Automatic Control
EE 418
Lecture 3

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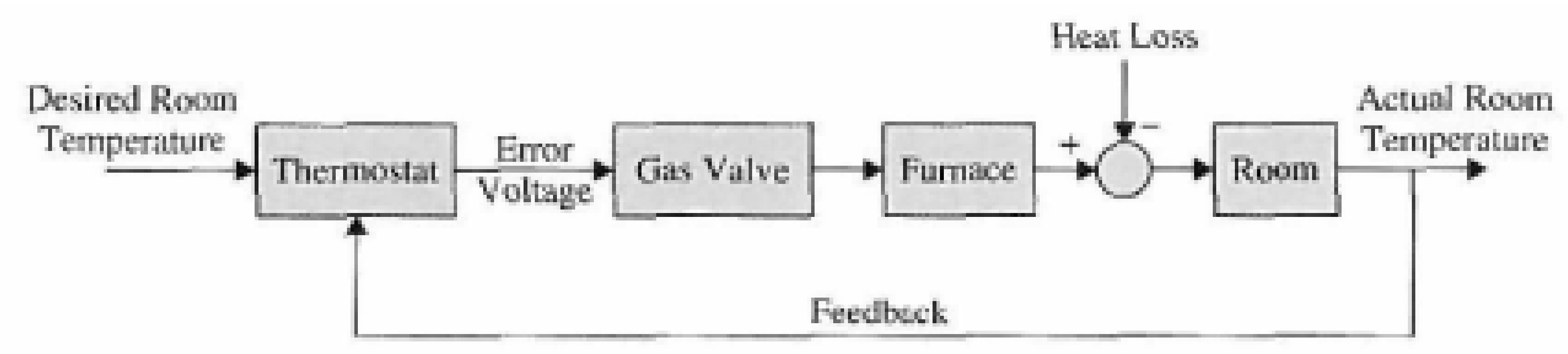
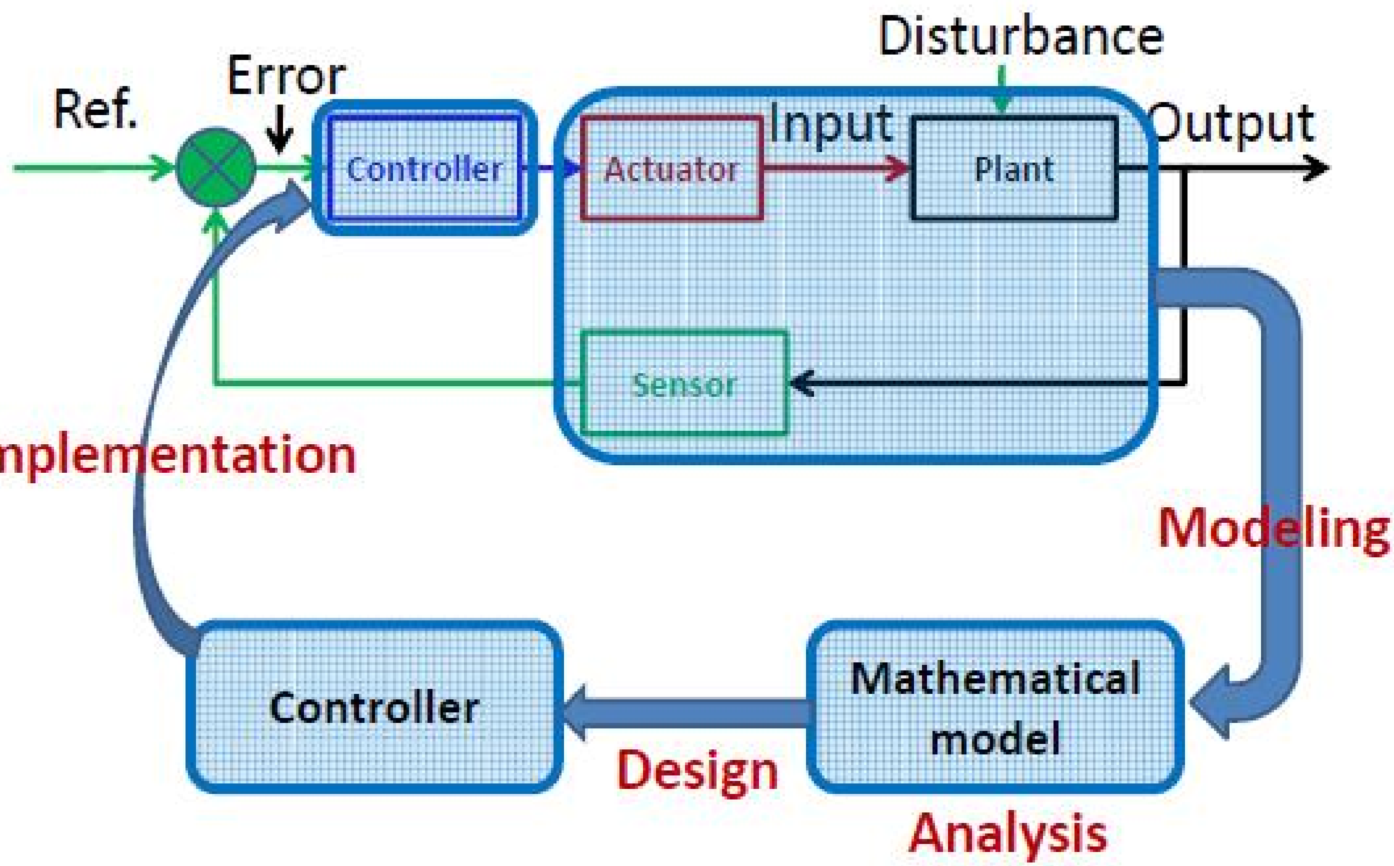
BLOCK DIAGRAMS

The **block diagram** modeling may provide control engineers with a better understanding of the composition and interconnection of the components of a system. Or it can be used, together with transfer functions, to describe the cause-and-effect relationships throughout the system. For example, consider a simplified block diagram representation of the heating system in your lecture room, shown in Fig



