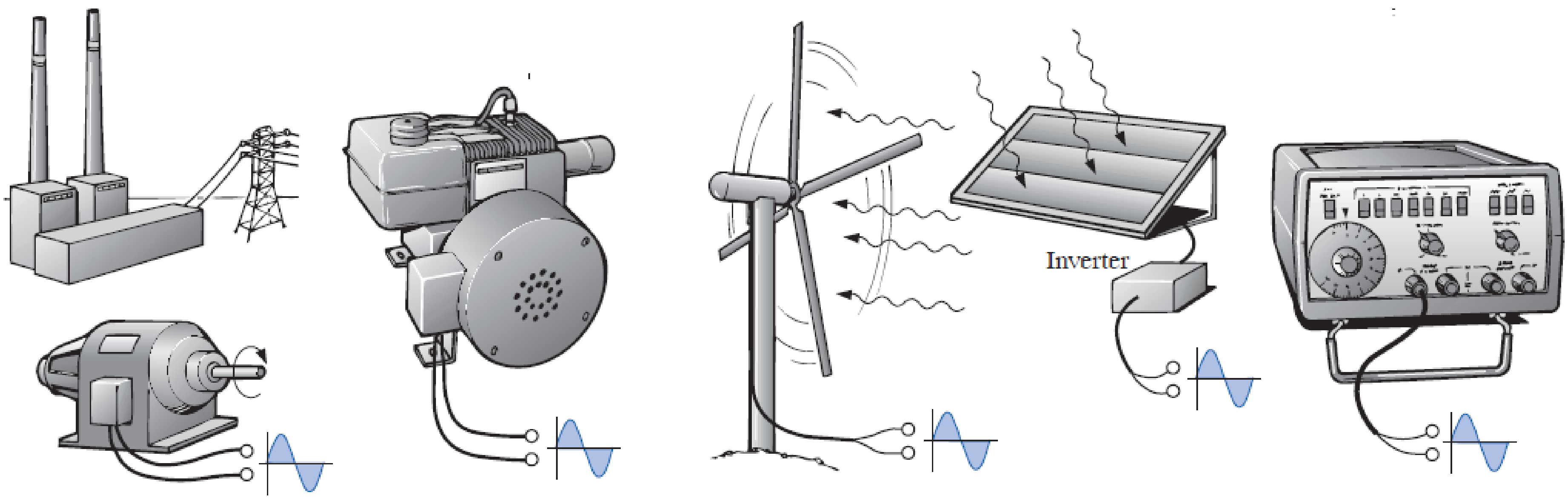
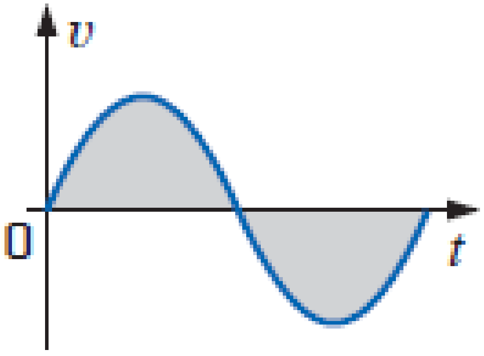
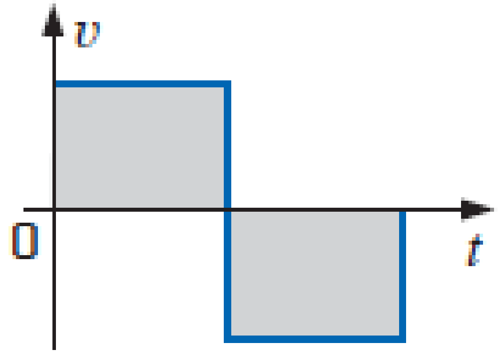
Alternating Current Circuits



