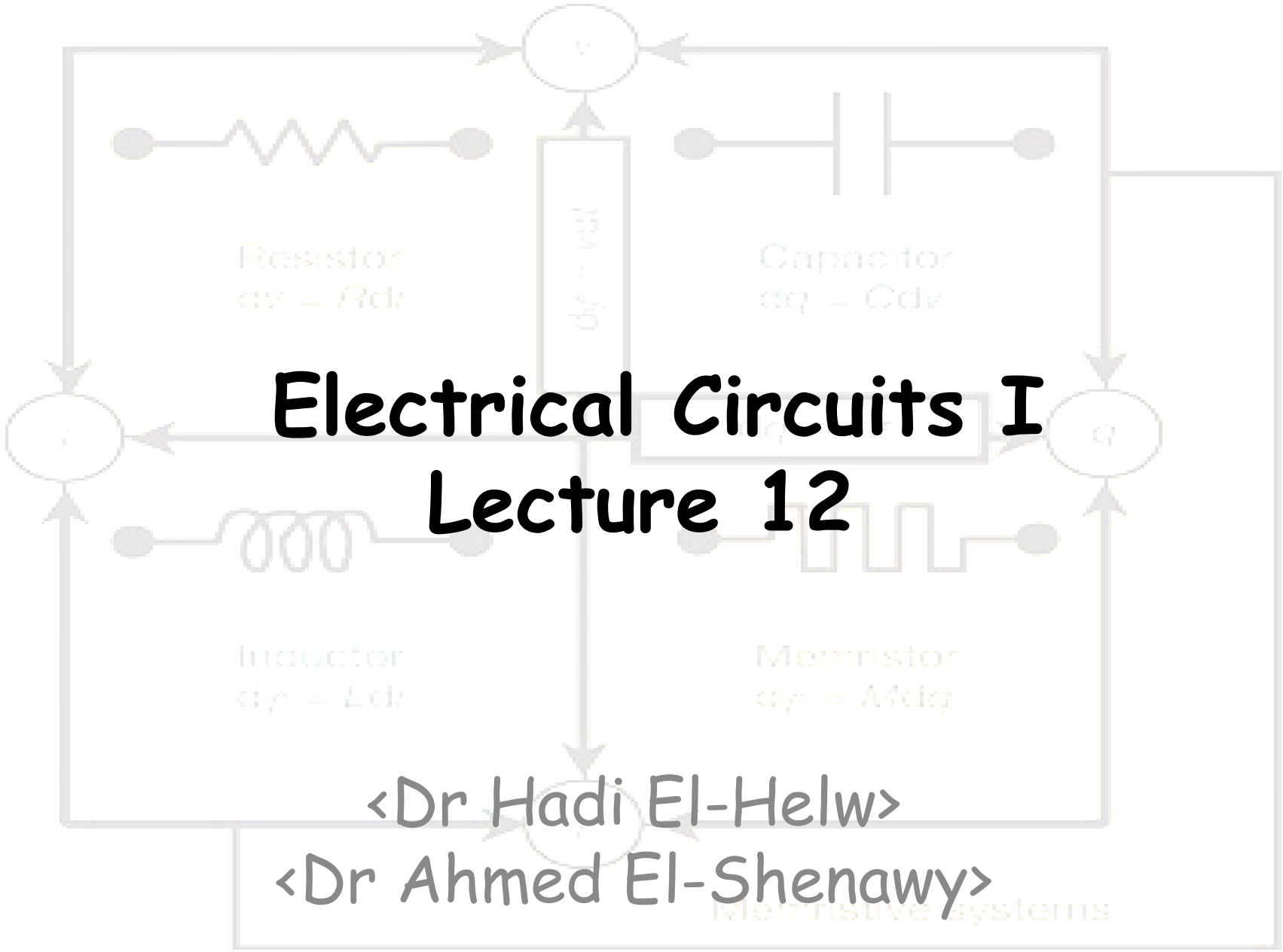

Electrical Circuits I
Lecture 12

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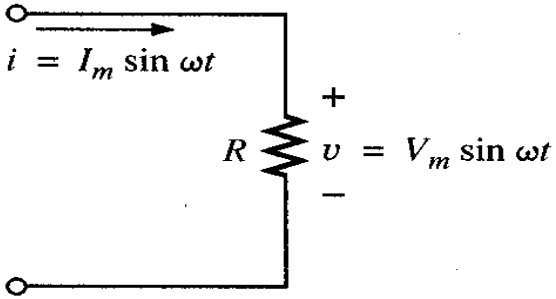
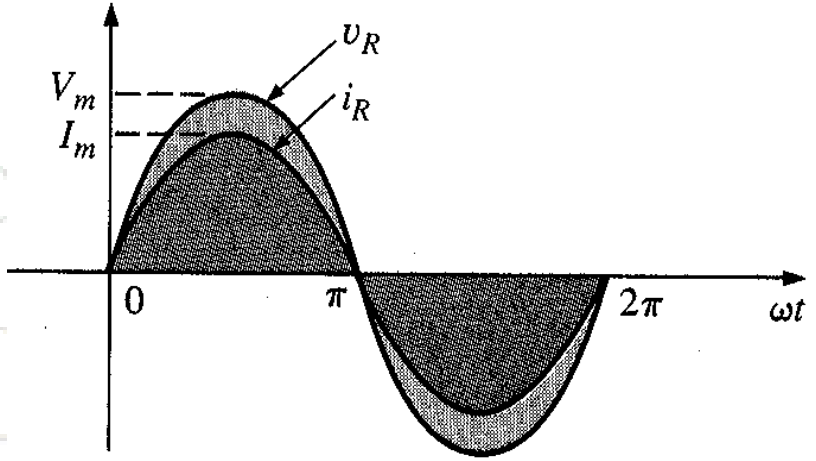
The response of the basic elements R,L, and C to a sinusoidal voltage or current

Resistance

- The resistance is unaffected by the frequency of the applied sinusoidal voltage or current
- For purely resistive circuit V and I were in phase

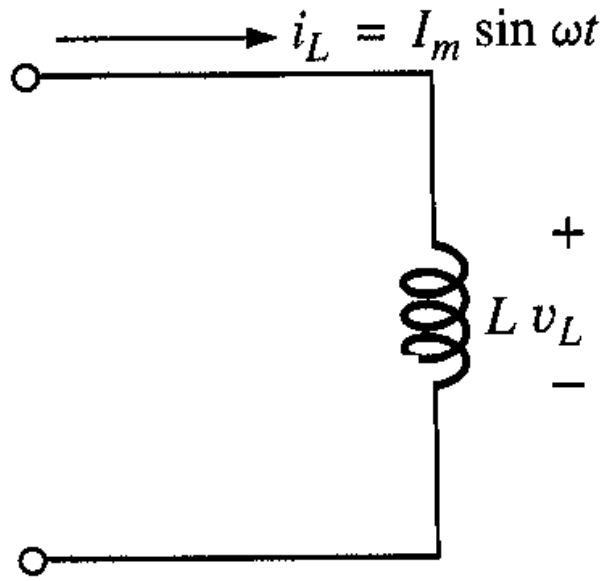
$$I_m = \frac{V_m}{R} \quad \text{or} \quad V_m = I_m R$$





Inductance

➢For the inductor the voltage across an inductor is directly related to the rate of change of current through the coil. Consequently, the higher the frequency, the greater will be the rate of change of current through the coil, and the greater the magnitude of the voltage