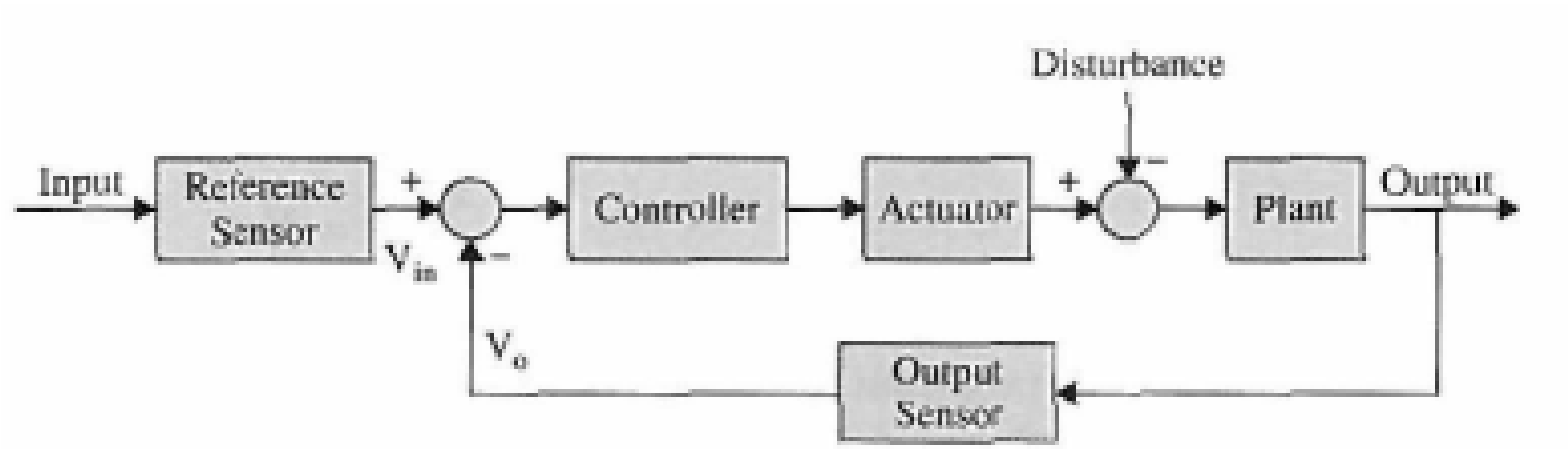
Lecture Objectives

1. To study block diagrams, their components, and their underlying mathematics.
2. To obtain transfer function of systems through block diagram manipulation and reduction.

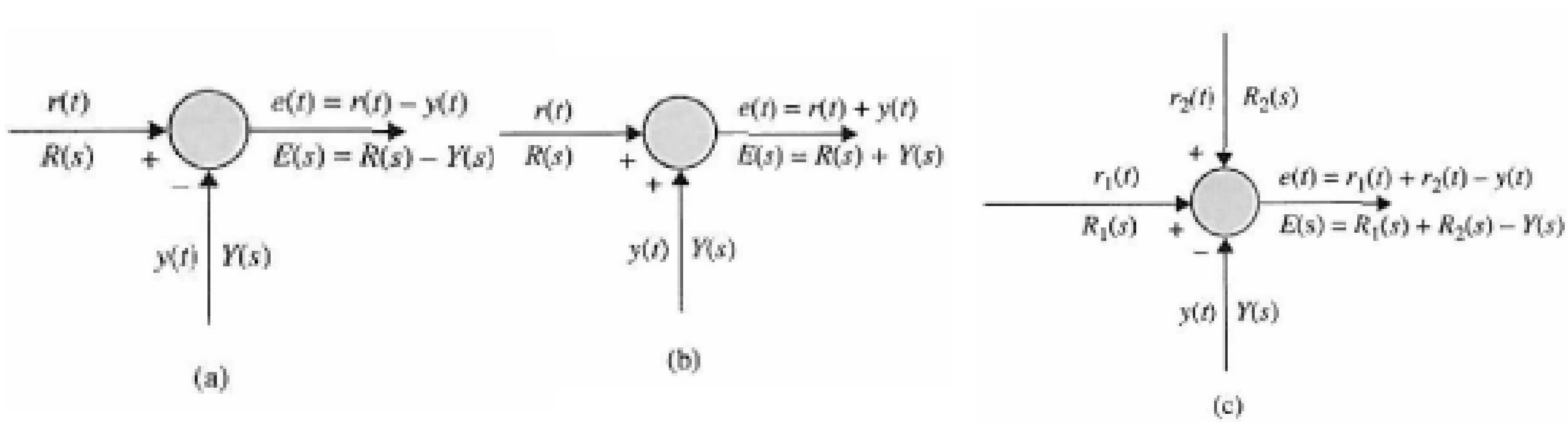


Typical Elements of Block Diagrams in Control Systems

- Comparators
- Blocks representing individual component transfer functions, including:
  - Reference sensor (or input sensor)
  - Output sensor
  - Actuator
  - Controller
  - Plant (the component whose variables are to be controlled)
- Input or reference signals
- Output signals
- Disturbance signal
- Feedback loops

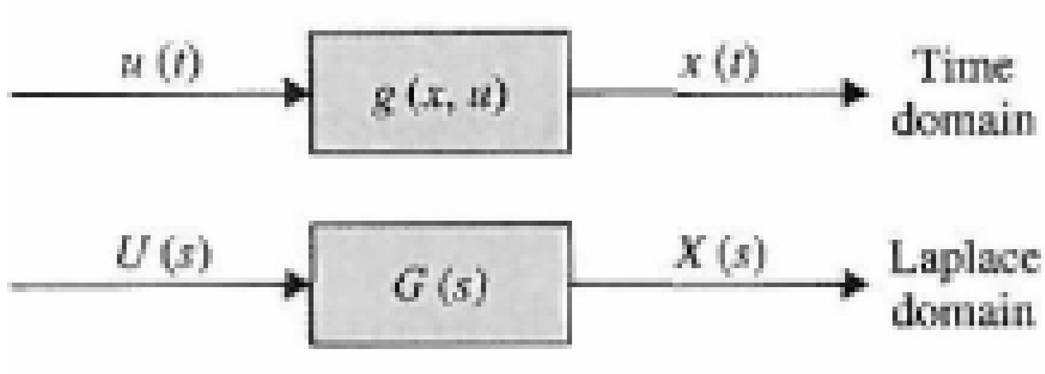


One of the important components of a control system is the sensing and the electronic device that acts as a junction point for signal comparisons otherwise known as a **comparator**



