Control System II
EE 412

Lecture 1

Dr. Mostafa Abdel-geliel

Course Contents

- State Space (SS) modeling of linear systems
- SS Representation from system Block Diagram
- SS from Differential equation
  - phase variables form
  - Canonical form
  - Parallel
  - Cascade
- State transition matrix and its properties
- Eigen values and Eigen Vectors
- SS solution
- Controllability and Observability
- Pole Placement and state feedback
- Nonlinear system
- Common non linearity
- Describing function method
- Limit cycle
- The phase plane method

Intended Learning Outcomes (ILO)

A- Knowledge and understanding

- A.4. Principles of design including elements design, process and/or a system related to specific disciplines.
- A.5. Methodologies of solving engineering problems, data collection and interpretation.
- A.15. Principles of operation and performance specifications of electrical and electromechanical engineering systems
- A.27. Analysis, design and implementation of various methods of control using analogue and digital control systems
- A.31. Formulate the problem, realizing the requirements and identifying the Constraints.

ILO (cont.)

B- Intellectual skills

- B.1 Select appropriate mathematical and computer-based methods for modeling and analyzing problems

  B.2 Select appropriate solutions for engineering problems based on analytical thinking

  B.3 Think in a creative and innovative way in problem solving and design

  B.8 Select and appraise appropriate ICT tools to a variety of engineering problems

  B.11 Analyze results of numerical models and assess their limitations

  B.19 Design computer programs to analyze and simulate different electrical systems components and control applications

ILO (cont.)

C- Professional and Practical Skills

- C.1 Apply knowledge of mathematics, science, information technology, design, business context and engineering practice integrally to solve engineering problems

  C.5 Use computational facilities and techniques, measuring instruments, workshops and laboratory equipment to design experiments, collect, analyze and interpret results

  C.6 Use a wide range of analytical tools, techniques, equipment, and software packages pertaining to the discipline and develop required computer programs

  C.13 Design and perform experiments, as well as analyze and interpret experimental results related to electrical power and machines systems

  C.14 Use laboratory and field equipment competently and safely

  C.16 Specify and evaluate manufacturing of components and equipment related to electrical power and machines

ILO (cont.)

B- General skills

- D.3 Communicate effectively

  D.4 Demonstrate efficient IT capabilities

State Space Definition

- Steps of control system design
  - Modeling: Equation of motion of the system
  - Analysis: test system behavior
  - Design: design a controller to achieve the required specification
  - Implementation: Build the designed controller
  - Validation and tuning: test the overall sysetm
- In SS :Modeling, analysis and design in time domain

SS-Definition

- In the classical control theory, the system model is represented by a transfer function
- The analysis and control tool is based on classical methods such as root locus and Bode
- It is restricted to single input/single output system
- It depends only the information of input and output and it does not use any knowledge of the interior structure of the plant,
- It allows only limited control of the closed-loop behavior using feedback control is used

- Modern control theory solves many of the limitations by using a much “richer” description of the plant dynamics.
- The so-called state-space description provide the dynamics as a set of coupled first-order differential equations in a set of internal variables known as state variables, together with a set of algebraic equations that combine the state variables into physical output variables.

