

College of Computing and Information Technology



Lecturer: Dr. Nahla Belal
Course: Computing Algorithms (CS312)
TA:



Sheet 1

1. Fill in the following table by selecting one of the following: $f(n) = O(g(n))$, $f(n) = \Omega(g(n))$, or $f(n) = \Theta(g(n))$. ($\log n = \log_2 n$).

$f(n)$	$g(n)$	
n^3	n^6	
n^3	$n^3 \log n$	
n^3	$2^{n \log n}$	
n^3	$n2^n$	
n^3	2^{2^n}	
n^6	$n^3 \log n$	
n^6	$2^{n \log n}$	
n^6	$n2^n$	
n^6	2^{2^n}	
$n^3 \log n$	$2^{n \log n}$	
$n^3 \log n$	$n2^n$	
$n^3 \log n$	2^{2^n}	
$2^{n \log n}$	$n2^n$	
$2^{n \log n}$	2^{2^n}	
$n2^n$	2^{2^n}	

2. Prove the following:

- $\frac{n^2}{10} + 2^n = O(2^n)$
- $21 + \frac{1}{n} = O(1)$
- $10 \log 3^n = O(n)$
- $\log n^3 = O(\log n)$
- $(O(n) + 10^2) = O(n)$

3. What is the time complexity of the following algorithms:

```
a. for(int i =0 ; i < =n ; i++)
    for(int j =1; j<= i * i; j++)
        if (j % i == 0)
            for(int k = 0; k<j; k++)
                sum++;
```

Ans: $O(n^4)$

```
b. int x=0;
   for(int i=4*n; i>=1; i--)
       x=x+2*i;
```

Ans: $O(n)$

```
c. int z=0;
   int x=0;
   for (int i=1; i<=n; i=i*3){
       z = z+5;
       z++;
       x = 2*x;
   }
```

Ans: $O(\log_3 n)$

```
d. int y=0;
   for(int j=1; j*j<=n; j++)
       y++;
```

Ans: $O(n^{1/2})$

```
e. int b=0;
   for(int i=n; i>0; i--)
       for(int j=0; j<i; j++)
           b=b+5;
```

Ans: $O(n^2)$

```
f. int y=1;
   int j=0;
   for(j=1; j<=2n; j=j+2)
       y=y+i;
   int s=0;
   for(i=1; i<=j; i++)
       s++;
```

Ans: $O(n)$

```

g. int b=0;
   for(int i=0; i<n; i++)
       for(int j=0; j<i*n; j++)
           b=b+5;

```

Ans: $O(n^3)$

```

h. int x=0;
   for(int i=1; i<=n; i=i*3){
       if(i%2 != 0)
           for(int j=0; j<i; j++)
               x++;
   }

```

Ans: $O(n)$

```

i. int t=0;
   for(int i=1; i<=n; i++)
       for(int j=0; j*j<4*n; j++)
           for(int k=1; k*k<=9*n; k++)
               t++;

```

Ans: $n*2(n^{1/2})*3(n^{1/2}) = O(n^2)$

```

j. int a = 0;
   int k = n*n;
   while(k > 1){
       for (int j=0; j<n*n; j++)
           { a++; }
       k = k/2;
   }

```

Ans: $\log(n^2)*n^2 = O(n^2 \log n)$

```

k. int i=0, j=0, y=0, s=0;
   for(j=0; j<n+1; j++)
       y=y+j;
   for(i=1; i<=y; i++)
       s++;

```

Ans: $O(n^2)$

4. What is the time complexity of the following algorithm:

```

1   m = a
2   i = 1
3   while ( i <= n ) do
4       m = m * b
5       for i = 1 to m do
6           S
7       end for
8   end while

```

9 **end**

S is $O(1)$.

5. Describe an $O(n \log n)$ -time algorithm that, given a set S of n integers and another integer k , determines whether or not there exists two elements in S whose sum is exactly k .
6. Describe an $O(\log n)$ -time algorithm that finds x^n .
7. Describe an algorithm that performs matrix transposition for an $n \times n$ matrix. Transposition is defined as interchange of the elements a_{ij} and a_{ji} , for $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, n-1$. Analyze the time and space complexity of your algorithm.