Arab Academy for Science, Technology and Maritime Transport
College of Engineering and Technology
Department of Basic and Applied Science

Mathematics (2) BA124

Lectures No.1-2

Revision-Definition of Indefinite Integrals

Standard Integrals

Integrals of the form $\int \frac{F'(x)}{F(x)}dx$ and $\int \frac{F'(x)}{\sqrt{F(x)}}dx$

Integration of Trigonometric

and

Exponential Functions

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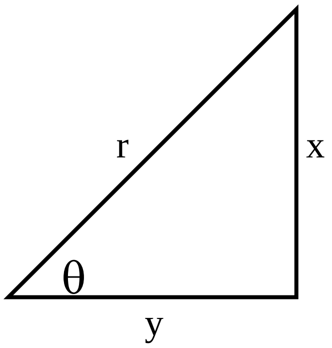
Revision

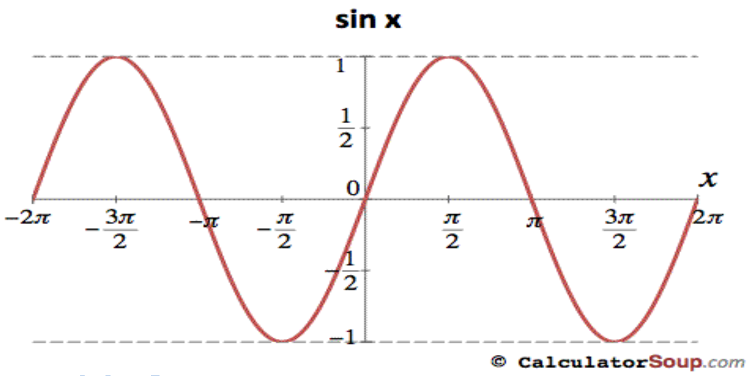


(I) Pythagoras' theorem:

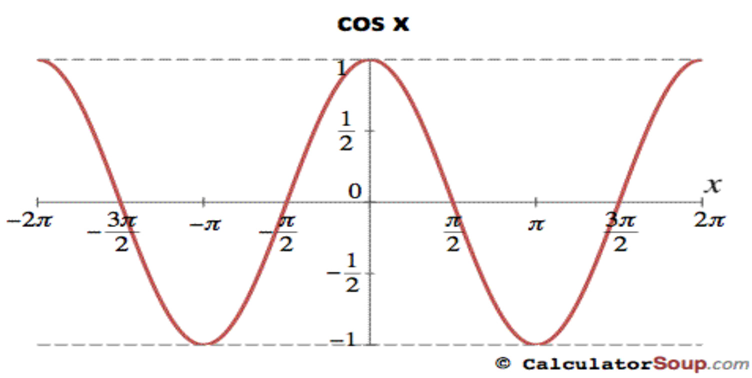
$$x^2 + y^2 = r^2$$

(II) Trigonometric Functions:

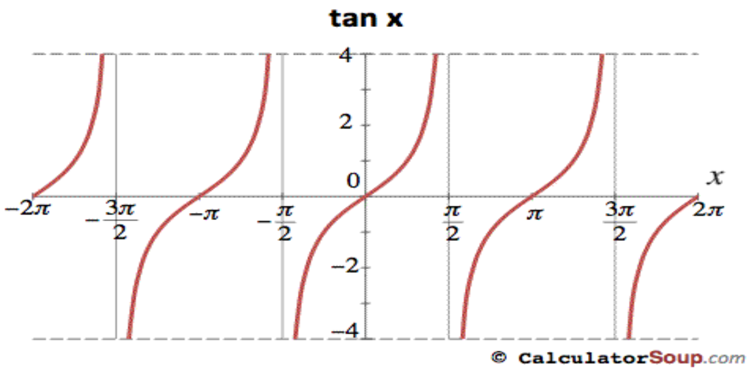
(1) $\sin\theta = \dfrac{x}{r}$



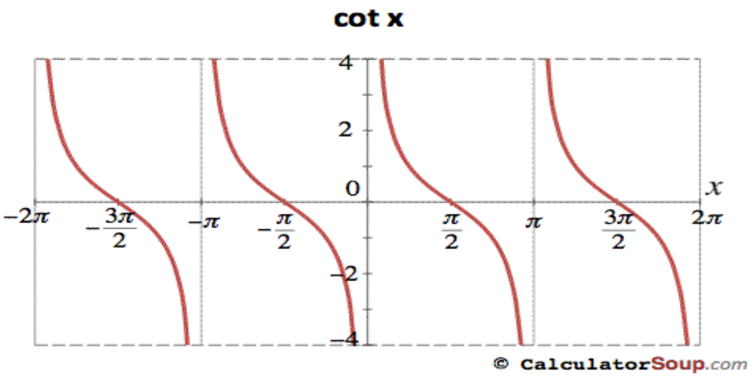
(2) $\cos\theta = \dfrac{y}{r}$



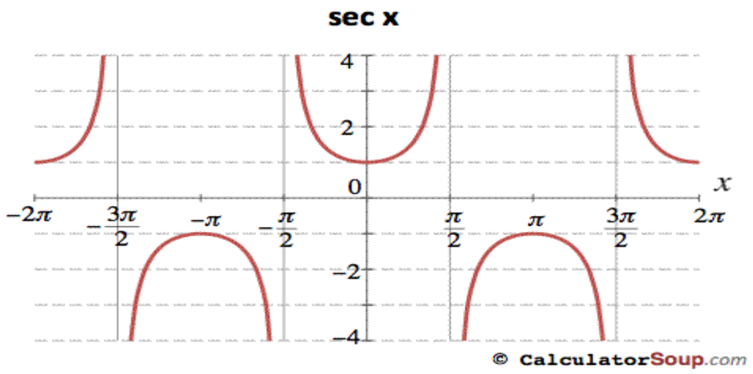
(3) $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{x}{y}$



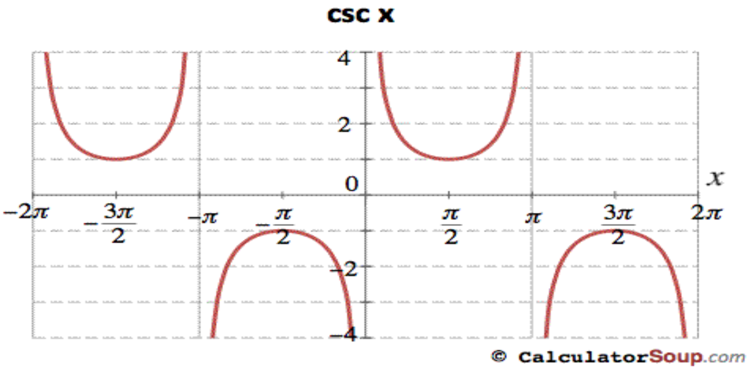
(4) $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta} = \dfrac{y}{x}$



(5) $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{r}{y}$



(6) $\mathbf{cosec\,\theta = \frac{1}{\sin\theta} = \frac{r}{x}}$



(III)Trigonometry

(1) $\cos^2\theta + \sin^2\theta = 1$

$\div \cos^2\theta$ $\quad$ $\div \sin^2\theta$

(2) $1 + \tan^2\theta = \sec^2\theta$

(3) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

(4) $\sin 2\theta = 2\sin\theta\cos\theta$

(5) $\sin 6\theta = 2\sin 3\theta\cos 3\theta$

(6) $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

(7) $\cos^2\theta = \frac{1}{2}[1 + \cos 2\theta]$

(8) $\sin^2\theta = \frac{1}{2}[1 - \cos 2\theta]$

(9) $\cos^2\theta - \sin^2\theta = \cos 2\theta$

(IV) Hyperbolic Functions:

(1) $$\sinh ax = \frac{e^{ax} - e^{-ax}}{2}$$

(2) $$\cosh ax = \frac{e^{ax} + e^{-ax}}{2}$$

(3) $$\tanh ax = \frac{\sinh ax}{\cosh ax} = \frac{e^{ax} - e^{-ax}}{e^{ax} + e^{-ax}}$$

(4) $$\coth ax = \frac{1}{\tanh ax} = \frac{\cosh ax}{\sinh ax} = \frac{e^{ax} + e^{-ax}}{e^{ax} - e^{-ax}}$$