SS-Definition

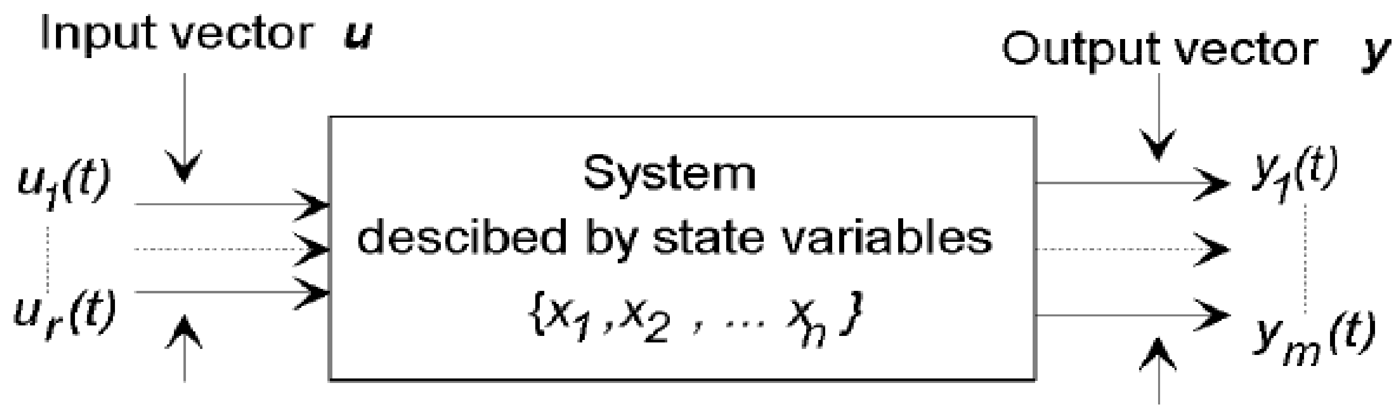
- The Philosophy of SS based on transforming the equation of motions of order *n* (highest derivative order) into an *n* equation of 1^{st} order
- State variable represents storage element in the system which leads to derivative equation between its input and output; it could be a physical or mathematical variables
- # of state=#of storage elements=order of the system
- For example if a system is represented by

$$\frac{d^3y}{dt^3} + 7\frac{d^2y}{dt^2} + 19\frac{dy}{dt} + 13y = 13\frac{du}{dt} + 26u$$

- This system of order 3 then it has 3 state and 3 storage elements

SS-Definition

- The concept of the state of a dynamic system refers to a minimum set of variables, known as state variables, that fully describe the system and its response to any given set of inputs



The state variables are an *internal* description of the system which completely characterize the system state at any time *t*, and from which any output variables *yi*(*t*) may be computed.

The State Equations

A standard form for the state equations is used throughout system dynamics. In the standard form the mathematical description of the system is expressed as a set of *n* coupled first-order ordinary differential equations, known as the *state equations*,

in which the time derivative of each state variable is expressed in terms of the state variables $x1(t), \ldots, xn(t)$ and the system inputs $u1(t), \ldots, ur(t)$.

$$
\begin{aligned}
\dot{x}_1 &= f_1(\mathbf{x}, \mathbf{u}, t) \\
\dot{x}_2 &= f_2(\mathbf{x}, \mathbf{u}, t) \\
\vdots &= \vdots \\
\dot{x}_n &= f_n(\mathbf{x}, \mathbf{u}, t)
\end{aligned}
$$

It is common to express the state equations in a vector form, in which the set of n state variables is written as a *state vector* $\mathbf{x}(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T$, and the set of r inputs is written as an input vector $\mathbf{u}(t) = [u_1(t), u_2(t), \ldots, u_r(t)]^T$. Each state variable is a time varying component of the column vector $\mathbf{x}(t)$.

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}, t).$$

where $\mathbf{f}(\mathbf{x}, \mathbf{u}, t)$ is a *vector* function with n components $f_i(\mathbf{x}, \mathbf{u}, t)$.

In this note we restrict attention primarily to a description of systems that are *linear* and *time-invariant* (LTI), that is systems described by linear differential equations with constant coefficients.

$$\begin{array}{rcl}
\dot{x}_1 & = & a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n + b_{11}u_1 + \ldots + b_{1r}u_r \\
\dot{x}_2 & = & a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n + b_{21}u_1 + \ldots + b_{2r}u_r \\
\vdots & & \vdots \\
\dot{x}_n & = & a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n + b_{n1}u_1 + \ldots + b_{nr}u_r
\end{array}$$

$$
\begin{aligned}
\dot{x}_1 &= a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n + b_{11}u_1 + \ldots + b_{1r}u_r \\
\dot{x}_2 &= a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n + b_{21}u_1 + \ldots + b_{2r}u_r \\
\vdots &\qquad \vdots \\
\dot{x}_n &= a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n + b_{n1}u_1 + \ldots + b_{nr}u_r
\end{aligned}
$$

where the coefficients a_{ij} and b_{ij} are constants that describe the system. This set of n equations defines the derivatives of the state variables to be a weighted sum of the state variables and the system inputs.

