

## Raman Gain and Raman Gain Coefficient for $\text{SiO}_2$ , $\text{GeO}_2$ , $\text{B}_2\text{O}_3$ and $\text{P}_2\text{O}_5$ Glasses

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### Abstract

Fiber Raman amplification using the transmission line is a promising technology to increase the repeater distance as well as the capacity of the communication systems. Because of the growing importance of fiber Raman amplification, it is desired to predict the magnitude and shape of Raman gain coefficient and Raman gain spectrum for different materials. In this paper, Raman cross section, Raman gain coefficient and Raman gain is investigated for  $\text{SiO}_2$ ,  $\text{GeO}_2$ ,  $\text{B}_2\text{O}_3$  and  $\text{P}_2\text{O}_5$ .

**Keywords:** Raman fiber amplifier (FRA), stimulated Raman scattering (SRS), Raman gain coefficient, Raman gain.

### 1. Introduction

The amplification of optical signals by stimulated Raman scattering in Raman fiber amplifiers is an attractive proposition for future telecommunications systems [1]. There have been a number of theoretical and experimental investigations of this phenomenon in recent years using silica based single mode optical fibers [2]. Raman amplification offers the prospect of high gain, high saturation power and polarization insensitivity in a fiber device. Combined with a smooth gain spectrum and relatively wide bandwidth, these properties are ideal for use as a power amplifier in high speed, wavelength division multiplexed transmission systems.

Different materials; namely,  $\text{SiO}_2$ ,  $\text{GeO}_2$ ,  $\text{B}_2\text{O}_3$  and  $\text{P}_2\text{O}_5$  are considered to model and investigate Raman amplifier properties including: Raman cross section, Raman gain coefficient and Raman gain.

### 2. Theory

When a weak signal is launched to an optical fiber with a strong pump, it will be amplified due to the Raman interaction between the pump and the signal, i.e. SRS. In the continuous wave (CW) case, the signal amplification is described by the following equations in terms of power transfer from the pump to the signal [3]

$$\frac{dP_s}{dz} = g_s P_p P_s - \alpha_s P_s, \quad (1)$$

and

$$\frac{dP_p}{dz} = \frac{\omega_p}{\omega_s} g_u P_p P_s - \alpha_p P_p. \quad (2)$$

Raman interaction transfers the pump power,  $P_p$ , to the signal,  $P_s$ .  $P_p$  and  $P_s$  are the powers of waves at frequencies  $\omega_p$  and  $\omega_s$ , and both are functions of propagation distance  $z$ . The coefficients  $\alpha_p$  and  $\alpha_s$  stand for the fiber loss at  $\omega_p$  and  $\omega_s$ , and  $g_s$  represents the Raman gain coefficient in a fiber. The effective area for Raman gain is determined by mode size and the overlap between Stokes modes defined as [4]

$$A_{\text{eff}} = 2\pi \frac{\int \psi_p^2 r dr \int \psi_s^2 r dr}{\int \psi_p^2 \psi_s^2 r dr} \quad (3)$$

where  $\psi_p, \psi_s$  represent the mode fields at the pump and Stokes wavelengths.

### 2.1 Raman Cross Section

A weak Stokes signal launched into a fiber with a stronger pump will be amplified due to SRS as discussed before. The amplification of the signal is described through [5]

$$P_s(L) = P_s(0) \exp \left[ \frac{g_R(\nu) P_p L_{\text{eff}}}{K A_{\text{eff}}} - \alpha_s L \right] \quad (4)$$

and

$$g_R(\nu) = \sigma(\nu) \frac{\lambda_s^3}{c^2 h n(\nu)^2} \quad (5)$$

where  $c$  is the speed of light in free space,  $h$  is Planck's constant,  $\lambda_s$  is the Stokes wavelength,  $n(\nu)$  is the frequency dependent refractive index and  $K$  is the polarization factor which is 1 if the pump and signal waves are polarization matched or 2 if depolarized.