(5) $$\operatorname{sech} ax = \frac{1}{\cosh ax} = \frac{2}{e^{ax} + e^{-ax}}$$

(6) $$\operatorname{cosech} ax = \frac{1}{\sinh ax} = \frac{2}{e^{ax} - e^{-ax}}$$

(V) Differentiation:

y	$y' = \frac{dy}{dx}$
$y = u^n$	$y' = nu^{n-1}u'$
$y = \sin u$	$y' = \cos u.u'$
$y = \cos u$	$y' = -\sin u.u'$
$y = \tan u$	$y' = \sec^2 u.u'$
$y = \cot u$	$y' = -\operatorname{cosec}^2 u.u'$
$y = \sec u$	$y' = \sec u.\tan u.u'$
$y = \operatorname{cosec} u$	$y' = -\operatorname{cosec} u.\cot u.u'$
$y = \sin^{-1} u$	$y' = \frac{1}{\sqrt{1-u^2}}.u'$
$y = \cos^{-1} u$	$y' = -\frac{1}{\sqrt{1-u^2}}.u'$
$y = \tan^{-1} u$	$y' = \frac{1}{1+u^2}.u'$
$y = \cot^{-1} u$	$y' = -\frac{1}{1+u^2}.u'$
$y = \sec^{-1} u$	$y' = \frac{1}{u\sqrt{u^2-1}}.u'$

$y = \operatorname{cosec}^{-1} u$	$y' = -\dfrac{1}{u\sqrt{u^2-1}}.u'$
$y = e^u$	$y' = e^u.u'$
$y = a^u$	$y' = a^u.u'.\ln a$
$y = \ln u$	$y' = \dfrac{1}{u}.u'$
$y = \log_a u$	$y' = \dfrac{1}{u}.u'.\dfrac{1}{\ln a}$
$y = \sinh u$	$y' = \cosh u.u'$
$y = \cosh u$	$y' = \sinh u.u'$
$y = \tanh u$	$y' = \operatorname{sech}^2 u.u'$
$y = \coth u$	$y' = -\operatorname{cosech}^2 u.u'$
$y = \operatorname{sech} u$	$y' = -\operatorname{sech} u.\tanh u.u'$
$y = \operatorname{cosech} u$	$y' = -\operatorname{cosech} u.\coth u.u'$
$y = \sinh^{-1} u$	$y' = \dfrac{1}{\sqrt{u^2+1}}.u'$
$y = \cosh^{-1} u$	$y' = \dfrac{1}{\sqrt{u^2-1}}.u'$
$y = \tanh^{-1} u$	$y' = \dfrac{1}{1-u^2}.u'$
$y = \coth^{-1} u$	$y' = \dfrac{1}{1-u^2}.u'$
$y = \operatorname{sech}^{-1} u$	$y' = -\dfrac{1}{u\sqrt{1-u^2}}.u'$
$y = \operatorname{cosech}^{-1} u$	$y' = -\dfrac{1}{u\sqrt{1+u^2}}.u'$

Integration

Integration is the reverse of differentiation. In differentiation, we start with a function and proceed to find its differential coefficient. In integration, we start with the differential coefficient and have to work back to find the function which it has been derived.

Standard Integrals:

Every differential coefficient, when written in reverse, gives us an integral.

Say, $\frac{dy}{dx} = F(x)$, from which $dy = F(x)dx$

The function $F(x)$ is called the differential coefficient

You know that $d(y + C) = dy$, since C is a constant.
So,

$$d(y + C) = F(x)dx$$

Integrate both sides yields

$$y + C = \int F(x)dx$$

In other word,

$$y = \int F(x)dx + C$$

(C is a constant. You can write it C or $-C$)

where,

y : is the antiderivative of $F(x)$

$\int$: is the integration symbol

$F(x)$: is the differential coefficient

x : is the variable of integration

C : is the constant of integration

What about the constant of integration "C"?

You know that $\dfrac{d}{dx}\left(x^4 + 5x + 3\right) = 4x^3 + 5$

If we had to find $\int \left(4x^3 + 5\right)dx$; without knowing the past history of the function, we should have no indication of the size of the constant term involved. All we can do is to indicate the constant term by a symbol C.