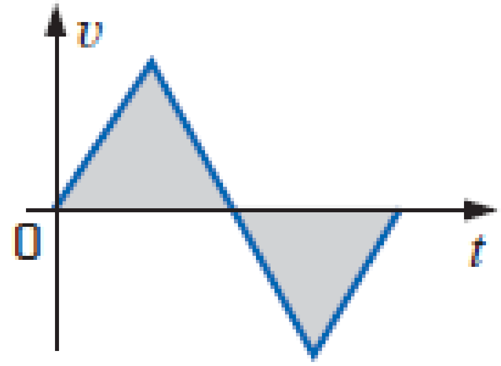
Alternating Waveforms



Sinusoidal

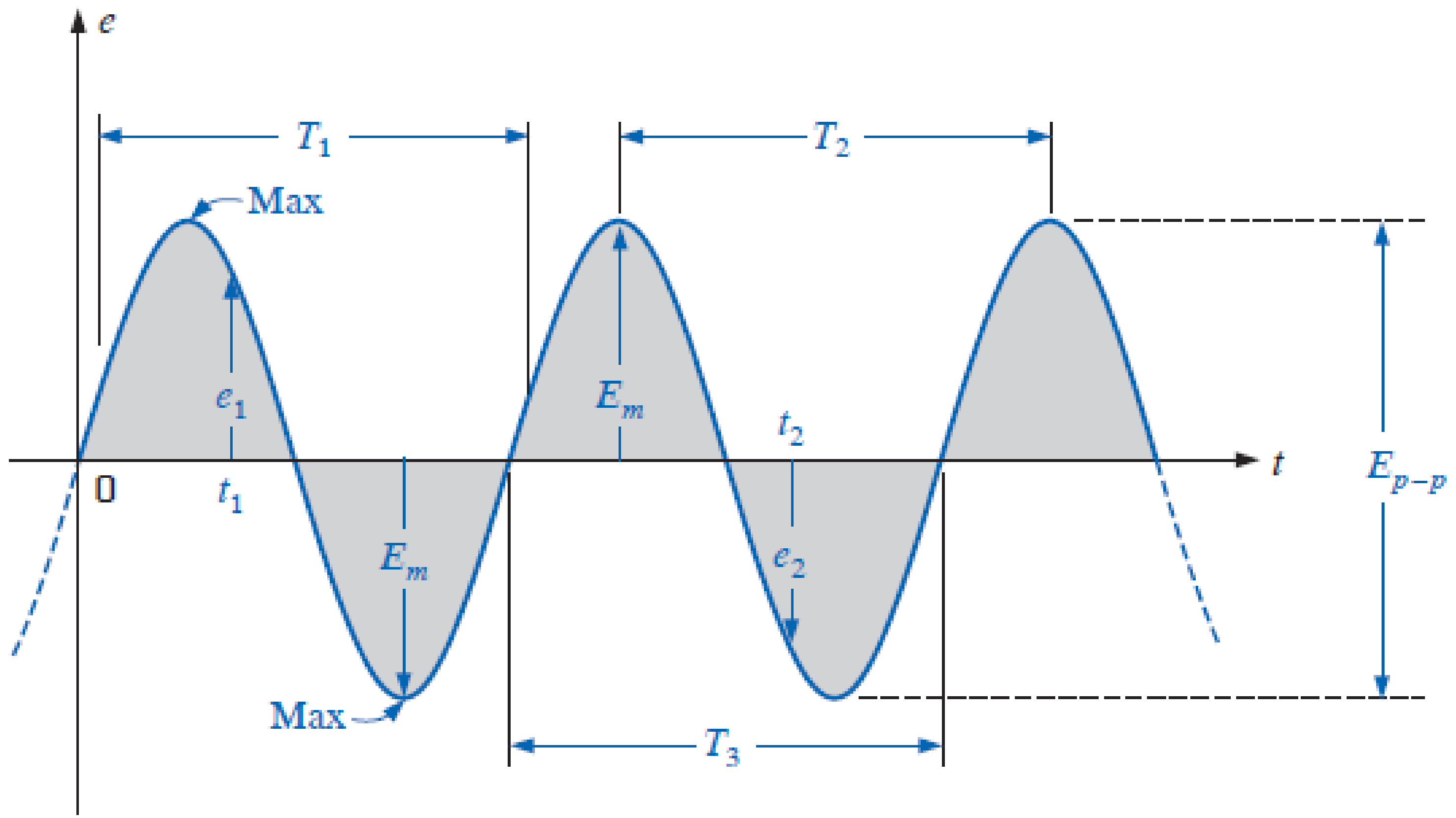


Square wave



Triangular wave

Important parameters for a sinusoidal voltage



Definitions

- **Waveform:** The path traced by a quantity, such as the voltage plotted as a function of some variable such as time.

- **Instantaneous value:** The magnitude of a waveform at any instant of time; denoted by lowercase letters (e_1, e_2).