The spontaneous zero Kelvin Raman cross section  $\sigma_0(\nu)$  is defined as the ratio of radiated power at the Stokes wavelengths to the pump power at temperature 0 K which can be obtained with the measured Raman cross-section  $\sigma_\nu(\nu)$  at temperature  $T$  K and the thermal population factor  $N(\Delta\nu, T)$  [6]

$$\sigma_0(\nu) = \sigma_\nu(\nu) / (N(\Delta\nu, T) + 1) \quad (6)$$

and

$$N(\Delta\nu, T) = \frac{1}{\exp \left[ \frac{hc\Delta\nu}{K_B T} - 1 \right]} \quad (7)$$

The change in Raman cross-section with  $\text{GeO}_2$ -doping mol% ( $x_{\text{GeO}_2}$ ) in pure silica is given by [7]

$$\sigma_0(x_{\text{GeO}_2}, \nu) = \sigma_0(\text{SiO}_2, \nu) + C(\nu) \cdot x_{\text{GeO}_2} \quad (8)$$

where  $\sigma_0(\text{SiO}_2, \nu)$  and  $\sigma_0(x_{\text{GeO}_2}, \nu)$ , respectively, represent the Raman cross-sections of pure silica and Ge-doped silica, normalized with respect of the peak Raman cross-section of pure silica and  $C(\nu)$  is a linear regression factor whose values at different frequency shifts are obtained from Ref. [8].

### 2.2 Linear Regression Factor

Now, if  $\sigma_0(x_{\text{GeO}_2}, \nu)$  and  $\sigma_0(\text{SiO}_2, \nu)$  represent the non-normalized cross-sections and  $\sigma_\nu(\text{SiO}_2, \nu)$  represents the silica peak cross section, then Eq. (8) can be written as [9]

$$\frac{\sigma_0(x_{\text{GeO}_2}, \nu)}{\sigma_{0,p}(\text{SiO}_2, \nu)} = \frac{\sigma_0(\text{SiO}_2, \nu)}{\sigma_{0,p}(\text{SiO}_2, \nu)} + C(\nu) \cdot x_{\text{GeO}_2} \quad (9)$$

### 2.3 Raman Gain Coefficient

Based on Eq. (5), the Raman gain coefficients  $g_R(\text{SiO}_2, \nu)$  and  $g_R \sigma_0(x_{\text{GeO}_2}, \nu)$  for pure silica and for  $\text{GeO}_2$ -doped silica can be respectively written as [10]

$$g_R(x_{\text{GeO}_2}, \nu) = \sigma_0(x_{\text{GeO}_2}, \nu) \cdot \frac{\lambda_s^3}{c^2 h n^2} \quad (10)$$

and

$$g_R(\text{SiO}_2, \nu) = \sigma_0(\text{SiO}_2, \nu) \cdot \frac{\lambda_s^3}{c^2 h n^2} \quad (11)$$

But, ideally there would be a  $C_{\perp}(v)$  for the perpendicular gain. As an approximation, and following Eq. (9), the perpendicular gain of GeO<sub>2</sub>-doped silica approximation would be

$$\sigma_{0\perp}(x_{GeO_2}, v) = \sigma_{0\perp}(SiO_2, v) + C_{\perp}(v) \cdot x_{GeO_2} \cdot \sigma_{0\perp}(SiO_2, v), \quad (12)$$

where  $C_{\perp}(v)$  would be normalized to its peak value. Thus, the total Raman gain coefficient of GeO<sub>2</sub>-doped silica for the average of random polarization states becomes [11]

$$g_{TOT}(x_{GeO_2}, v) = \frac{n_2^2}{2n_1^2} [A + B], \quad (13)$$

with

$$A = g_{\parallel}(SiO_2, v) + g_{\perp}(SiO_2, v), \quad (14)$$

and

$$B = [C_{\parallel}(v) + C_{\perp}(v)] x_{GeO_2} \cdot g_{\parallel}(SiO_2, v) \cdot \frac{\lambda_s^3}{\lambda_{speak}^3}. \quad (15)$$

Also, because the Raman gain coefficient varies linearly with signal frequency, the depolarized Raman gain coefficient obtained through Eq. (13) at a Stokes wavelength of 1  $\mu\text{m}$  needs to be converted into the one at actual Stokes wavelengths  $\lambda_s$  by dividing the Raman gain coefficient by the Stokes wavelength. So, Eq. (13) can be modified to [12]

$$g_{TOT}(x_{GeO_2}, v) = \frac{n_2^2}{2n_1^2} \left[ \frac{A}{\lambda_s} + \frac{B}{\lambda_{speak}} \right]. \quad (16)$$