$$
\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & & & \vdots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} b_{11} & \ldots & b_{1r} \\ b_{21} & & b_{2r} \\ \vdots & & \vdots \\ b_{n1} & \ldots & b_{nr} \end{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ u_r \end{bmatrix}
$$

$$
\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}
$$

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

where the state vector **x** is a column vector of length *n*, the input vector **u** is a column vector of length *r*, **A** is an $n \times n$ square matrix of the constant coefficients a_{ij}, and **B** is an $n \times r$ matrix of the coefficients b_{ij} that weight the inputs.

A system ***output*** **is** defined to be any system variable of interest. A description of a physical system in terms of a set of state variables does not necessarily include all of the variables of direct engineering interest.

An important property of the linear state equation description is that all system variables may be represented by a linear combination of the state variables *xi* and the system inputs *ui*.

An arbitrary output variable in a system of order *n* with *r* inputs may be written:

$$y(t) = c_1x_1 + c_2x_2 + \ldots + c_nx_n + d_1u_1 + \ldots + d_ru_r$$

$$\begin{aligned}
y_1 &= c_{11}x_1 + c_{12}x_2 + \ldots + c_{1n}x_n + d_{11}u_1 + \ldots + d_{1r}u_r \\
y_2 &= c_{21}x_1 + c_{22}x_2 + \ldots + c_{2n}x_n + d_{21}u_1 + \ldots + d_{2r}u_r \\
\vdots & \quad \vdots \\
y_m &= c_{m1}x_1 + c_{m2}x_2 + \ldots + c_{mn}x_n + d_{m1}u_1 + \ldots + d_{mr}u_r
\end{aligned}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \ldots & c_{1n} \\ c_{21} & c_{22} & \ldots & c_{2n} \\ \vdots & & & \vdots \\ c_{m1} & c_{m2} & \ldots & c_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} d_{11} & \ldots & d_{1r} \\ d_{21} & & d_{2r} \\ \vdots & & \vdots \\ d_{m1} & \ldots & d_{mr} \end{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ u_r \end{bmatrix}$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1n} \\ c_{21} & c_{22} & \dots & c_{2n} \\ \vdots & & & \vdots \\ c_{m1} & c_{m2} & \dots & c_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} d_{11} & \dots & d_{1r} \\ d_{21} & & d_{2r} \\ \vdots & & \vdots \\ d_{m1} & \dots & d_{mr} \end{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ u_r \end{bmatrix}$$

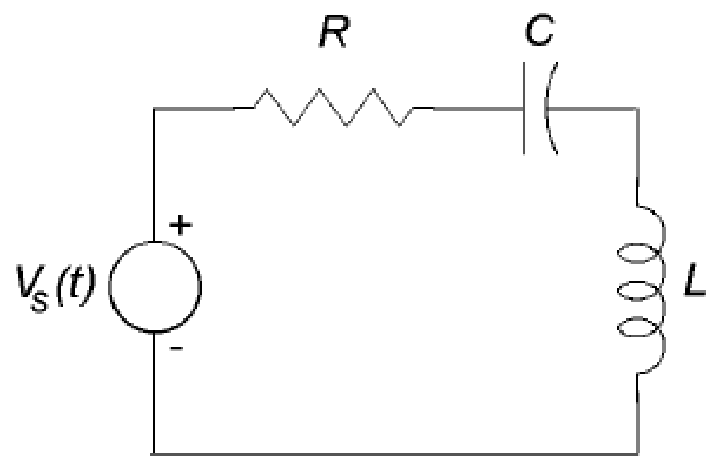
$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}$$

where **y** is a column vector of the output variables $y_i(t)$, **C** is an $m \times n$ matrix of the constant coefficients c_{ij} that weight the state variables, and D is an $m \times r$ matrix of the constant coefficients d_{ij} that weight the system inputs. For many physical systems the matrix **D** is the null matrix, and the output equation reduces to a simple weighted combination of the state variables:

$$\mathbf{y} = \mathbf{C}\mathbf{x}.$$

Example

Find the State equations for the series R-L-C electric circuit shown in



Solution:
capacitor voltage $v_C(t)$ and the inductor current $i_L(t)$ are state variables

$$\begin{bmatrix} \dot{v}_c \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} 0 & 1/C \\ -1/L & -R/L \end{bmatrix} \begin{bmatrix} v_c \\ i_L \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} V_{in}.$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} v_c \\ i_L \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} V_{in}$$