$$v_L = L\frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{d}{dt}(I_m \sin \varpi t) = \varpi I_m \cos \varpi t$$

$$v_L = L\frac{di_L}{dt} = L(\varpi I_m \cos \varpi t) = \varpi L I_m \cos \varpi t$$

or

$$v_L = V_m \sin(\varpi t + 90) \quad \text{Where} \quad V_m = \varpi L I_m$$

- Therefore, for the inductor, v_L leads i_L by 90°, or i_L lags v_L by 90°

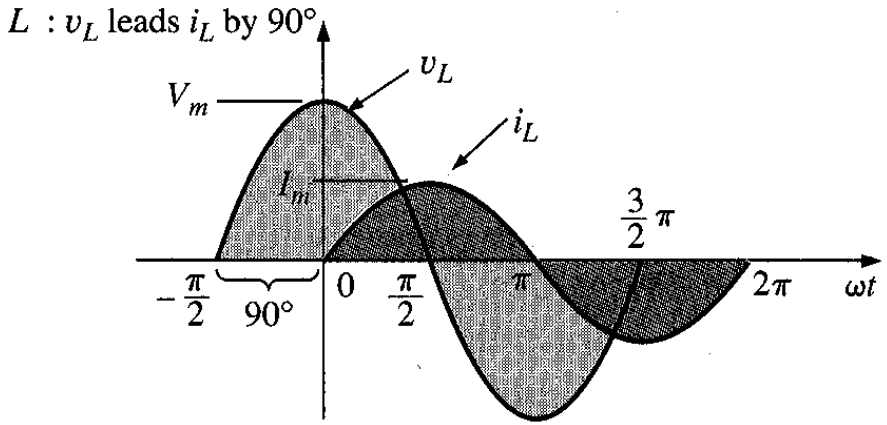
- The quantity ωL, called the reactance of an inductor, is symbolically represented by X_L and is measured in ohms

$$X_L = \omega L = 2\pi f L \quad \text{(ohms)}$$

- In an ohm's law format, its magnitude can be determined from:

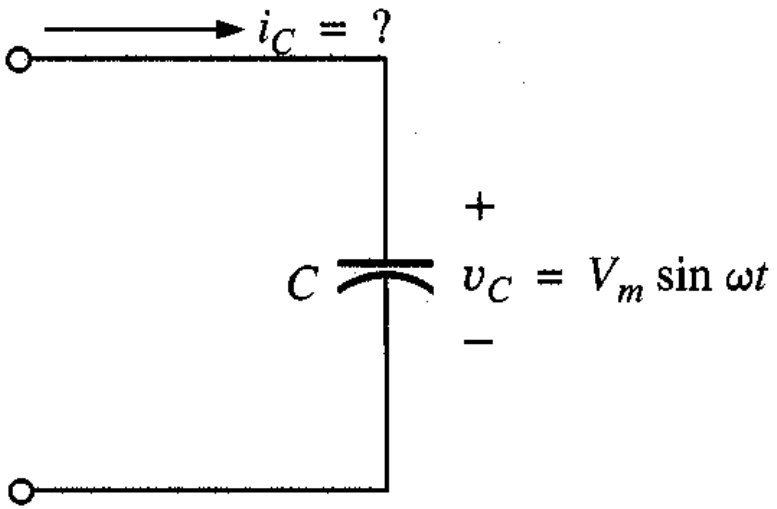
$$X_L = \frac{V_m}{I_m}$$

- Inductive reactance is opposition to the flow of current, which results in the continual interchange of energy between the source and the magnetic field of the inductor



Capacitance

The voltage across the capacitor is limited by the rate at which charge can be deposited on, or released by, the plates of the capacitor during the charging and discharging phases, respectively. In other words, an instantaneous change in voltage across a capacitor is opposed by the fact that there is an element of time required to deposit charge on the plates of a capacitor



➢The fundamental equation relating the voltage across a capacitor to the current of a capacitor is:

$$i_c = c\frac{dv_c}{dt}$$

applying differentiation

$$\frac{dv_c}{dt} = \frac{d}{dt}(v_m \sin \varpi t) = \varpi V_m \cos \varpi t$$

Therefore

$$i_c = c\frac{dv_c}{dt} = c(\varpi V_m \cos \varpi t) = \varpi c V_m \cos \varpi t$$