## 2.4 Raman Gain

In the absence of saturation, the gain in dB of an optical fiber of length  $L$ , being pumped by a laser with power  $P$ , can be expressed as [13]

$$G(v) = 4.34 \left[ \frac{g(v)PL}{A_{\text{eff}}} (1 - e^{-\alpha L}) - \alpha(v)L \right], \quad (17)$$

where  $\alpha$  and  $\alpha(v)$  are the optical attenuations at the pump and signal wavelengths which differ in energy by an amount  $h\nu$ . The effective area,  $A_{\text{eff}}$ , of the core of a step-index single mode fiber in the Gaussian approximation is given by  $\pi[\omega^2/2 + \omega'(v)^2/2]$  where  $\omega$  and  $\omega'(v)$  are the mode field radius at the pump and signal wavelengths, respectively. These may be calculated from the wavelength of the LP<sub>11</sub> cutoff for a step-index fiber.

## 3. Results and Discussion

Using Eq. (8), Raman cross section spectrum in the wavelength range 1.5 - 1.6  $\mu\text{m}$  and over the frequency shift range from 0 - 700  $\text{cm}^{-1}$  is obtained for SiO<sub>2</sub>, GeO<sub>2</sub>, B<sub>2</sub>O<sub>3</sub> and P<sub>2</sub>O<sub>5</sub>. These materials have refractive indices 1.46, 1.60, 1.48 and 1.55 respectively. Raman gain coefficient spectrum is also studied over the same wavelength range using Eqs. (14) - (16) for the same materials. The Raman gain coefficient consists of parallel and perpendicular gain coefficients. This is followed by the study of Raman gain at different fiber lengths 10, 20, 30, 40 km, through Eq. (17), at a pump power 400 mW. The procedure of Raman gain calculation is then repeated at pump powers 350, 400, 450 and 500 mW at a constant fiber length of 20 km over the same wavelength range.

The numerical values used to perform calculations are the following [9]: the relative refractive index difference,  $\Delta = 0.016$ , the germania doping ratio,  $x = 0.0$ , the effective area,  $A = 1.96 \times 10^{-11} \text{ m}^2$ , the optical attenuation  $\alpha$  and  $\alpha(v)$  are taken 0.3 and 0.2 dB/km, respectively, pump power wavelength  $\lambda_{pump} = 1.54 \mu\text{m}$ , core diameter = 8.2  $\mu\text{m}$  and the wavelength range is 1.5 - 1.6  $\mu\text{m}$ .

### 3.1 SiO<sub>2</sub> Raman Amplifier Parameters

#### 3.1.1 Raman cross section spectrum

SiO<sub>2</sub> Raman cross section can be obtained with the measured Raman cross section at temperature T K and the thermal population factor  $N(\Delta\nu, T)$ , Eq.(7). The obtained results are displayed in Fig. 1.

The Raman cross section increases with the frequency shift reaching its peak value ( $1 \text{ cm}^2$ ) at  $440 \text{ cm}^{-1}$ , in consistence with the results found in Ref. [13]. This glass is suitable in fabricating a CW fiber Raman oscillator which was tunable from 230 to  $530 \text{ cm}^{-1}$ . The SiO<sub>2</sub> has the lowest refractive index and Raman cross section among the considered materials with values 1.46 and  $1 \text{ cm}^2$ , respectively. If Raman cross section is studied with wavelength, Fig. 2, it is found that it reaches its peak value ( $1 \text{ cm}^2$ ) at  $1.55 \mu\text{m}$  which is expected as this is the minimum loss wavelength.

#### 3.1.2 Raman gain coefficients spectrum

The SiO<sub>2</sub> Raman gain coefficient consists of parallel and perpendicular Raman gain coefficients. The perpendicular Raman gain coefficient is approximately zero. The SiO<sub>2</sub> Raman gain coefficient spectrum over wavelength range 1.5-1.6  $\mu\text{m}$  is shown in Fig. 3 reaching its peak value  $3.8 \times 10^{-14} \text{ m}^2/\text{W}$  at  $1.55 \mu\text{m}$  which is expected, corresponding to peak cross section.