So,

$$\int \left(4x^3 + 5\right)dx = x^4 + 5x + C$$

Basic Rules:

(1) $\int d\square = \square + C$

(2) $\int K\,d\square = K\int d\square = K\square + C$,   *i.e.* $\int 2\,dx = 2x + C$

(3) $\int x^n dx = \dfrac{x^{n+1}}{n+1} + C$, provided that $n \neq -1$

$$\textit{i.e.} \int x^4\, dx = \frac{x^5}{5} + C$$

If $n = -1$, rule No.3 can be written in the following form

(4) $\int x^{-1}dx = \int \dfrac{1}{x}\,dx = \ln|x| + C$

In general,

(5) $\int \square^n d\square = \dfrac{\square^{n+1}}{n+1} + C$, provided that $n \neq -1$

I think you know that, $\int \cos x \, dx = \sin x + C.$

What about $\int \cos x \, d\cos x$ ***?***

Look at the integration variable. It's $\cos x$ not x.

So, by applying rule No.5, we get

$$\int \cos x \, d\cos x = \frac{(\cos x)^2}{2} + C$$

(6) Functions of a linear function of x

$$\int (ax+b)^n dx = \frac{1}{a}\int (ax+b)^n d(ax+b) = \frac{(ax+b)^{n+1}}{a(n+1)} + C$$

since, $d(ax+b) = adx$

Example:

$$\int (3x+4)^4 dx = \frac{(3x+4)^5}{3*5} + C = \frac{(3x+4)^5}{15} + C$$

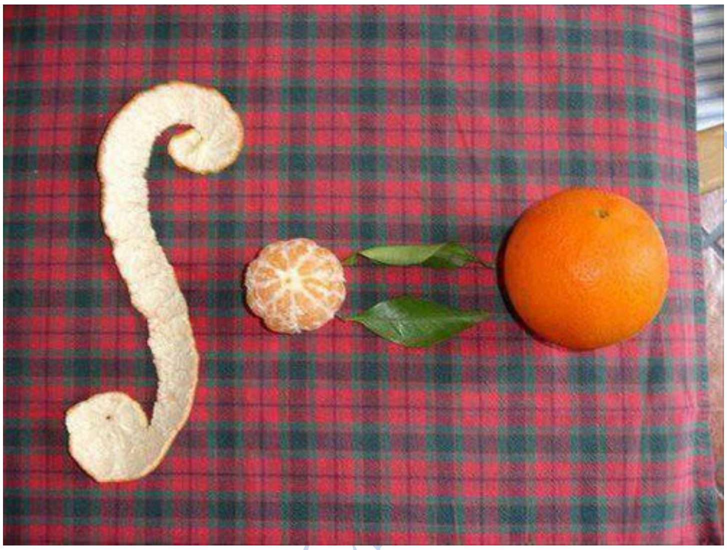
In general:

(7) $\int (a\,\square + b)^n d\,\square \quad = \frac{(a\,\square + b)^{n+1}}{a(n+1)} + C$

(8) $\int [F(x) \pm g(x)]dx = \int F(x)dx \pm \int g(x)dx$

Note:

For integrals in the form $\int \frac{F(x)}{g(x)} dx$, the degree of $F(x)$ must be less than that of $g(x)$. If it is more than or equal the degree of $g(x)$, you have to make long division.



معنى Integration: الحصول علي الكل من الجزء

Sheet (1)

No.1 $$\int (x + x^2 + 3)\, dx$$

Solution: $$\int (x + x^2 + 3)\, dx = \frac{1}{2}x^2 + \frac{1}{3}x^3 + 3x + C$$

No.2 $$\int \frac{dx}{x\sqrt{x}}$$

Solution: $$\int \frac{dx}{x\sqrt{x}} = \int x^{-\frac{3}{2}} dx = \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + C = -\frac{2}{\sqrt{x}} + C$$

No.3 $$\int \left[\frac{1}{x^2} + \frac{4}{x\sqrt{x}} - 2\right] dx$$

Solution:

$$\int \left[\frac{1}{x^2} + \frac{4}{x\sqrt{x}} - 2\right] dx = \int \left(x^{-2} + 4x^{-\frac{3}{2}} - 2 \right) dx$$

$$= \frac{x^{-1}}{-1} + 4\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} - 2x + C = -\frac{1}{x} - \frac{8}{\sqrt{x}} - 2x + C$$

