Design in Frequency Domain

Lect. 9

Presented by

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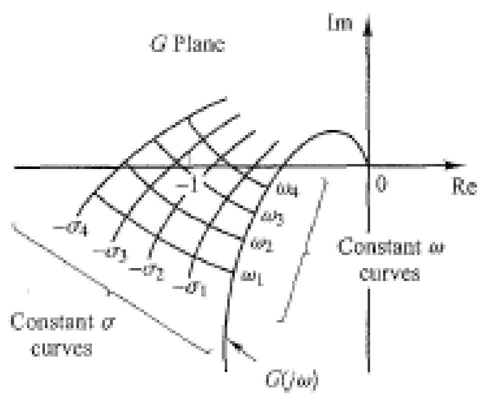
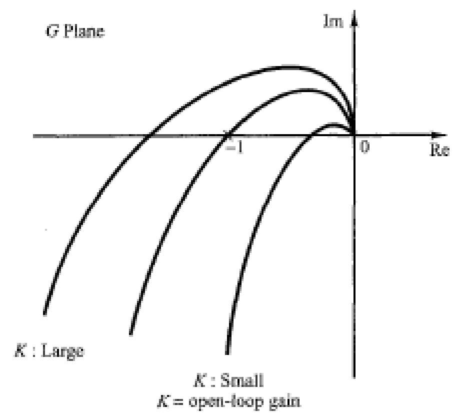
Relative Stability

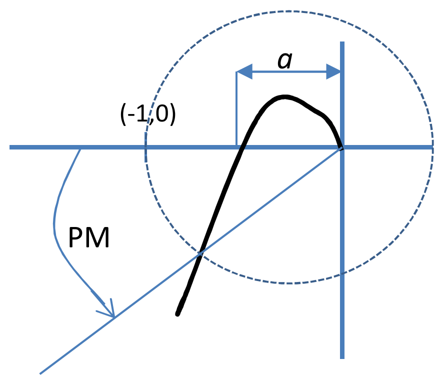
- Stability Margins
  - What is the worst perturbation of the transfer function that will make the system marginally stable?
  - Marginal stability for open loop stable systems is when the contour goes through the point (-1,0).