Prove

Appling KVL on the circuit

$$v_s(t) = R * i + v_c + L\frac{di}{dt} \ldots (1)$$

The relation of capacitor voltage and current $i = c\frac{dv_c}{dt}$

$$then$$

$$\dot{x}_1 = \frac{dv_c}{dt} = \frac{1}{c}i = \frac{1}{c}x_2$$

$$from\ equation\ (1)$$

$$\dot{x}_2 = \frac{di}{dt} = -v_c - R * i + v_s(t)$$

$$\dot{x}_2 = \frac{1}{L}[-x_1 - R * x_2 + u(t)]$$

$$y = v_c = x_1$$

Example

Draw a direct form realization of a block diagram, and write the state equations in phase variable form, for a system with the differential equation

$$\frac{d^3y}{dt^3} + 7\frac{d^2y}{dt^2} + 19\frac{dy}{dt} + 13y = 13\frac{du}{dt} + 26u$$

Solution

we define state variables as

$$x_1 = y,\ x_2 = \dot{y},\ and\ x_3 = \ddot{y} + 13u,$$

then the state space representation is

$$\dot{x}_1 = \dot{y} = x_2$$

$$\dot{x}_2 = \ddot{y} = x_3 - 13u$$

$$\dot{x}_3 = \dddot{y} - 13\dot{u} = -7\ddot{y} - 19\dot{y} - 13y + 26u$$

$$= -7(x_3 - 13u) - 19x_2 - 13x_1 + 26u$$

$$= -7x_3 - 19x_2 - 13x_1 + 117u$$

$$y = x_1$$

Then the model will be

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -13 & -19 & -7 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ -13 \\ 117 \end{bmatrix} u(t)$$

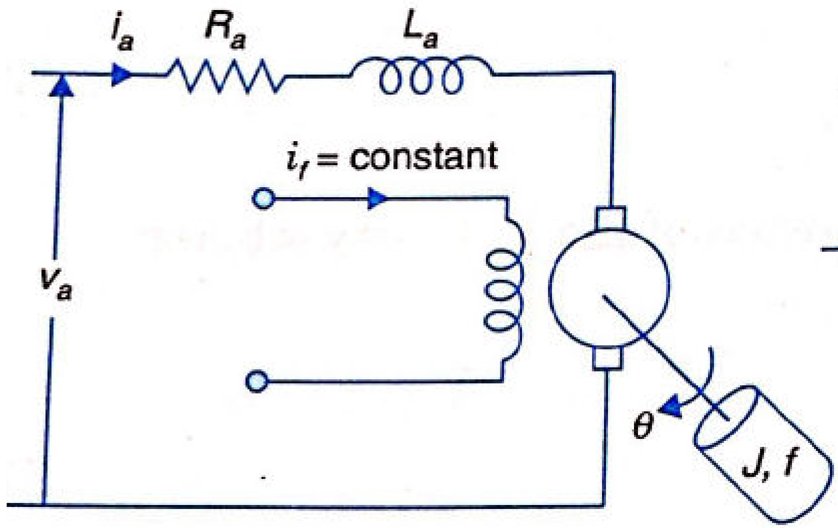
$$y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \mathbf{x}(t) + [0] u(t)$$