Note that the addition and subtraction operations are linear, so the input and output variables of these block diagram elements can be time-domain variables or Laplace-transform variables.

$$X(s) = G(s)\,U(s)$$



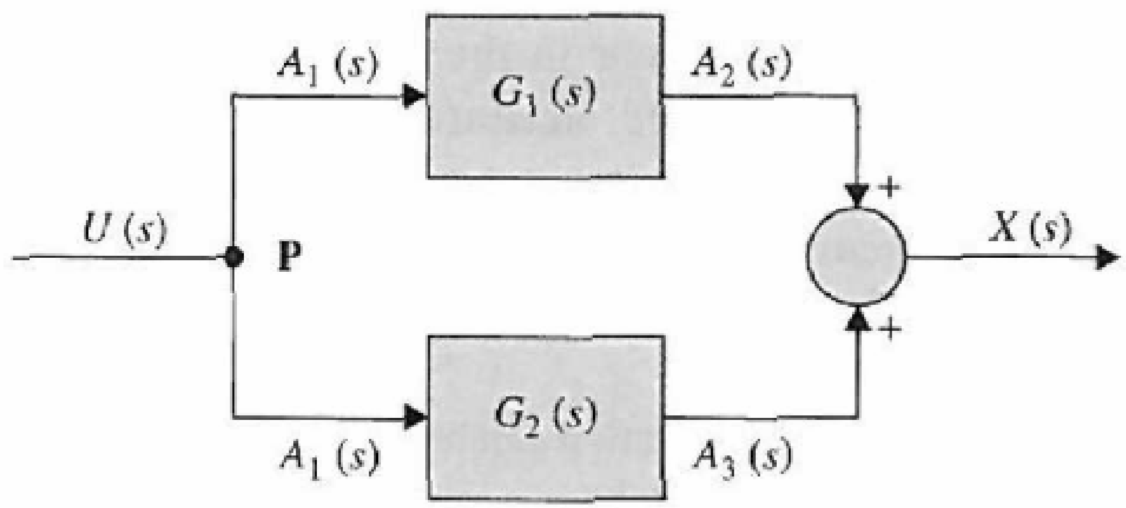
If signal *X(s)* is t h e output and signal *U(s)* denotes the input, the transfer function of the block in Fig. is