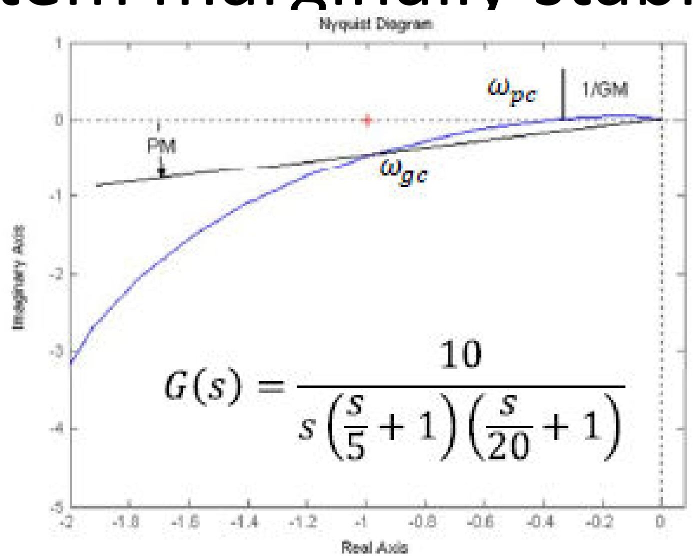




Gain and Phase Margins

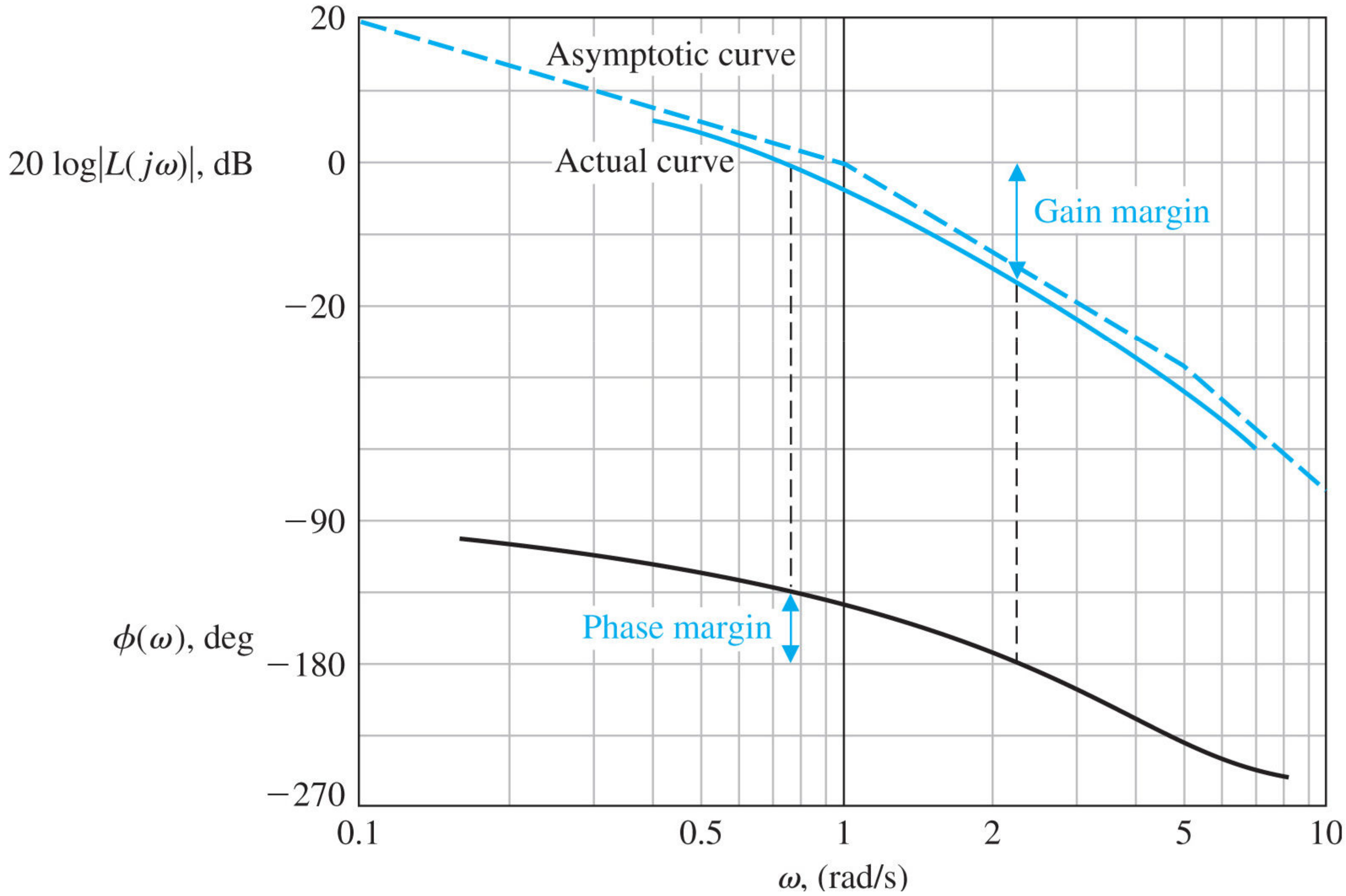
- Gain Margin: gain perturbation that makes the system marginally stable.
- Phase Margin: negative phase perturbation that makes the system marginally stable.





