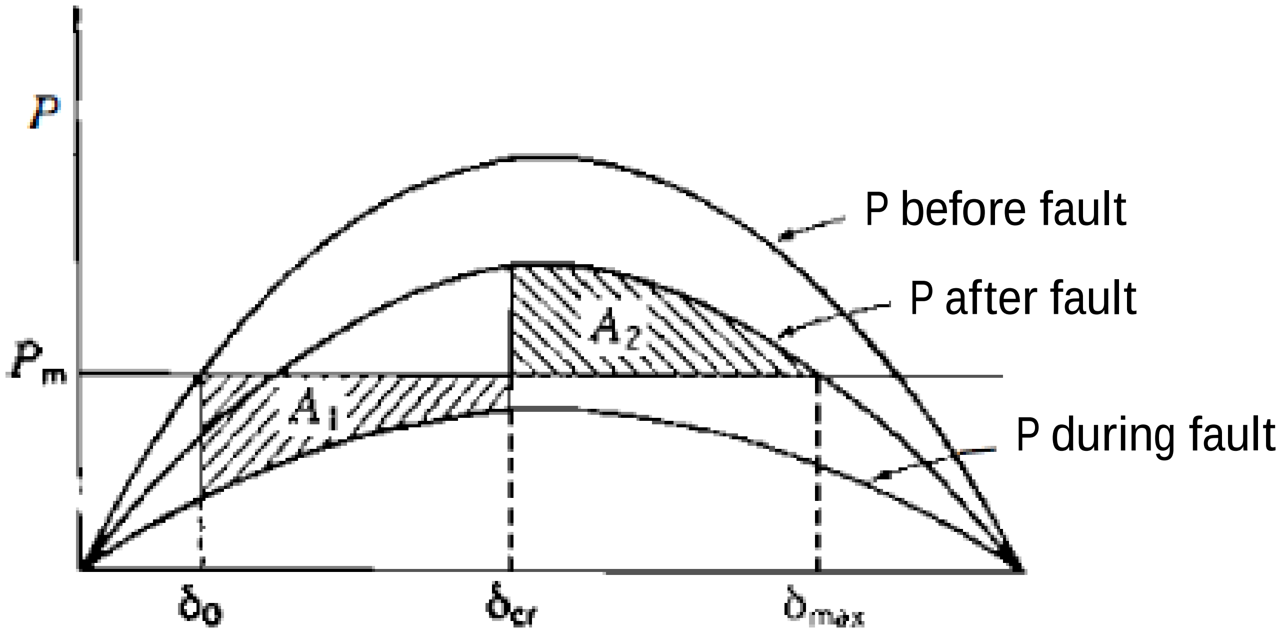
The power angle equation during three phase fault at middle of the lower transmission line is

$P = 0.808 \sin \partial$

The power angle equation after removal of the transmission line is

$P = 1.5 \sin \partial$

And the mechanical power is 1 p.u.


P
P_m
A_1
A_2
P before fault
P after fault
P during fault
δ_0
δ_{cr}
δ_{max}

$$\delta_0 = 28.44° = 0.496 \text{ rad}$$

$$\delta_{max} = 180° - \sin^{-1}\left[\frac{1.000}{1.500}\right] = 138.190° = 2.412 \text{ rad}$$

$$\cos \delta_{cr} = \frac{\left(\frac{1.0}{2.10}\right)(2.412 - 0.496) + 0.714 \cos(138.19°) - 0.385 \cos(28.44°)}{0.714 - 0.385}$$

$$= 0.127$$

Hence,

$$\delta_{cr} = 82.726°$$