- **Peak amplitude:** The maximum value of a waveform as measured from its *average, or mean, value, denoted by uppercase letters*
-
- **Peak value:** The maximum instantaneous value of a function as measured from the zero-volt level. For the pervious waveform ,the peak amplitude and peak value are the same, since the average value of the function is zero volts.

Definitions (2)

- **Peak-to-peak value:** Denoted by $E_{p\text{-}p}$ *or* $V_{p\text{-}p}$, *the full voltage* between positive and negative peaks of the waveform, that is, the sum of the magnitude of the positive and negative peaks.

- **Periodic waveform:** A waveform that continually repeats itself after the same time interval.

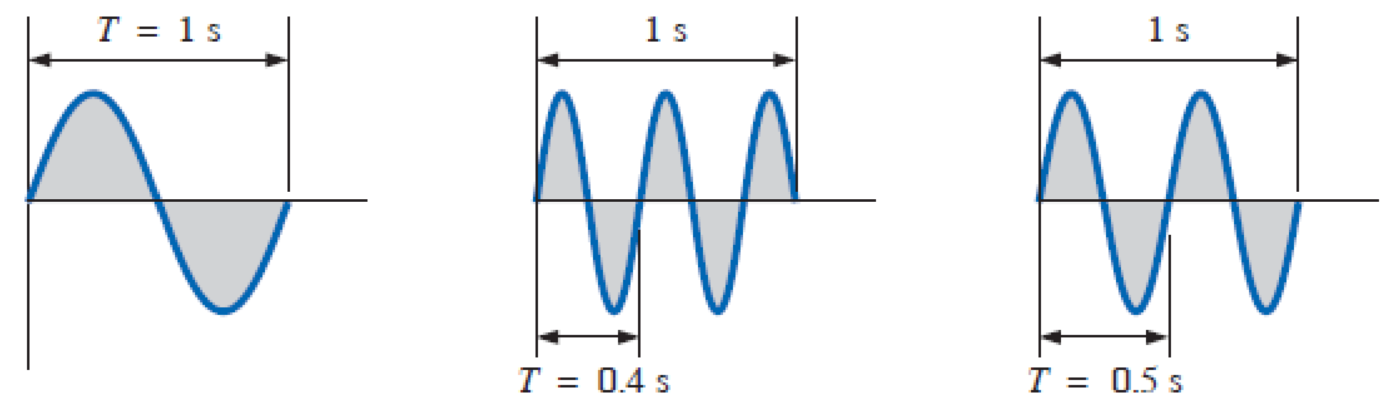
- **Period (*T*):** *The time interval between successive repetitions of a* periodic waveform.

- **Cycle**: The portion of a waveform contained in *one period of time.*

Frequency

- **Frequency (*f*): *The number of cycles that occur in 1 s.***
- The unit of measure for frequency is the *hertz (Hz), where*

1 hertz (Hz) = 1 cycle per second (c/s)



(a) (b) (c)

EXAMPLE 13.1 Find the period of a periodic waveform with a frequency of

a. 60 Hz.

b. 1000 Hz.

Solutions:

a. $T = \frac{1}{f} = \frac{1}{60\text{ Hz}} \cong 0.01667\text{ s or }\mathbf{16.67\text{ ms}}$

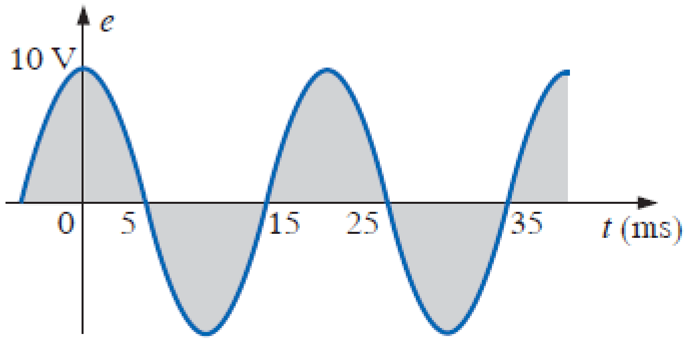
(a recurring value since 60 Hz is so prevalent)

b. $T = \frac{1}{f} = \frac{1}{1000\text{ Hz}} = 10^{-3}\text{ s} = \mathbf{1\text{ ms}}$

EXAMPLE 13.2 Determine the frequency of the waveform of Fig. 13.8.