There are many parameters required to calculate the Raman gain coefficient such as  $\Delta$ . The relative refractive index difference between the core and cladding is approximately constant which is equal to 0.03. There is no doping ratio and the linear regression factor varies over the frequency shift range from 0 to  $700 \text{ cm}^{-1}$  [7].

#### 3.1.3 Raman gain spectrum at different fiber lengths

The SiO<sub>2</sub> Raman gain is investigated at different fiber lengths as shown in Fig. 4 the (10, 20, 30 and 40 km) having corresponding gain peaks of (15, 30, 38 and 50 dB) and 3-dB bandwidths of (18, 15, 13 and 7 nm), respectively. It is interesting to note that the bandwidth of Raman amplifier decreases with the fiber length. This because at wavelengths away from the peak of Raman gain spectrum, as the Raman gain coefficient becomes smaller the effect of optical attenuation of the signal becomes more important with gain passing through a maximum value at shorter fiber lengths as the Raman gain coefficient decreases. For all the investigated lengths, the peak gain occurs (as expected) at  $1.55 \mu\text{m}$ .

#### 3.1.4 Raman gain spectrum at different pump power

Raman gain is calculated at different pump powers 350, 400, 450 and 500 mW at a constant fiber length of 20 km over wavelength range from 1.5 to 1.6  $\mu\text{m}$ . The results obtained are displayed in Fig. 5. It is clear that Raman gain increases with the pump power as expected from Eq. (17). SiO<sub>2</sub> Raman fiber amplifier has the largest gain among different materials. So, it is much interesting as a power amplifier.

### 3.2 GeO<sub>2</sub> Raman Amplifier Parameters

The same procedure is repeated for GeO<sub>2</sub>. A peak of  $7.4 \text{ cm}^2$  cross section is noticed at  $420 \text{ cm}^{-1}$  as shown in Fig. 6. This value of frequency shift corresponds to  $1.54 \mu\text{m}$  which is a low loss optical telecommunications window.

Raman gain coefficient is displayed over wavelength range 1.5 - 1.6  $\mu\text{m}$  in Fig. 7. It reaches its peak value  $2.7 \times 10^{-14} \text{ m}^2/\text{W}$  at  $1.54 \mu\text{m}$ , as expected. Raman gain is investigated for GeO<sub>2</sub> at different fiber lengths (10, 20, 30 and 40 km) at a constant pump power of 400 mW in Fig. 8.

GeO<sub>2</sub> Raman gain is studied at different pump powers (500, 450, 400 and 350 mW) in Fig. 9 showing an increase with the pumping power. The peak values are depicted as (17, 11, 8 and 5 dB) respectively at a constant fiber length of 20 km. It is interesting to note that for all the reported pump powers the peak gain occurs, as expected, at  $1.54 \mu\text{m}$  [7].

### 3.3 B<sub>2</sub>O<sub>3</sub> Raman Amplifier Parameters

B<sub>2</sub>O<sub>3</sub> is studied over the wavelength range 1.5 - 1.6 μm. Raman cross section is found reaching its peak value (4.7 cm<sup>2</sup>), while the gain coefficient also has its peak value, Fig. 11, (1.9×10<sup>-14</sup> m/W) at 1.56 μm. B<sub>2</sub>O<sub>3</sub> Raman gain over wavelength range from 1.5 to 1.6 μm at different fiber lengths (40, 30, 20 and 10 km) and constant pump power of 400 mW is shown in Fig. 12.

Figure 13 displays B<sub>2</sub>O<sub>3</sub> Raman gain at different pumping powers. It is obvious that Raman gain increases with the pump power while the 3-dB decreases. The pump powers used are 500, 450, 400 and 350 mW showing corresponding peaks of 13, 8, 4 and 3 dB and bandwidths of 11, 16, 18 and 20nm, respectively.

### 3.4 P<sub>2</sub>O<sub>5</sub> Raman Amplifier Parameters

Similarly, the previous procedure was repeated for the P<sub>2</sub>O<sub>5</sub> fiber Raman amplifier. It is found that the peak Raman cross section is 4.9 cm<sup>2</sup> at 640 cm<sup>-1</sup>, peak Raman gain coefficient is 1.93×10<sup>-14</sup> m/W at 1.59 μm and Raman gain peaks are 13, 9, 6 and 2 dB at 500, 450, 400 and 350 mW pump powers, respectively.