No.4 $$\int \left[\frac{1}{\sqrt[4]{x}} + \frac{3}{x}\right] dx$$

Solution:

$$\int \left[\frac{1}{\sqrt[4]{x}} + \frac{3}{x}\right] dx = \int \left(x^{-\frac{1}{4}} + \frac{3}{x} \right) dx = \frac{x^{\frac{3}{4}}}{\frac{3}{4}} + 3\ln|x| + C = \frac{4}{3}x^{\frac{3}{4}} + 3\ln|x| + C$$

No.5 $$\int \frac{(x+5)^2}{x}\,dx$$

Solution:

$$\int \frac{(x+5)^2}{x}\,dx = \int\left(\frac{x^2+10x+25}{x}\right)dx = \int\left(x+10+\frac{25}{x}\right)dx$$

$$= \frac{1}{2}x^2 + 10x + 25\ln|x| + C$$

No.6 $$\int\left[\frac{1}{x^3}-\frac{2}{x^2}\right]dx$$

Solution:

$$\int\left[\frac{1}{x^3}-\frac{2}{x^2}\right]dx = \int\left(x^{-3}-2x^{-2}\right)dx = \frac{x^{-2}}{-2} - 2\frac{x^{-1}}{-1} + C = -\frac{1}{2x^2} + \frac{2}{x} + C$$

No.7 $$\int (2x+5)^5\,dx$$

Solution:

$$\int (2x+5)^5\,dx = \frac{(2x+5)^6}{6*2} + C = \frac{(2x+5)^6}{12} + C$$

Example: What about $\int (2x^2+5)^2\,dx$?

Wrong Solution:

$$\int (2x^2+5)^2\,dx = \frac{(2x^2+5)^3}{3*2} + C = \frac{(2x^2+5)^3}{6} + C$$

You cannot apply rule No.6 because the integration variable is x *not* x^2

True Solution:

$$\int (2x^2+5)^2\,dx = \int\left(4x^4+20x^2+25\right)dx = \frac{4}{5}x^5 + \frac{20}{3}x^3 + 25x + C$$

Rules:

(1) $$\int \frac{F'(x)}{F(x)}\,dx = \ln|F(x)| + C$$

Recall: $\frac{d}{dx}(\ln x) = \frac{1}{x}$ ⇒ $\therefore \int \frac{1}{x}\,dx = \ln|x| + C$

$\frac{d}{dx}(x) = 1$

(2) $$\int \frac{F'(x)}{\sqrt{F(x)}}\,dx = 2\sqrt{F(x)} + C$$

Sheet (2):

No.1 $$\int \frac{e^x}{1+e^x}\,dx$$

Solution: $$\int \frac{e^x}{1+e^x}\,dx = \ln|1+e^x| + C$$

Since: $\frac{d}{dx}(1+e^x) = e^x$

No.2 $$\int \frac{x^2}{x^3+5}\,dx$$

Solution: $$\int \frac{x^2}{x^3+5}\,dx = \frac{1}{3}\int \frac{3x^2}{x^3+5}\,dx = \frac{1}{3}\ln|x^3+5| + C$$

No.3 $$\int \frac{x}{\sqrt{x^2-3}}\,dx$$

Solution: $$\int \frac{x}{\sqrt{x^2-3}}\,dx = \frac{1}{2}\int \frac{2x}{\sqrt{x^2-3}}\,dx = \frac{1}{2}.2\sqrt{x^2-3} + C = \sqrt{x^2-3} + C$$

No.4 $\int \frac{dx}{x\ln x}$

Solution: $$\int \frac{dx}{x\ln x} = \int \frac{\frac{1}{x}}{\ln x}dx = \ln\left|\ln x\right| + C$$

No.5 $\int \frac{dx}{x\ln x \ln\ln x}$

Solution: $$\int \frac{dx}{x\ln x\ln\ln x} = \int \frac{\frac{1}{x\ln x}}{\ln\ln x}dx = \ln\left|\ln\ln x\right| + C$$

No.6 $\int \frac{dx}{x\ln x^2}$

Solution: $$\int \frac{dx}{x\ln x^2} = \int \frac{dx}{2x\ln x} = \frac{1}{2}\int \frac{\frac{1}{x}}{\ln x} = dx = \frac{1}{2}\ln\left|\ln x\right| + C$$