$$G(s) = \frac{X(s)}{U(s)}$$

Block Diagram Properties

Cascaded blocks

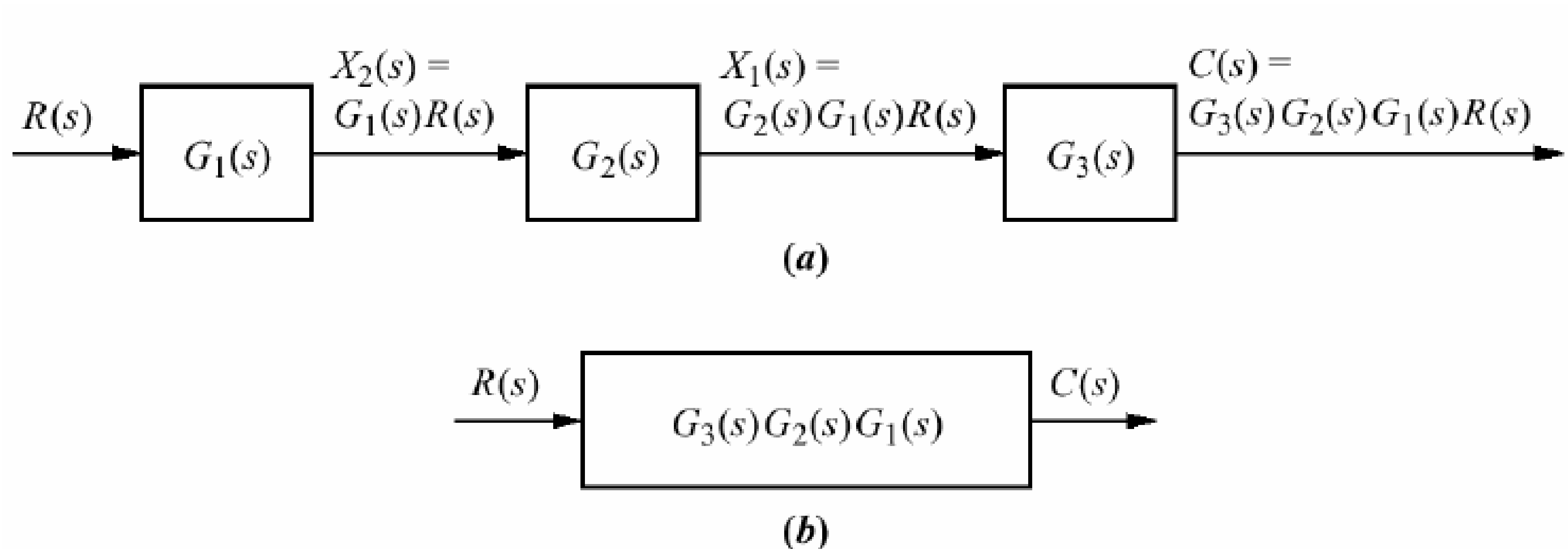
$$G(s) = G_1(s)G_2(s)$$



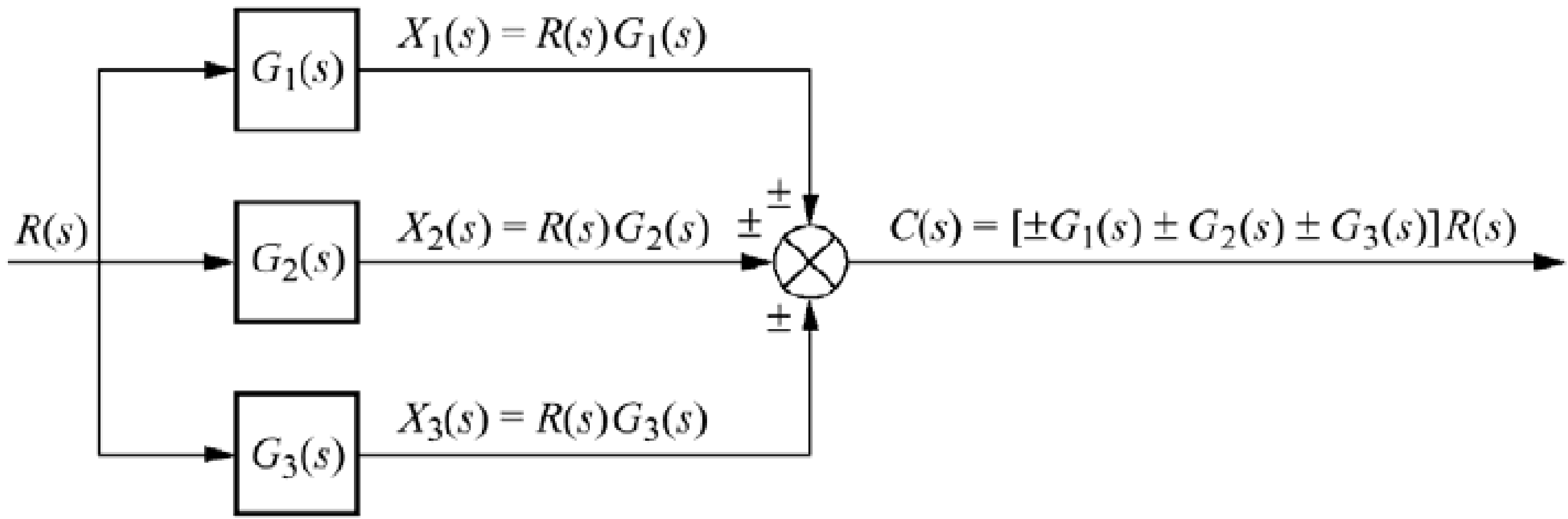
$$\ddot{x}(t) + 2\zeta\omega_n\dot{x}(t) + \omega_n^2 x(t) = \omega_n^2 u(t)$$

TO BLOCK DIAGRAM

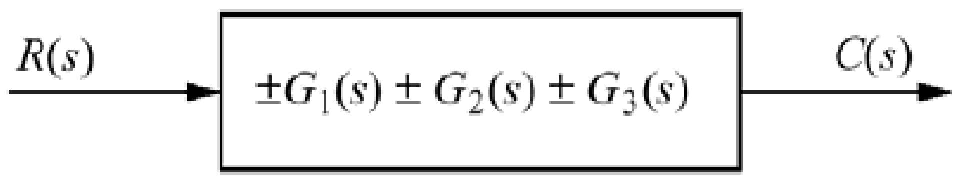
Block Diagram Properties



Block Diagram Properties

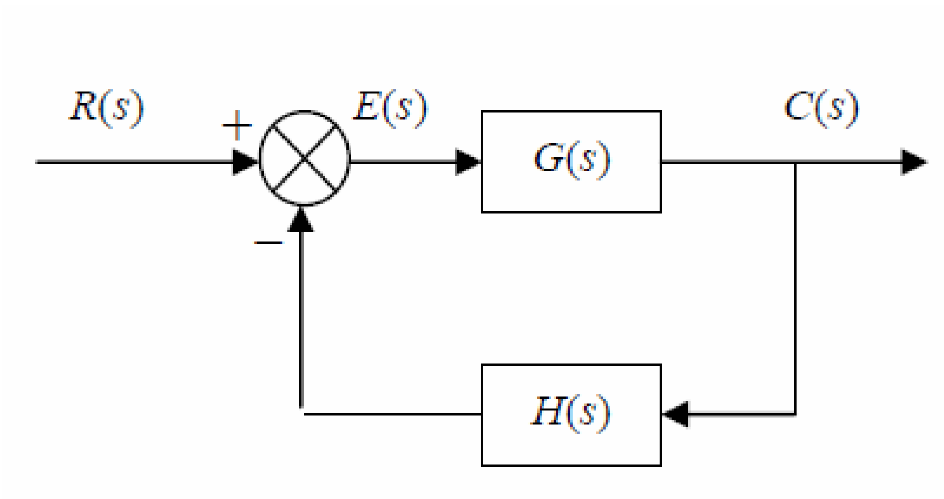


(*a*)



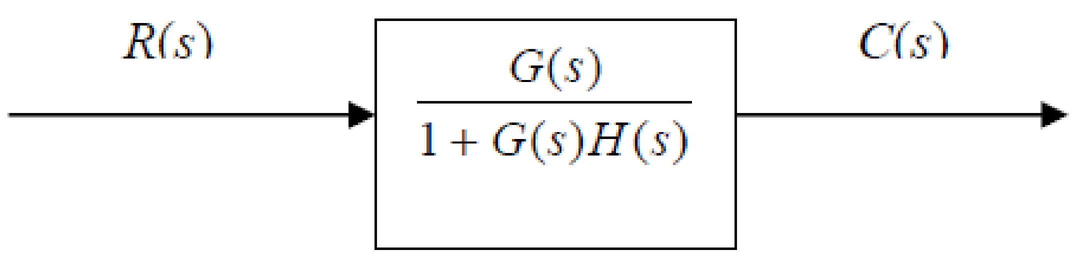
(*b*)