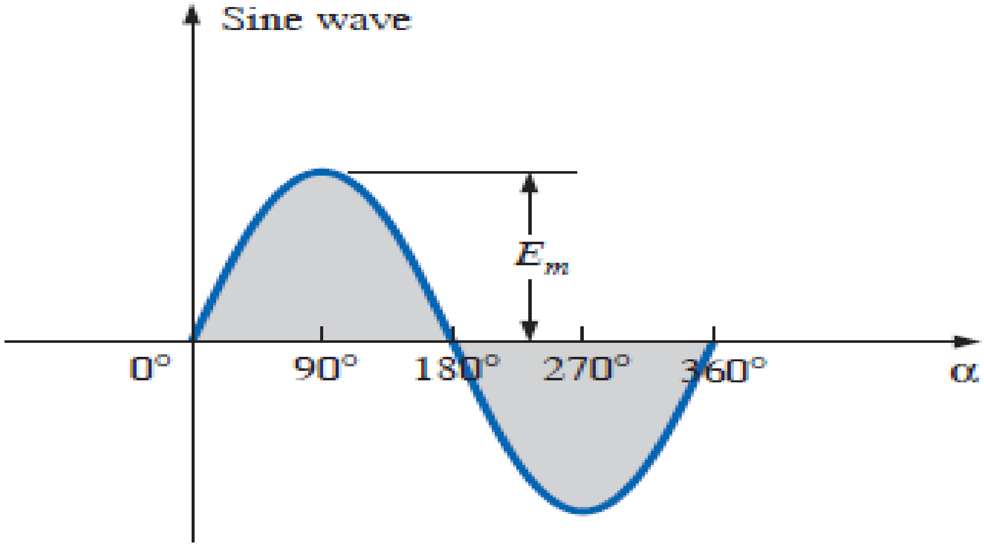
Solution: From the figure, $T = (25\text{ ms} - 5\text{ ms}) = 20\text{ ms}$, and

$$f = \frac{1}{T} = \frac{1}{20 \times 10^{-3}\text{ s}} = \mathbf{50\text{ Hz}}$$



THE SINE WAVE

- ***The sinusoidal waveform is the only alternating waveform whose shape is unaffected by the response characteristics of R, L, and C elements.***



$$2\pi \text{ rad} = 360°$$

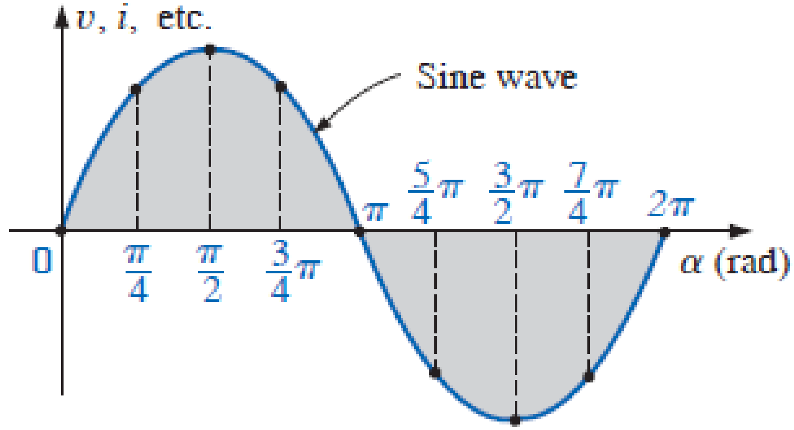
$$1 \text{ rad} = 57.296° \cong 57.3°$$

The quantity π is the ratio of the circumference of a circle to its diameter.

$$\pi = 3.14159\ 26535\ 89793\ 23846\ 26433\ .\ .\ .$$

$$\text{Radians} = \left(\frac{\pi}{180°}\right) \times (\text{degrees})$$

$$\text{Degrees} = \left(\frac{180°}{\pi}\right) \times (\text{radians})$$



$$90°: \quad \text{Radians} = \frac{\pi}{180°}(90°) = \frac{\pi}{2}\ \text{rad}$$

$$30°: \quad \text{Radians} = \frac{\pi}{180°}(30°) = \frac{\pi}{6}\ \text{rad}$$

$$\frac{\pi}{3}\ \text{rad}: \quad \text{Degrees} = \frac{180°}{\pi}\left(\frac{\pi}{3}\right) = 60°$$

$$\frac{3\pi}{2}\ \text{rad}: \quad \text{Degrees} = \frac{180°}{\pi}\left(\frac{3\pi}{2}\right) = 270°$$

$$\text{Angular velocity} = \frac{\text{distance (degrees or radians)}}{\text{time (seconds)}}$$

$$\omega = \frac{\alpha}{t}$$

$$\alpha = \omega t$$

$$\omega = \frac{2\pi}{T} \qquad \text{(rad/s)}$$

$$\omega = 2\pi f \qquad \text{(rad/s)}$$

EXAMPLE 13.4 Determine the angular velocity of a sine wave having a frequency of 60 Hz.