where

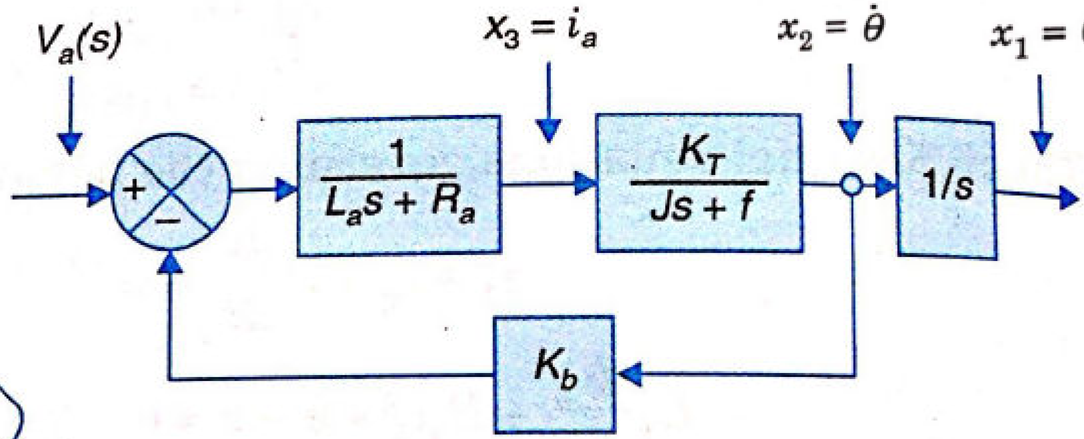
$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -13 & -19 & -7 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ -13 \\ 117 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \qquad D = 0$$

Electro Mechanical System

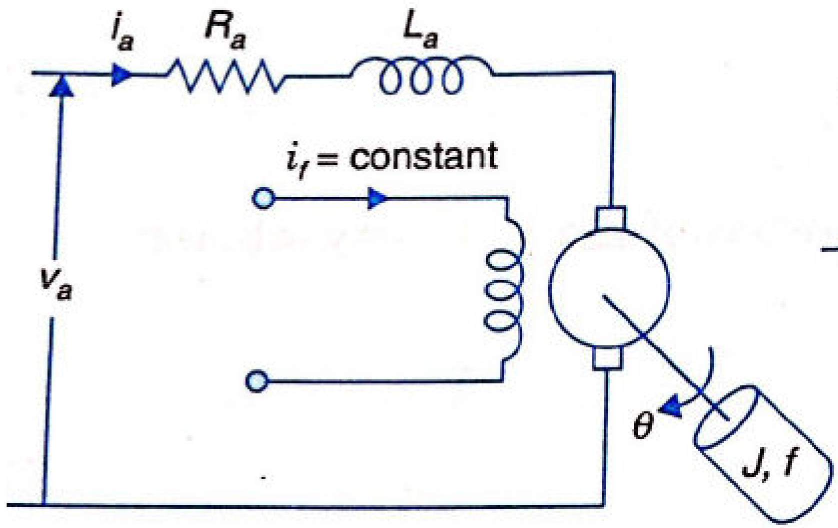


(a)

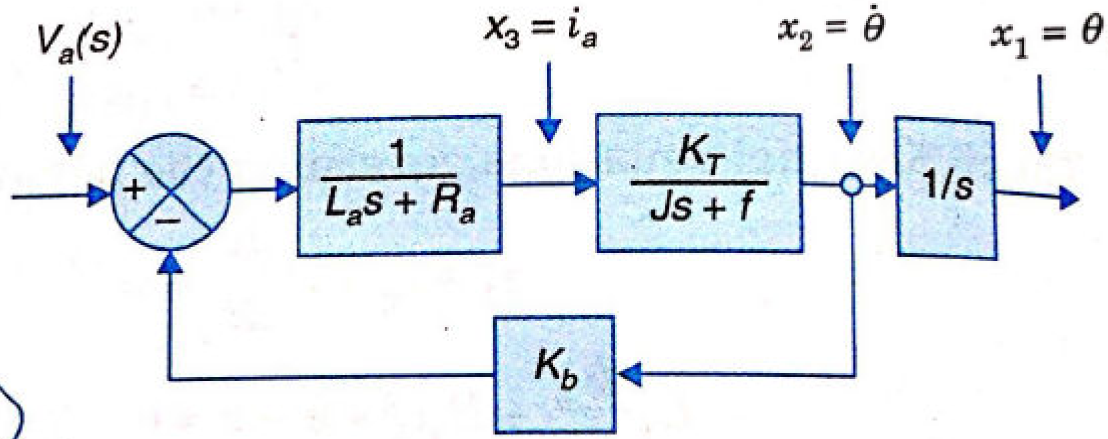


(b)

$$x_1 = \theta\,;\, x_2 = \dot{\theta}\,;\ \text{and}\ x_3 = i_a$$



(a)