## 4. Conclusion

In this work, Raman cross section, Raman gain coefficient and Raman gain are modeled and investigated for different materials; namely, SiO<sub>2</sub>, GeO<sub>2</sub>, B<sub>2</sub>O<sub>3</sub> and P<sub>2</sub>O<sub>5</sub>. It is found that, SiO<sub>2</sub> and GeO<sub>2</sub> Raman amplifiers offer higher Raman gain and smaller bandwidths than other Raman amplifiers. So, both are suitable to use as power amplifiers. B<sub>2</sub>O<sub>3</sub> and P<sub>2</sub>O<sub>5</sub> Raman amplifiers offer larger bandwidths and smaller gain and hence, both are recommended in wavelength division multiplexing.

## 5. References

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Figure 1 Raman cross section vs frequency shift.

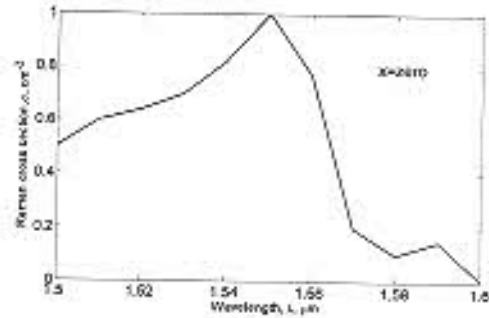


Figure 2 Raman cross section vs wavelength.

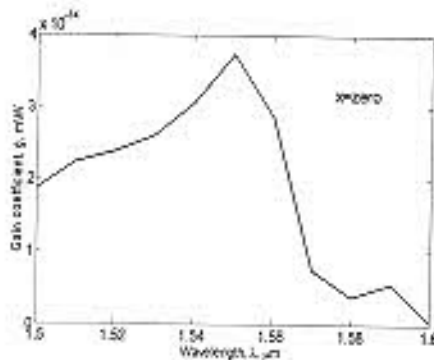


Figure 3 Raman gain coefficient vs wavelength.

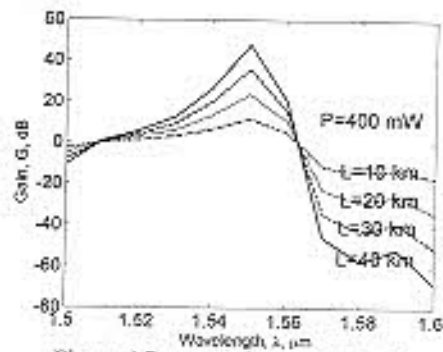


Figure 4 Raman gain vs wavelength.

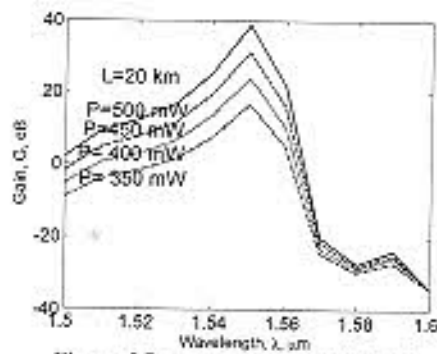


Figure 5 Raman gain vs wavelength.

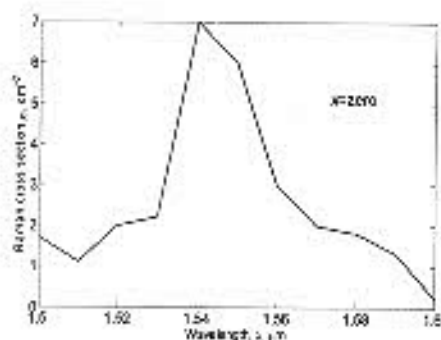


Figure 6 Raman cross section vs wavelength.

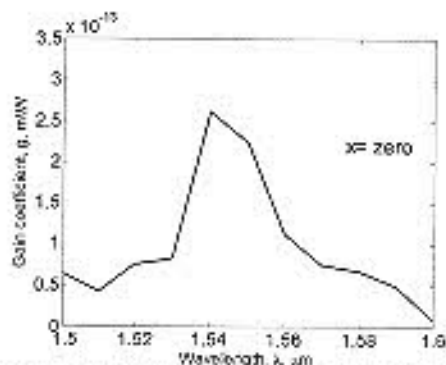


Figure 7 Raman gain coefficient Vs wavelength.