Solution:

$$\omega = 2\pi f = (2\pi)(60\ \text{Hz}) \cong \mathbf{377\ rad/s}$$

(a recurring value due to 60-Hz predominance)

EXAMPLE 13.6 Given $\omega = 200$ rad/s, determine how long it will take the sinusoidal waveform to pass through an angle of 90°.

Solution: Eq. (13.10): $\alpha = \omega t$, and

$$t = \frac{\alpha}{\omega}$$

However, α must be substituted as $\pi/2$ (= 90°) since ω is in radians per second:

$$t = \frac{\alpha}{\omega} = \frac{\pi/2 \text{ rad}}{200 \text{ rad/s}} = \frac{\pi}{400} \text{ s} = \mathbf{7.85\ ms}$$

EXAMPLE 13.7 Find the angle through which a sinusoidal waveform of 60 Hz will pass in a period of 5 ms.

Solution: Eq. (13.11): $\alpha = \omega t$, or

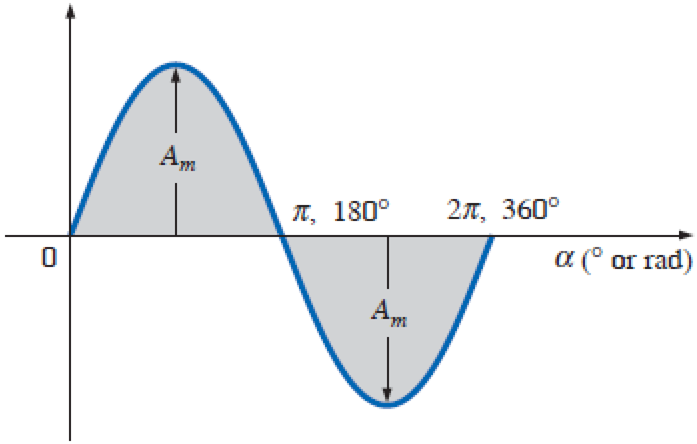
$$\alpha = 2\pi f t = (2\pi)(60 \text{ Hz})(5 \times 10^{-3} \text{ s}) = \mathbf{1.885\ rad}$$

If not careful, one might be tempted to interpret the answer as 1.885°. However,

$$\alpha\,(°) = \frac{180°}{\pi \text{ rad}}(1.885 \text{ rad}) = \mathbf{108°}$$

GENERAL FORMAT FOR THE SINUSOIDAL VOLTAGE OR CURRENT

$$A_m \sin \alpha$$



EXAMPLE 13.8 Given $e = 5 \sin \alpha$, determine e at $\alpha = 40°$ and $\alpha = 0.8\pi$.

Solution: For $\alpha = 40°$,

$$e = 5 \sin 40° = 5(0.6428) = \mathbf{3.214\ V}$$

For $\alpha = 0.8\pi$,

$$\alpha\ (°) = \frac{180°}{\pi}(0.8\pi) = 144°$$

and

$$e = 5 \sin 144° = 5(0.5878) = \mathbf{2.939\ V}$$

EXAMPLE 13.9

a. Determine the angle at which the magnitude of the sinusoidal function $v = 10 \sin 377t$ is 4 V.

b. Determine the time at which the magnitude is attained.

Solutions:

a. Eq. (13.15):

$$\alpha_1 = \sin^{-1}\frac{v}{E_m} = \sin^{-1}\frac{4\text{ V}}{10\text{ V}} = \sin^{-1} 0.4 = \mathbf{23.578^\circ}$$

However, Figure 13.19 reveals that the magnitude of 4 V (positive) will be attained at two points between 0° and 180°. The second intersection is determined by

$$\alpha_2 = 180^\circ - 23.578^\circ = \mathbf{156.422^\circ}$$

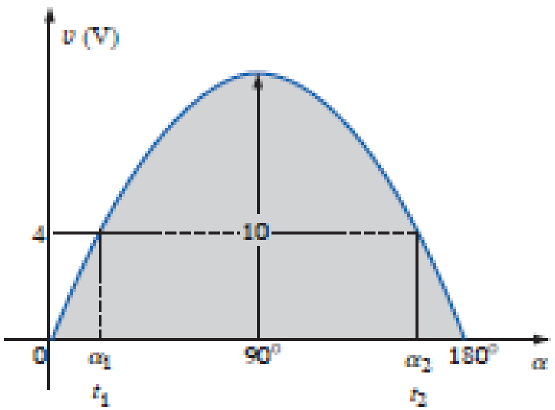


FIG. 13.19
Example 13.9.