(b)

$$\frac{1}{s}, \frac{K_T}{Js+f}\ \text{and}\ \frac{1}{L_a s + R_a}$$

$$\dot{x}_1 = x_2$$

$$J\dot{x}_2 + f x_2 = K_T x_3$$

$$V_a - K_b x_2 = R_a x_3 + L_a \dot{x}_3$$

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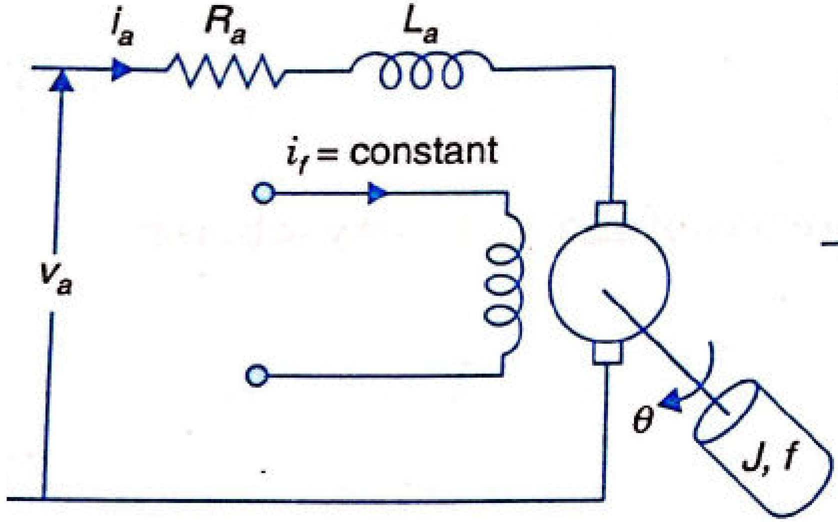
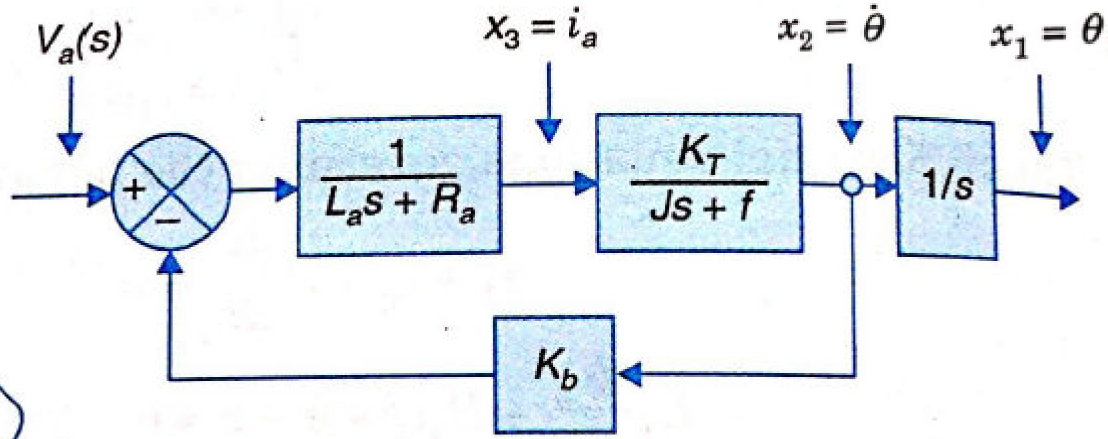


(a)



(b)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -f/J & K_T/J \\ 0 & -K_b/L_a & -R_a/L_a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/L_a \end{bmatrix} v_a \qquad y = \theta = x_1 = [1 \quad 0 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$