Block Diagram Properties



$$E(s) = R(s) - C(s)H(s)$$

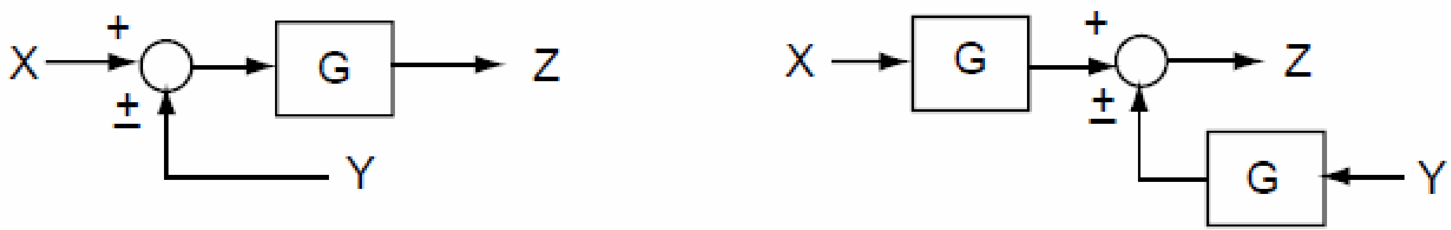
$$C(s) = R(s)G(s) - C(s)H(s)G(s)$$



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

Block Diagram Properties

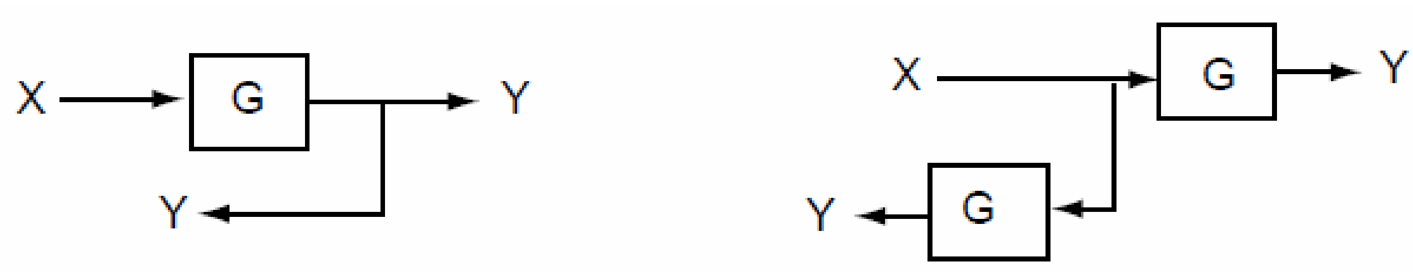
Moving a summer behind a block



Moving a summer ahead of a block