No.7 $\int \frac{\tan^2 x}{\tan x - x}dx$

Solution: $$\int \frac{\tan^2 x}{\tan x - x}dx = \int \frac{\sec^2 x - 1}{\tan x - x}dx = \ln\left|\tan x - x\right| + C$$

No.8 $\displaystyle\int \frac{dx}{\cos^2 x\sqrt{3\tan x-1}}$

Solution:

$$\int \frac{dx}{\cos^2 x\sqrt{3\tan x-1}} = \frac{1}{3}\int \frac{3\sec^2 x}{\sqrt{3\tan x-1}}dx = \frac{1}{3}.2\sqrt{3\tan x-1}+C$$

$$= \frac{2}{3}\sqrt{3\tan x-1}+C$$

No.9 $\displaystyle\int \frac{\sin 2x}{a^2+b^2\sin^2 x}\,dx$

Solution: $\displaystyle\frac{d}{dx}\left(a^2+b^2\sin^2 x\right) = 2b^2\sin x\cos x = b^2\sin 2x$

$$\therefore \int \frac{\sin 2x}{a^2+b^2\sin^2 x}\,dx = \frac{1}{b^2}\int \frac{b^2\sin 2x}{a^2+b^2\sin^2 x}\,dx = \frac{1}{b^2}\ln\left|a^2+b^2\sin^2 x\right|+C$$

No.10 $\displaystyle\int \frac{\sin 4x}{\sqrt{5+3\cos^2 2x}}\,dx$

Solution: $\displaystyle\frac{d}{dx}\left(5+3\cos^2 2x\right) = 6\cos 2x.(-2\sin 2x) = -6\sin 4x$

$$\therefore \int \frac{\sin 4x}{\sqrt{5+3\cos^2 2x}}dx = -\frac{1}{6}\int \frac{-6\sin 4x}{\sqrt{5+3\cos^2 2x}}dx$$

$$= -\frac{1}{6}.2\sqrt{5+3\cos^2 2x}+C = -\frac{1}{3}\sqrt{5+3\cos^2 2x}+C$$

No.11 $\int \frac{\ln x}{x\left[1+(\ln x)^2\right]}dx$

Solution: $\frac{d}{dx}\left(1+(\ln x)^2\right)=2\ln x.\frac{1}{x}$

$$\therefore \int \frac{\ln x}{x\left[1+(\ln x)^2\right]}dx=\frac{1}{2}\int \frac{2.\frac{\ln x}{x}}{\left[1+(\ln x)^2\right]}dx=\frac{1}{2}\ln\left|1+(\ln x)^2\right|+C$$

No.12 $\int \frac{dx}{1+e^x}$

Solution: $*\frac{e^{-x}}{e^{-x}}$

$$\therefore \int \frac{dx}{1+e^x}=\int \frac{e^{-x}}{e^{-x}+1}dx=-\int \frac{-e^{-x}}{e^{-x}+1}dx=-\ln\left|e^{-x}+1\right|+C$$

No.13 $\int \frac{\sin 4x}{1+\cos^2 2x}dx$

Solution: $\frac{d}{dx}\left(1+\cos^2 2x\right)=2\cos 2x.(-2\sin 2x)=-2\sin 4x$

$$\therefore \int \frac{\sin 4x}{(1+\cos^2 2x)}dx=-\frac{1}{2}\int \frac{-2\sin 4x}{(1+\cos^2 2x)}dx=-\frac{1}{2}.\ln\left|(1+\cos^2 2x)\right|+C$$

Integration of trigonometric functions:

Recall that, $\int (\cos x)d(\cos x) = \frac{1}{2}(\cos x)^2 + C$, but

$\int \cos x dx = \sin x + C$.

i.e

$$\int \cos \square \, d \square = \sin \square + C$$

$$\int \cos kx dx = \frac{1}{k}\int \cos kx \, d\,kx = \frac{\sin kx}{k} + C$$

Rules:

(1) $\int \cos ku \, du = \frac{\sin ku}{k} + C$

(2) $\int \sin ku \, du = -\frac{\cos ku}{k} + C$

(3) $\int \sec^2 ku \, du = \frac{\tan ku}{k} + C$

(4) $\int \operatorname{cosec}^2 ku \, du = -\frac{\cot ku}{k} + C$

(5) $\int \sec ku \tan ku \, du = \frac{\sec ku}{k} + C$

(6) $\int \operatorname{cosec} ku \cot ku \, du = -\frac{\operatorname{cosec} ku}{k} + C$