In general, therefore, keep in mind that Equations (13.15) and (13.16) will provide an angle with a magnitude between 0° and 90°.

b. Eq. (13.10): $\alpha = \omega t$, and so $t = \alpha/\omega$. However, α must be in radians. Thus,

$$\alpha\text{ (rad)} = \frac{\pi}{180^\circ}(23.578^\circ) = 0.411\text{ rad}$$

and

$$t_1 = \frac{\alpha}{\omega} = \frac{0.411\text{ rad}}{377\text{ rad/s}} = \mathbf{1.09\text{ ms}}$$

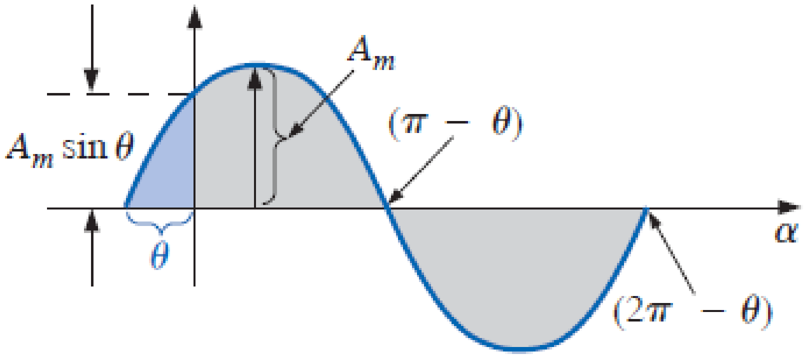
For the second intersection,

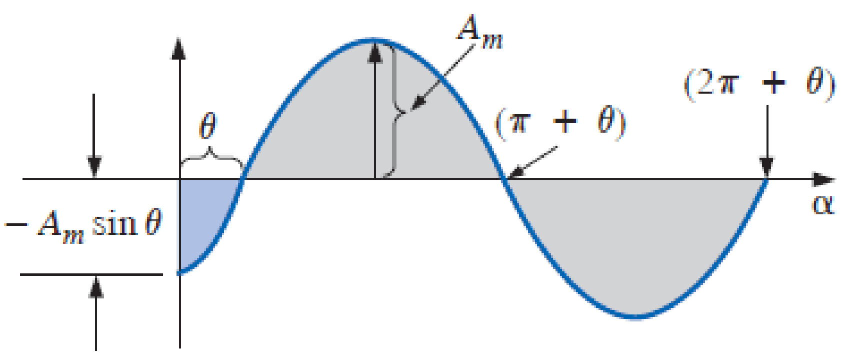
$$\alpha\text{ (rad)} = \frac{\pi}{180^\circ}(156.422^\circ) = 2.73\text{ rad}$$

$$t_2 = \frac{\alpha}{\omega} = \frac{2.73\text{ rad}}{377\text{ rad/s}} = \mathbf{7.24\text{ ms}}$$

PHASE RELATIONS

$$A_m \sin(\omega t \pm \theta)$$





$$A_m \sin(\omega t + \theta)$$

$$A_m \sin(\omega t - \theta)$$

Lead

Lag

$$\sin(\omega t + 90°) = \sin\left(\omega t + \frac{\pi}{2}\right) = \cos \omega t$$

$$\sin \omega t = \cos(\omega t - 90°) = \cos\left(\omega t - \frac{\pi}{2}\right)$$

$$\cos \alpha = \sin(\alpha + 90°)$$
$$\sin \alpha = \cos(\alpha - 90°)$$
$$-\sin \alpha = \sin(\alpha \pm 180°)$$
$$-\cos \alpha = \sin(\alpha + 270°) = \sin(\alpha - 90°)$$
etc.

