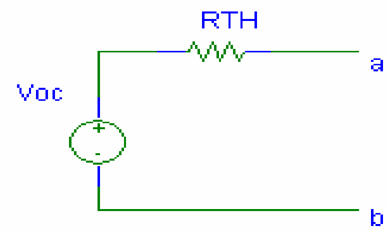
Electrical Circuits I
Lecture 9

<Dr Ahmed El-Shenawy>

Method of finding Thevenin equivalent Circuit

Number of method	**If the circuit contains:**	**Thevenin equivalent circuit** 
1	Resistors and independent sources only (case 1)	a) Connect an open circuit between terminals a and b. Find $v_{oc} = v_{ab}$ the voltage across the open circuit b) Deactivate the independent sources (replace independent voltage sources with short circuits and independent current sources with open circuits). Find R_{TH} by methods introduced in Chapter 2.
2	Resistors and independent and dependent sources (case 2) or case 1	a) Connect an open circuit between terminals a and b. Find $v_{oc} = v_{ab}$ the voltage across the open terminals. b) Connect a short circuit between terminals a and b. Find i_{sc}, the current directed from a to b in the short circuit. c) Calculate $R_{TH} = v_{oc}/i_{sc}$.
3	Resistors and dependent sources only.	a) Note that $v_{oc} = 0$ b) Connect a 1-A current source from terminal b to terminal a. Determine v_{ab} c) Then $R_{TH} = v_{ab}/1$.

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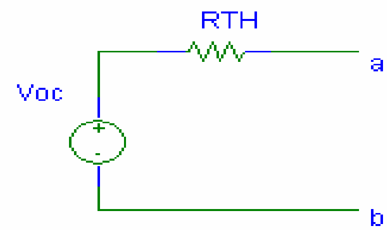
Number of method	**If the circuit contains:**	**Thevenin equivalent circuit** 
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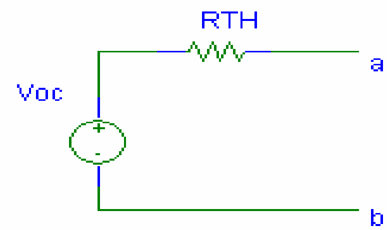
Number of method	**If the circuit contains:**	**Thevenin equivalent circuit** 
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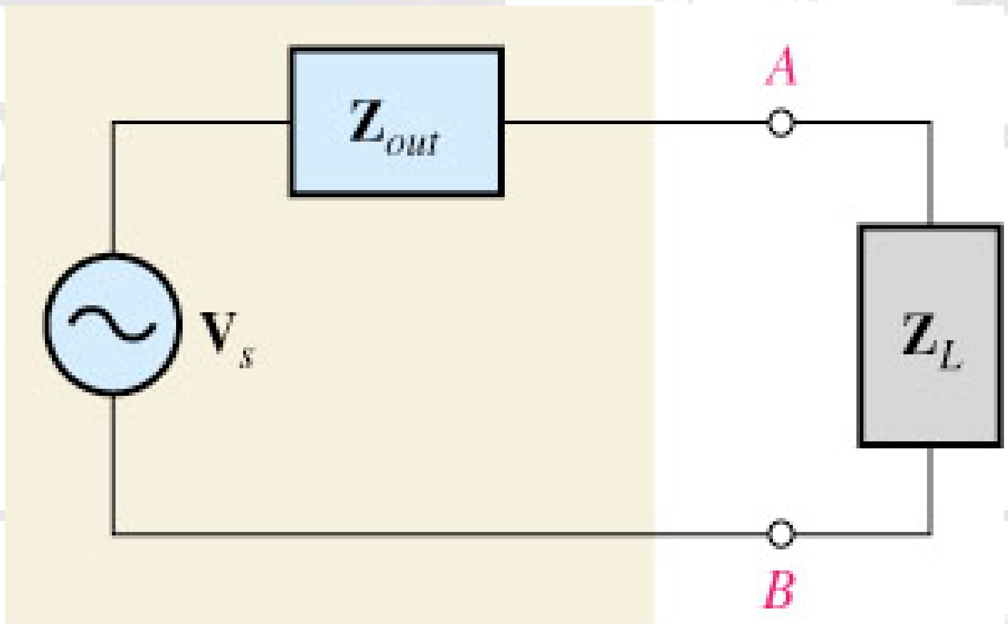
Maximum Power Transfer

It is desired to find the load R_L to be connected to the terminal between A and B which results in maximum power transferred.



Maximum Power Transfer

- To find the value of RL, the network is replaced by its Thevenin equivalent.
- Then :

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

- The RL that maximizes the above equation can be found by taking the first derivative of p with respect to RL and equate it to zero.