(7) $$\int \cot ku\, du = \frac{\ln|\sin ku|}{k} + C$$

How???????

$$\int \cot x\, dx = \int \frac{\cos x}{\sin x} dx = \ln|\sin x| + C$$

(8) $$\int \tan ku\, du = \frac{\ln|\sec ku|}{k} + C$$

How???????

$$\int \tan x\, dx = -\int \frac{-\sin x}{\cos x} dx = -\ln|\cos x| + C$$

$$= \ln(|\cos x|)^{-1} + C = \ln|\sec x| + C$$

(9) $$\int \sec ku\, du = \frac{\ln|\sec ku + \tan ku|}{k} + C$$

How???????

$$\int \sec x\, dx = \int \sec x . \frac{\sec x + \tan x}{\sec x + \tan x} dx = \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} dx$$

$$= \ln|\sec x + \tan x|$$

(10) $$\int \operatorname{cosec} ku\, du = \frac{\ln|\operatorname{cosec} ku - \cot ku|}{k} + C$$

How???????

$$\int \operatorname{cosec} x\, dx = \int \operatorname{cosec} x . \frac{\operatorname{cosec} x - \cot x}{\operatorname{cosec} x - \cot x} dx = \int \frac{\operatorname{cosec}^2 x - \operatorname{cosec} x \cot x}{\operatorname{cosec} x - \cot x} dx$$

$$= \ln|\operatorname{cosec} x - \cot x| + C$$

No.14 $\int \cos 5x\, dx$

Solution: $\int \cos 5x\, dx = \frac{\sin 5x}{5} + C$

No.15 $\int \frac{\sin x + \cos x}{\cos x} dx$

Solution: $\int \frac{\sin x + \cos x}{\cos x} dx = \int (\tan x + 1)\, dx = \ln|\sec x| + x + C$

No.16 $\int \sin(2x-3)\, dx$

Solution: $\int \sin(2x-3)\, dx = -\frac{\cos(2x-3)}{2} + C$

Integration of Exponential functions:

Recall that, $\frac{d}{dx}(e^x) = e^x \Longrightarrow \int e^x dx = e^x + C$

Recall that, $\frac{d}{dx}(a^x) = a^x . \ln a \Longrightarrow \int a^x dx = \frac{a^x}{\ln a} + C$

Rules:

(1) $\int e^{\square}\, d\square = e^{\square} + C$

(2) $\int e^{kx} dx = \frac{1}{k}\int e^{kx} d\,kx = \frac{e^{kx}}{k} + C$

(3) $\int a^{kx} dx = \frac{1}{k}\int a^{kx} d\,kx = \frac{a^{kx}}{k \ln a} + C$

No.17 $\int (e^{4x} + 5^{3x})dx$

Solution: $$\int (e^{4x} + 5^{3x})dx = \frac{e^{4x}}{4} + \frac{5^{3x}}{3\ln 5} + C$$

No.18 $\int (e^{2x} + e^{-2x})^2 \, dx$

Solution: $$\int (e^{2x} + e^{-2x})^2 \, dx = \int (e^{4x} + 2 + e^{-4x}) \, dx = \frac{e^{4x}}{4} + 2x + \frac{e^{-4x}}{-4} + C$$

No.19 $\int \frac{2^{x+1} - 5^{x+1}}{10^x} \, dx$

Solution:

$$\int \frac{2^{x+1} - 5^{x+1}}{10^x} \, dx = \int \left(\frac{2^{x+1}}{10^x} - \frac{5^{x+1}}{10^x} \right) dx = \int \left(\frac{2.2^x}{10^x} - \frac{5.5^x}{10^x} \right) dx$$

$$= \int \left(2\left(\frac{1}{5}\right)^x - 5\left(\frac{1}{2}\right)^x \right) dx = 2.\frac{\left(\frac{1}{5}\right)^x}{\ln\left|\frac{1}{5}\right|} - 5.\frac{\left(\frac{1}{2}\right)^x}{\ln\left|\frac{1}{2}\right|} + C$$