Block Diagram Properties

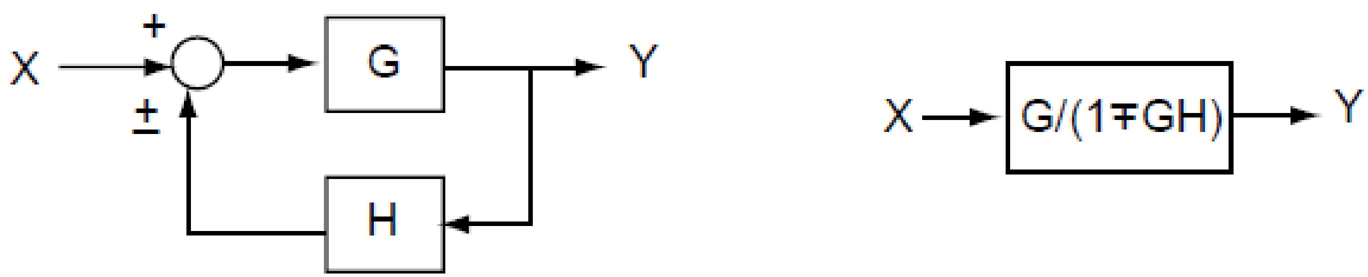
Moving a pickoff ahead of a block



Moving a pickoff behind a block

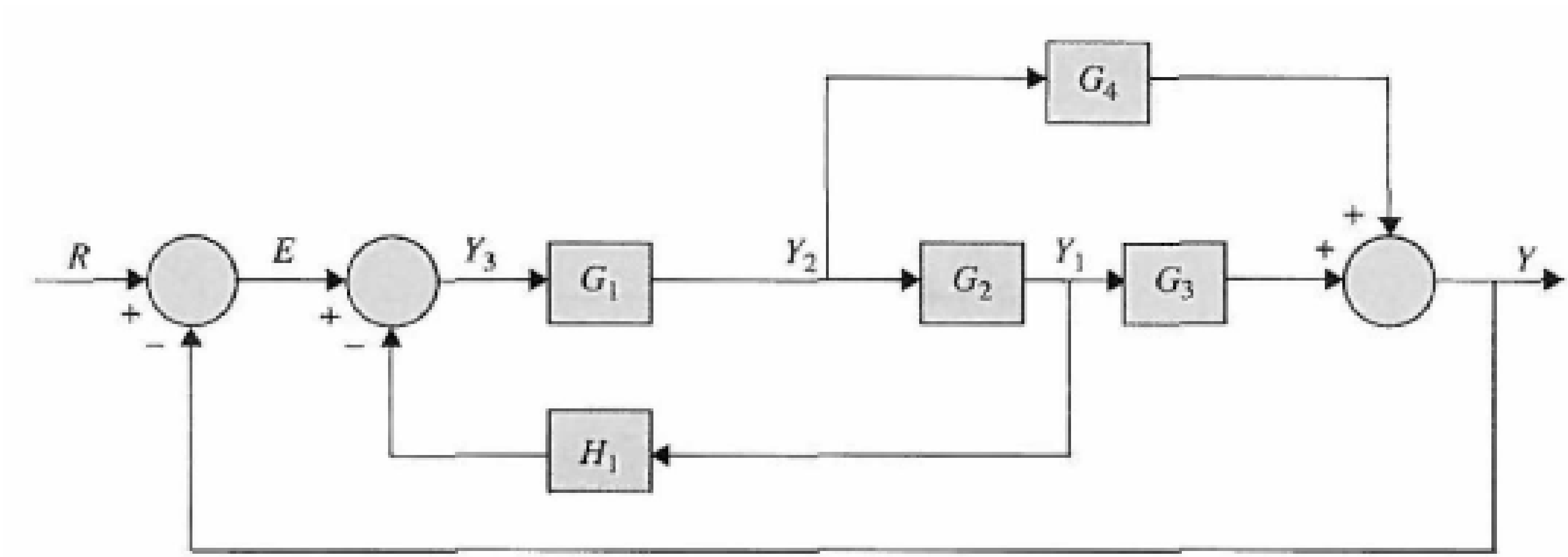
Block Diagram Properties

Eliminating a feedback loop

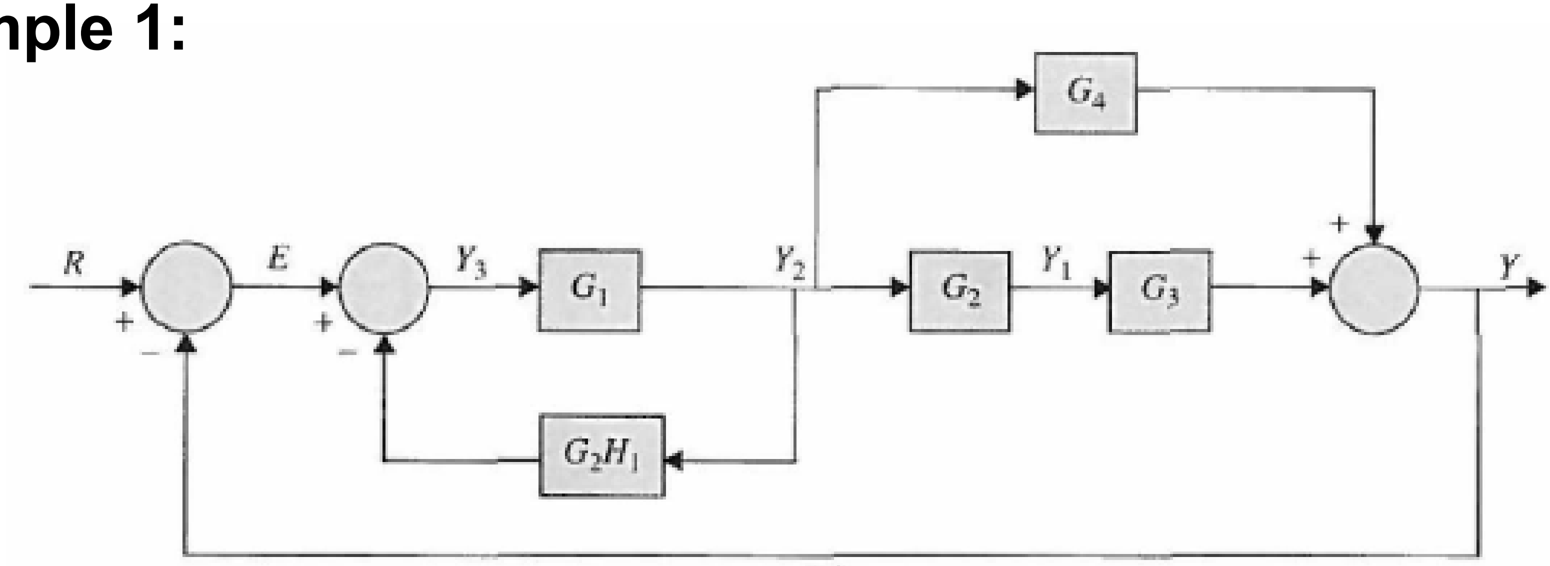


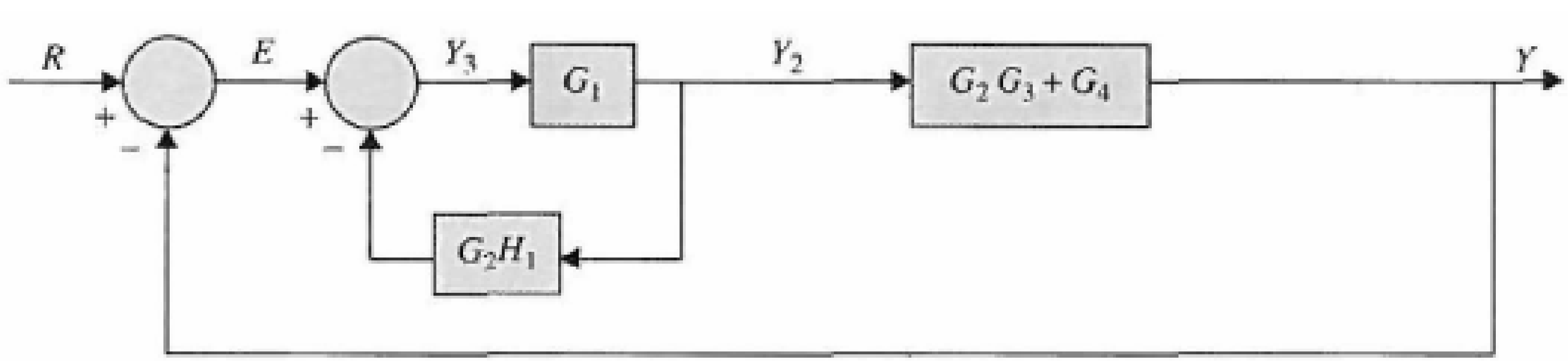
Example 1:

Find the input-output transfer function of the system shown in Figure below

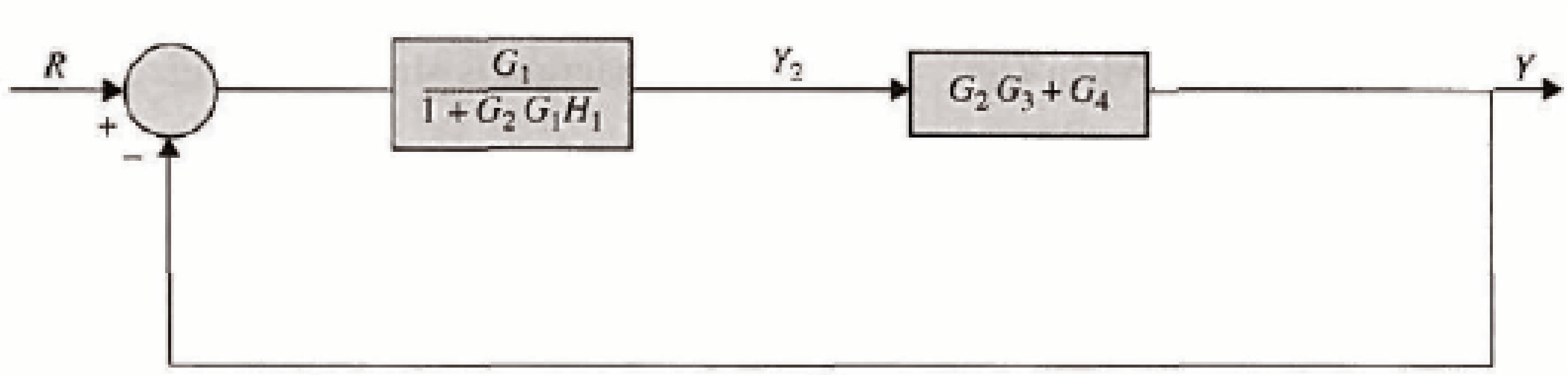


Example 1:



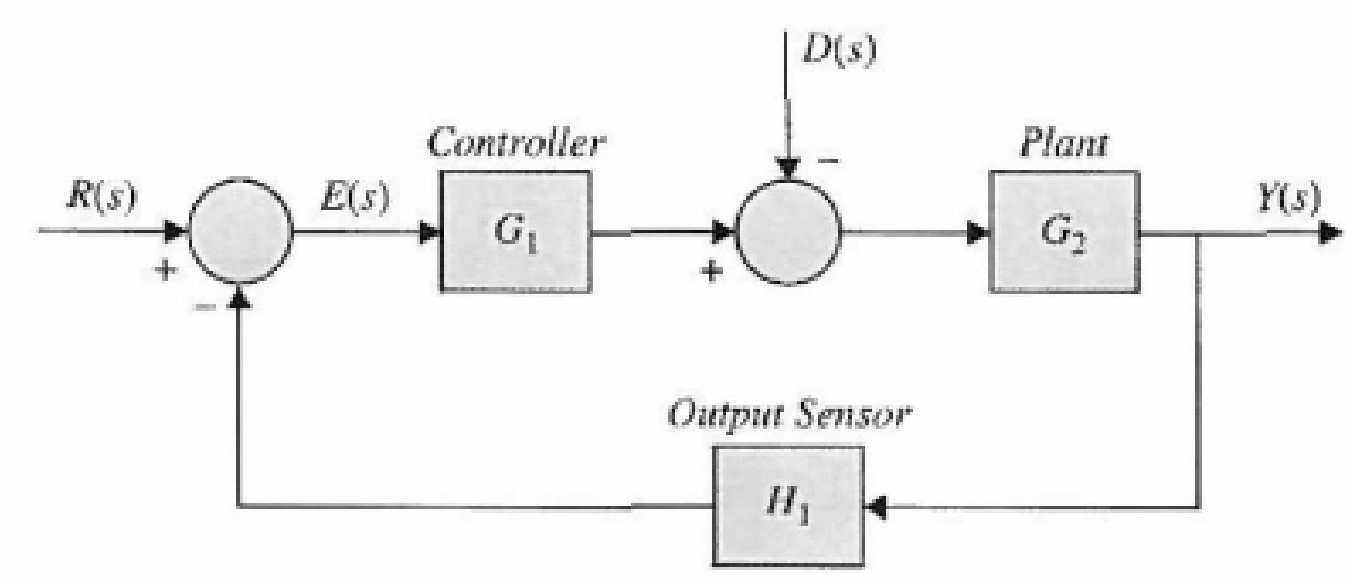


Example 1:



$$\frac{Y(s)}{E(s)} = \frac{G_1G_2G_3 + G_1G_4}{1 + G_2G_3H_1 + G_1G_2G_3 + G_1G_4}$$

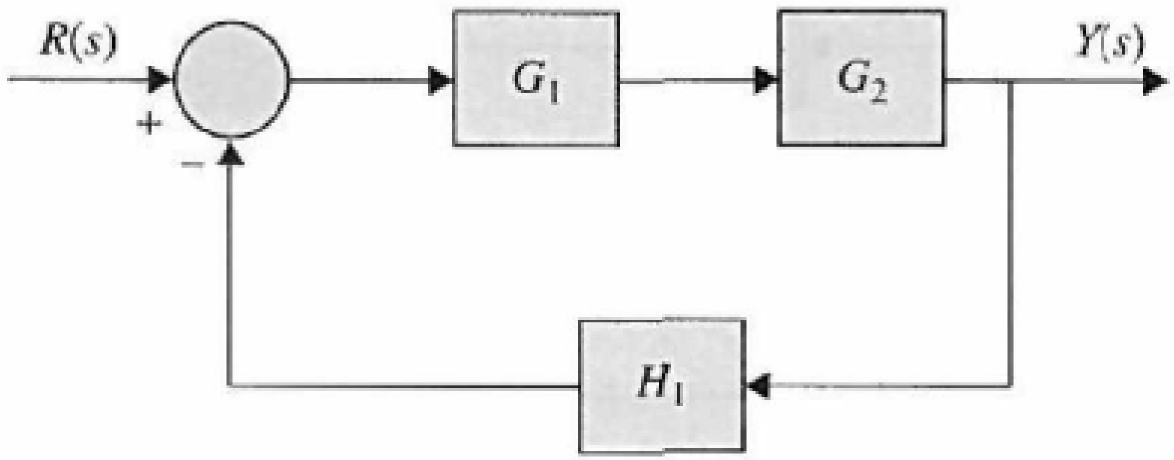
Example 2:



Super Position: For linear systems, the overall response of the system under multi-inputs is the summation of the responses due to the individual inputs, i.e., in this case,

$$Y_{total} = Y_R|_{D=0} + Y_D|_{R=0}$$

Example 2:

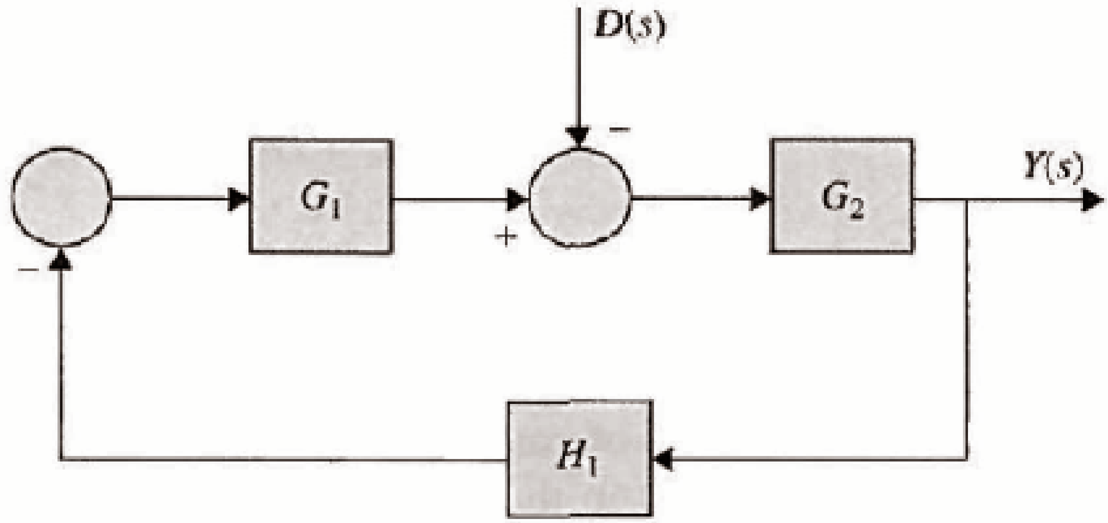


When $D(s) = 0$,

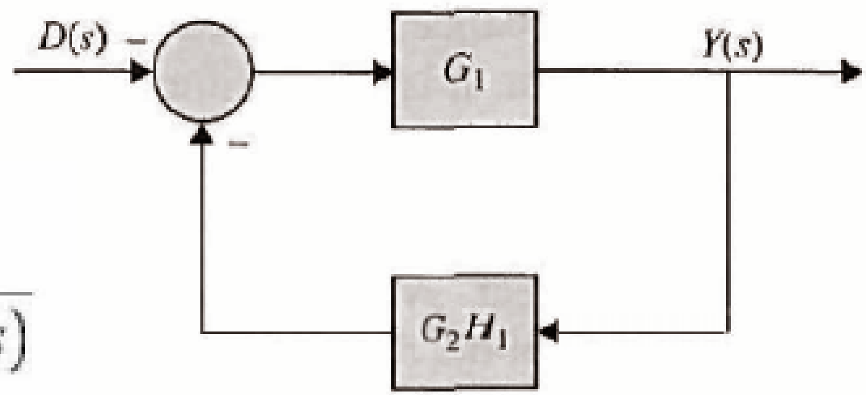
$$\frac{Y(s)}{R(s)} = \frac{G_1(s)\,G_2(s)}{1 + G_1(s)\,G_2\,H_1(s)}$$

Example 2:

When $R(s) = 0$,



(a)



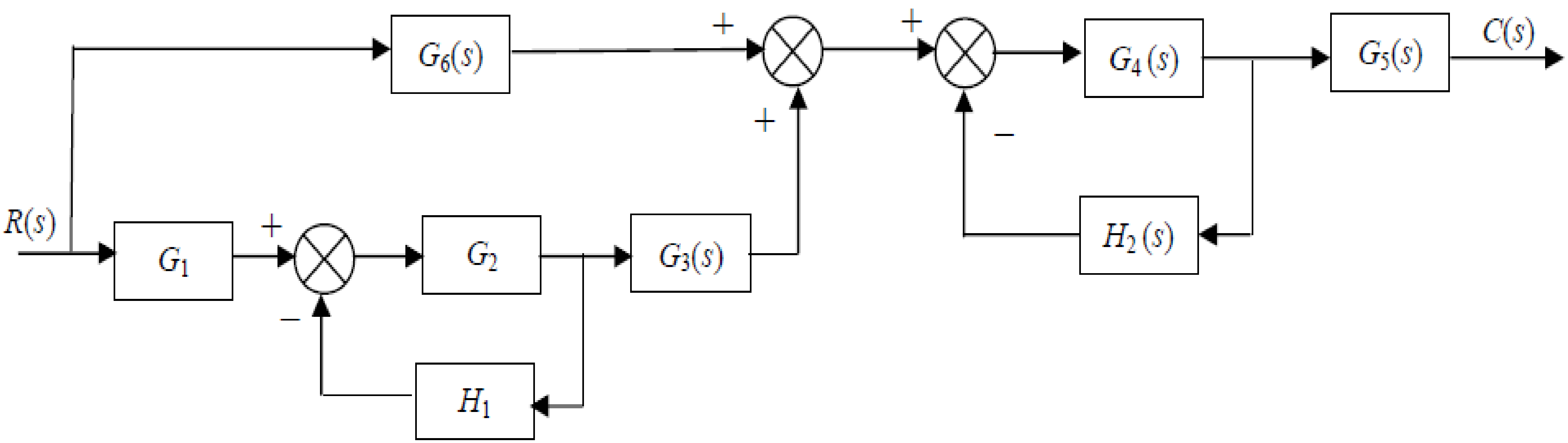
(b)

$$\frac{Y(s)}{D(s)} = \frac{-G_2(s)}{1 + G_1(s)\,G_2(s)\,H_1(s)}$$

Example 2:

$$Y_{total} = \left.\frac{Y(s)}{R(s)}\right|_{D=0} R(s) + \left.\frac{Y(s)}{D(s)}\right|_{R=0} D(s)$$

$$Y(s) = \frac{G_1 G_2}{1 + G_1 G_2 H_1} R(s) + \frac{-G_2}{1 + G_1 G_2 H_1} D(s)$$


$G_6(s)$
$G_4(s)$
$G_5(s)$
$C(s)$
$R(s)$
G_1
G_2
$G_3(s)$
$H_2(s)$
H_1
+
+
+
+
−
−