$$\sin(-\alpha) = -\sin \alpha$$
$$\cos(-\alpha) = \cos \alpha$$

EXAMPLE 13.12 What is the phase relationship between the sinusoidal waveforms of each of the following sets?

a. $v = 10 \sin(\omega t + 30°)$
   $i = 5 \sin(\omega t + 70°)$

b. $i = 15 \sin(\omega t + 60°)$
   $v = 10 \sin(\omega t - 20°)$

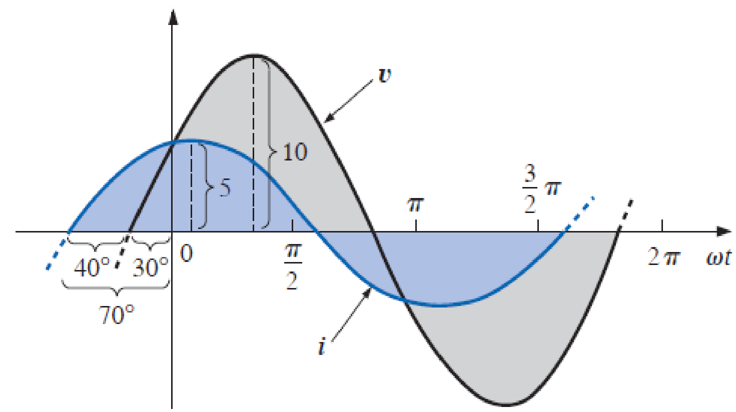
c. $i = 2 \cos(\omega t + 10°)$
   $v = 3 \sin(\omega t - 10°)$

d. $i = -\sin(\omega t + 30°)$
   $v = 2 \sin(\omega t + 10°)$

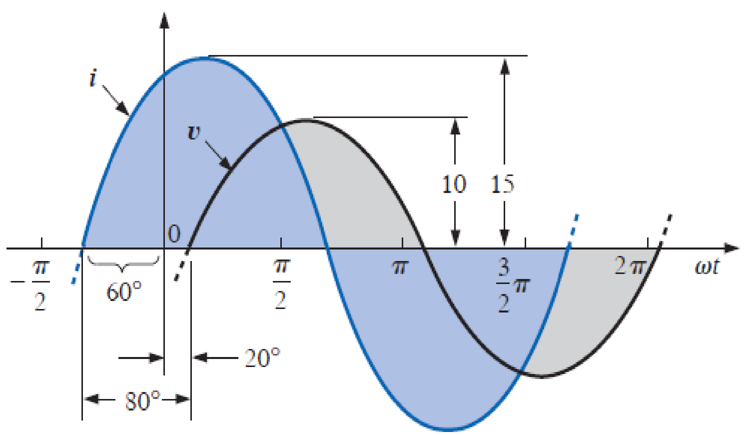
e. $i = -2 \cos(\omega t - 60°)$
   $v = 3 \sin(\omega t - 150°)$

Solutions:

a. See Fig. 13.27.
i leads v by 40°, or v lags i by 40°.



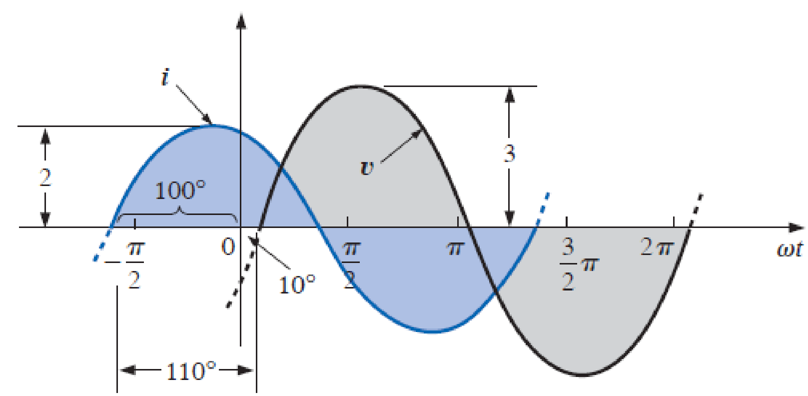
b. See Fig. 13.28.
i leads v by 80°, or v lags i by 80°.



c. See Fig. 13.29.

$$i = 2 \cos(\omega t + 10°) = 2 \sin(\omega t + 10° + 90°)$$
$$= 2 \sin(\omega t + 100°)$$

i leads v by 110°, or v lags i by 110°.



Thanks