Maximum Power Transfer

$$\frac{dp}{dR_L} = V_{Th}^2 \frac{(R_{Th}+R_L)^2 - R_L * 2(R_{Th}+R_L)}{(R_{Th}+R_L)^4}$$

By equating the above equation to zero:

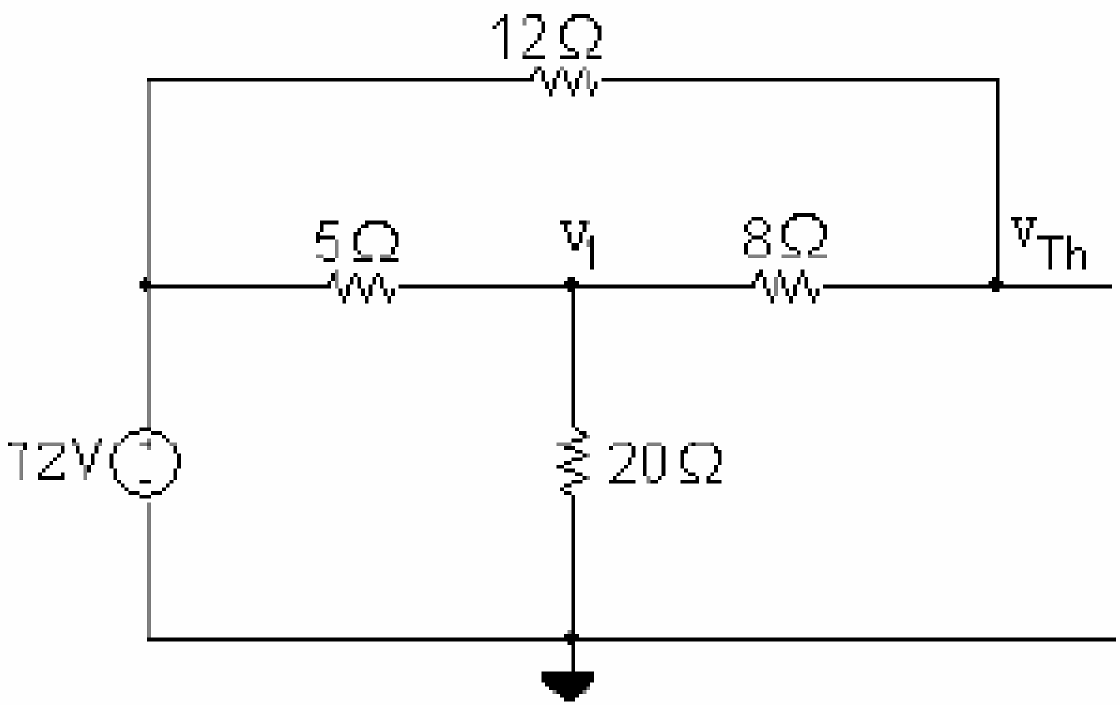
$$(R_{Th} + R_L)^2 = 2R_L(R_{Th} + R_L)$$

$$\Rightarrow R_L = R_{Th}$$

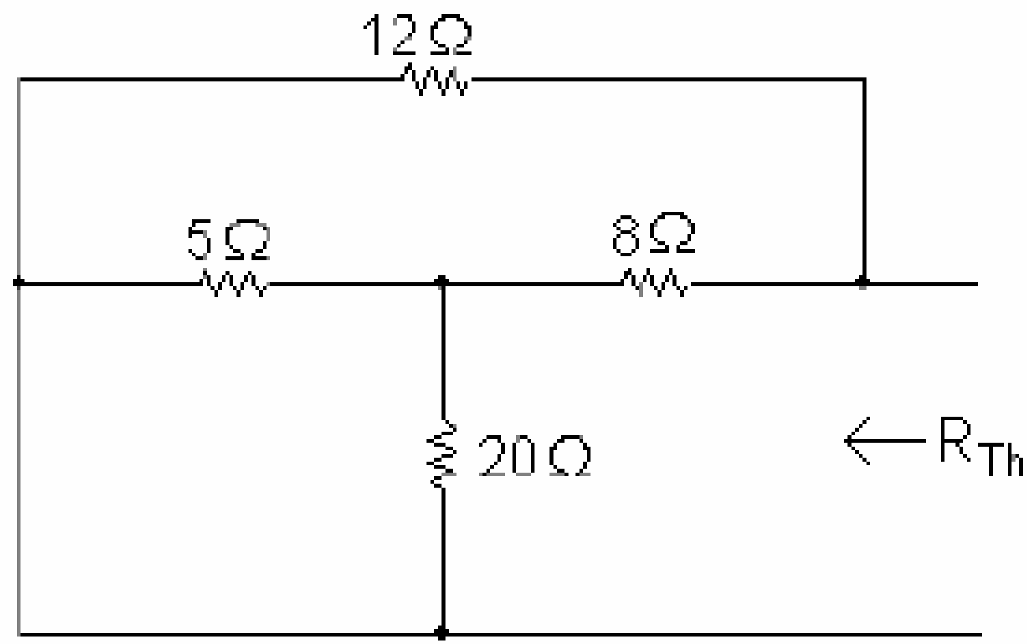
Then the maximum power transferred:

$$P_{\max} = \frac{V_{Th}^2 R_L}{(2R_L)^2} = \frac{V_{Th}^2}{4R_L}$$

Example

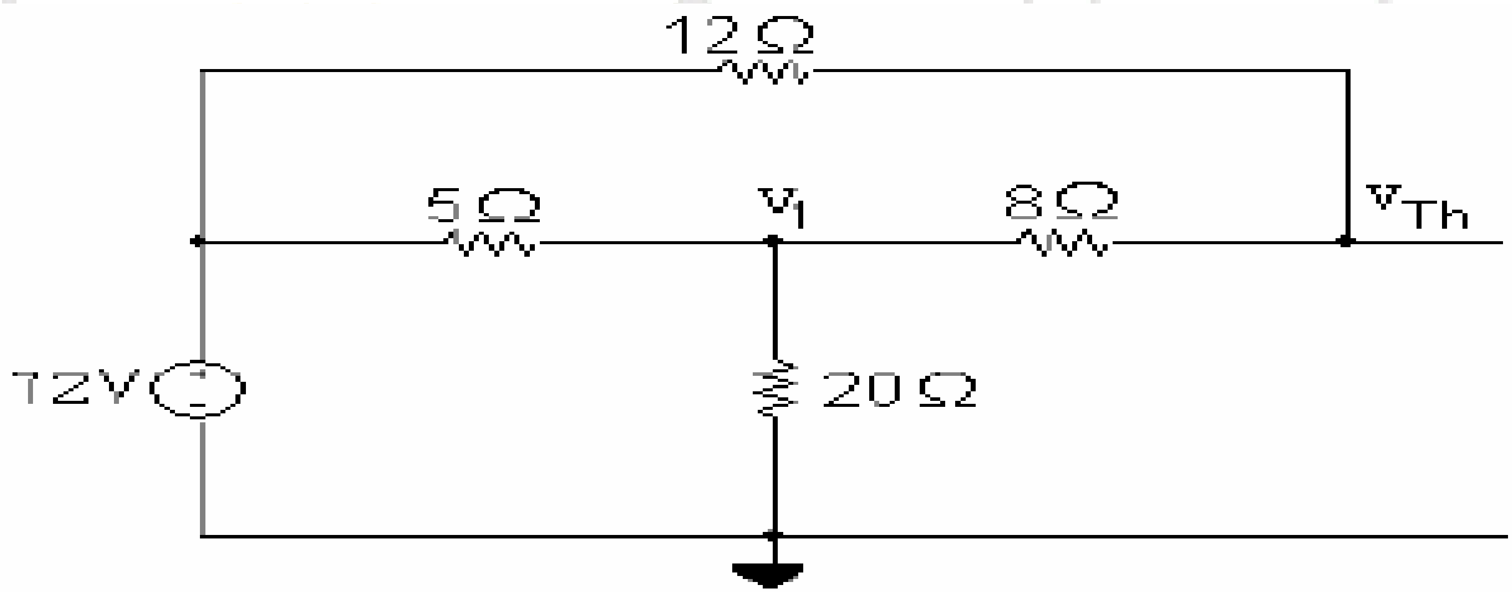


To find R_{Th}, replace the 72 V source with a short circuit:



Note that the $5\,\Omega$ and $20\,\Omega$ resistors are in parallel, with an equivalent resistance of $5||20 = 4\,\Omega$. The equivalent $4\,\Omega$ resistance is in series with the $8\,\Omega$ resistor for an equivalent resistance of $4 + 8 = 12\,\Omega$. Finally, the $12\,\Omega$ equivalent resistance is in parallel with the $12\,\Omega$ resistor, so $R_{Th} = 12||12 = 6\,\Omega$.

Use node voltage analysis to find v_{Th}. Begin by redrawing the circuit and labeling the node voltages:



The node voltage equations are

$$\frac{v_1 - 72}{5} + \frac{v_1}{20} + \frac{v_1 - v_{\text{Th}}}{8} = 0$$

$$\frac{v_{\text{Th}} - v_1}{8} + \frac{v_{\text{Th}} - 72}{12} = 0$$

Place these equations in standard form:

$$v_1\left(\frac{1}{5}+\frac{1}{20}+\frac{1}{8}\right) + v_{\mathrm{Th}}\left(-\frac{1}{8}\right) = \frac{72}{5}$$

$$v_1\left(-\frac{1}{8}\right) + v_{\mathrm{Th}}\left(\frac{1}{8}+\frac{1}{12}\right) = 6$$

Solving, $v_1 = 60$ V and $v_{\mathrm{Th}} = 64.8$ V. Therefore, the Thévenin equivalent circuit is a 64.8 V source in series with a 6 Ω resistor.

- RL must equal to RTh = 6 ohms
- The maximum power transferred=