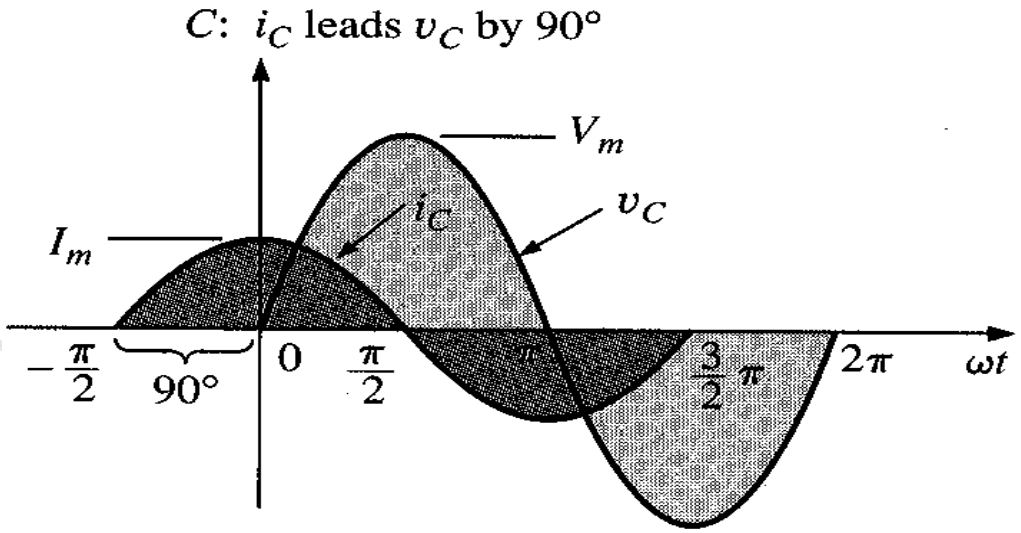
or

$$i_c = I_m \sin(\varpi t + 90) \quad \text{where} \quad I_m = \varpi C V_m$$

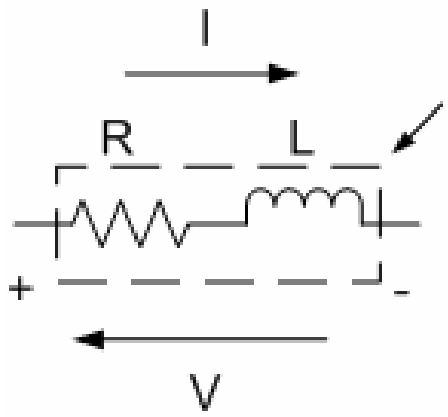
- For a capacitor i_c leads v_c by 90° or v_c lags i_c by 90°
- The quantity $1/\omega c$ called the reactance of a capacitor, is symbolically by X_c and is measured in ohm

$$X_c = \frac{1}{\varpi c}$$

- Capacitor reactance is the opposition to the flow of charge, which results in the continual interchange of energy between the source and the electric field of a capacitor.



Series RL circuit



Impedance

$$Z = R + JX_L$$

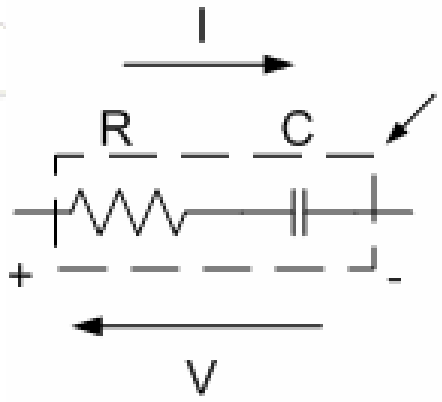
$$V = V_m \angle \phi_v \qquad I = I_m \angle \phi_i$$

Where

$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}}$$

$$\phi_i = \phi_v - \tan^{-1}\frac{\omega L}{R}$$

Series RC circuit



Impedance

$$Z = R - JX_c$$

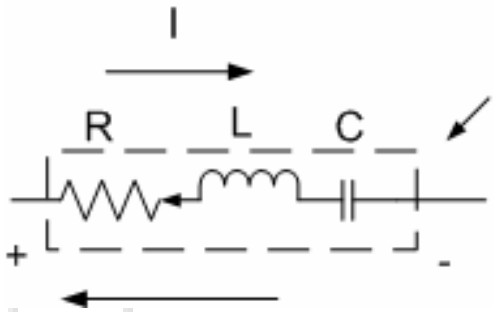
$$V = V_m \angle \phi_v \qquad I = I_m \angle \phi_i$$

Where

$$I_m = \frac{V_m}{\sqrt{R^2 + (\frac{1}{\omega C})^2}}$$

$$\phi_i = \phi_v - \tan^{-1}\frac{1}{R\omega C}$$

Series RLC circuit



Impedance

$$Z = R + J(X_L - X_c)$$

$$V = V_m \angle \phi_v \qquad I = I_m \angle \phi_i$$

Where

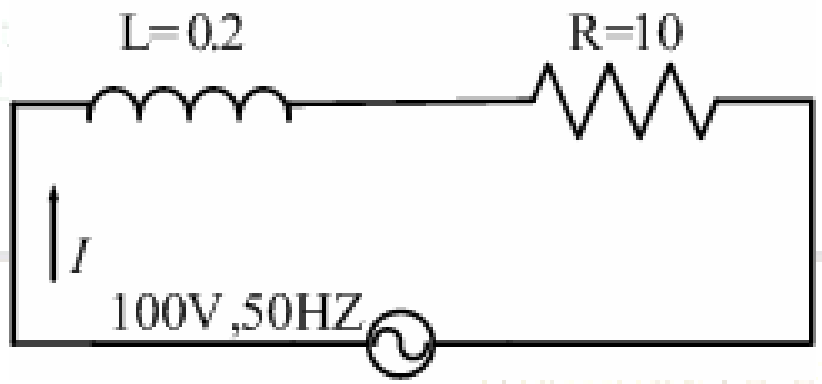
$$I_m = \frac{V_m}{\sqrt{R^2 + (\varpi L - \frac{1}{\varpi C})^2}}$$

$$\phi_i = \phi_v - \tan^{-1} \frac{\varpi L - 1/\varpi c}{R}$$

Example 1

A coil is connected to supply voltage as shown in the figure below calculate the following

1. the inductive reactance(X_L)
2. the total impedance (Z)
3. the current(I)
4. the phase angle Θ between the current and the supply voltage



Example 2

Two coils are connected in series across a voltage supply (V) as shown in the figure below. Calculate the value of the voltage V_1,V_2 across each coil. each coil has its own resistance