ω_{gc}/ω_{pc}=gain crossover ($M = 1$) /phase crossover ($\phi = -180°$)

Nyquist Diagram



20
0
−20
−90
−180
−270
20 log|L(jω)|, dB
φ(ω), deg
Asymptotic curve
Actual curve
Gain margin
Phase margin
0.1
0.5
1
2
5
10
ω, (rad/s)

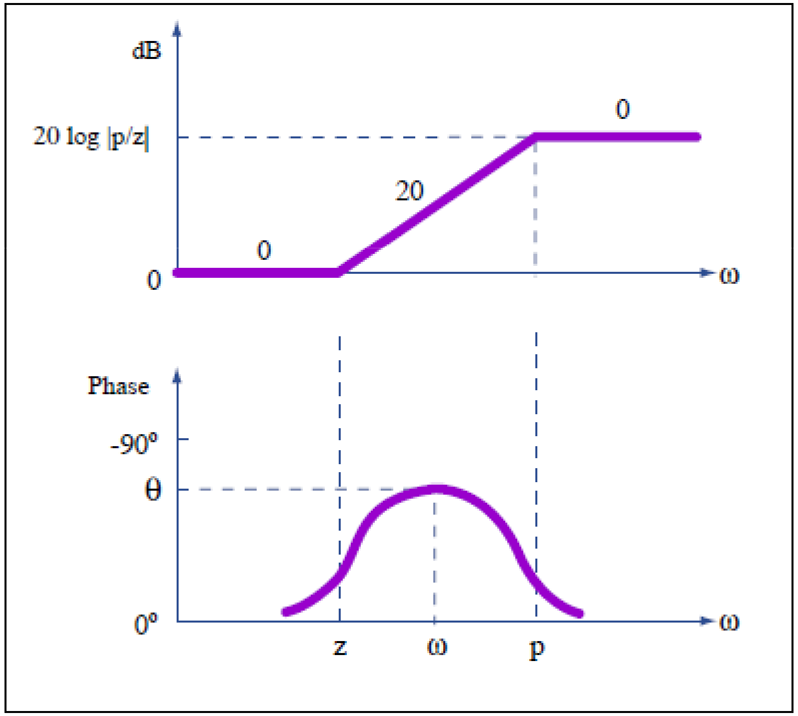
Design Compensator

- The design criteria is to adjust the gain and phase margin (GM and PM)
- GM and PM can be improved using either Lead or Lag compensator
- Discuss the difference between lead and lag properties!!!

Lead Compensator Design

- Maximum phase

$$G_c(s) = K_c \frac{s+z}{s+p}$$



$$\sin \phi_{\max} = \frac{1-\alpha}{1+\alpha}$$

where $\alpha = |z|/|p|$, which implies that

$$\alpha = \frac{1-\sin \phi_{\max}}{1+\sin \phi_{\max}}$$

- Frequency of the maximum phase addition is $\omega_{\max} = \sqrt{|z| \cdot |p|}$
  - Usually try to place this near ω_c
- High frequency gain increase is by $1/\alpha$
- So there is a compromise between wanting to add a large amount of phase (α small) and the tendency to generate large gains at high frequency
  - So try to keep $1/\alpha \leq 10$, which means $|p| \leq 10|z|$ and

$$\phi_{\max} \leq 60^\circ$$

Design lead procedure:

1- determine the required specification (error constant, PM and GM)

2- Draw Bode plot based on the error constant (gain)

3- Determine the system PM and GM

4- check the PMs of the system with the PMd

5- compute compensator phase angle

$$\phi_{\max} = PM_d - PM_s + \varepsilon$$

$$5 \le \varepsilon \le 12^o$$

6- compute *α:* $\alpha = \dfrac{1-\sin\phi_{\max}}{1+\sin\phi_{\max}}$

7- the new gain crossover frequency will be at LM $LM(\omega_m) = 10\log\alpha$

8- compute T where $\omega_m = \dfrac{1}{T\sqrt{\alpha}}$

9- let $z_c = \dfrac{1}{T}$ and $p_c = \dfrac{1}{\alpha T}$