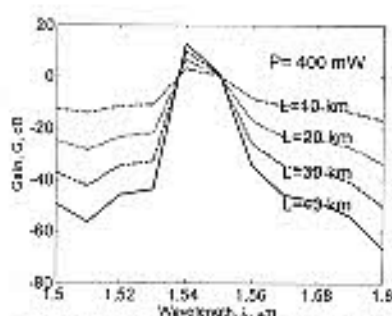


Figure 8 Raman gain vs wavelength.

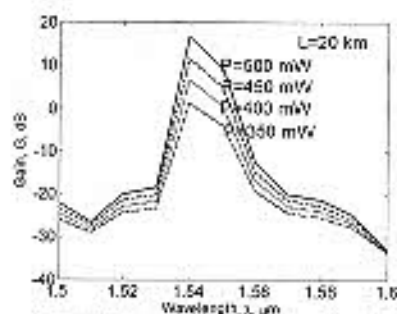


Figure 9 Raman gain vs wavelength.

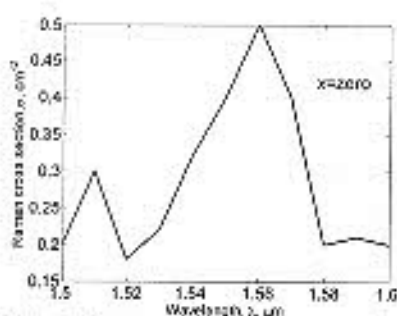


Figure 10 Raman cross section vs wavelength.

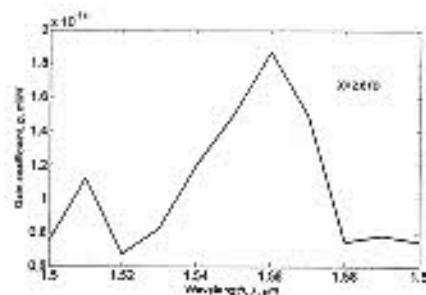


Figure 11 Raman gain coefficient vs wavelength.

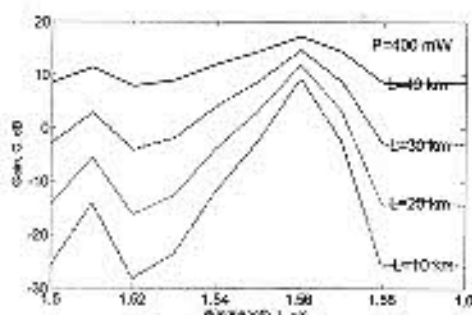


Figure 12 Raman gain vs wavelength.

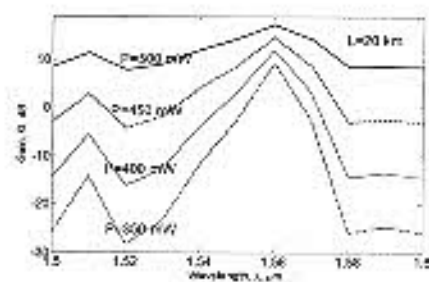


Figure 13 Raman gain vs wavelength.

## Raman Gain and Raman Gain Coefficient for SiO<sub>2</sub>, GeO<sub>2</sub>, B<sub>2</sub>O<sub>3</sub> and P<sub>2</sub>O<sub>5</sub> Glasses

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### المخلص العربي

تحدد قدرة جميع نظم الاتصالات تقريباً بالمدى الذي تنتشر موجة المعلومات خلا له قبل اضمحلالها وزوال القدرة على استقبالها بشكل غير مشوّش. وفي نظم الاتصالات البصرية بشكل خاص تعتبر عمليتا الامتصاص والتشتت للموجة الضوئية محدداً أساسياً للمدى المتاح من الأطوال الموجية. وعادة ما تقوم بالتغلب على عملية اضمحلال الموجة البصرية باستخدام المعيدات الكهروضوئية والتي كانت تعمل في بادئ الأمر على تحويل الموجة الضوئية المرسله إلى تيار كهربى يتم تكبيره بالمكبرات الإلكترونية ثم إعادة بثه محملاً بالموجة المقوّاة.

وتطور الأمر بعد ذلك إلى استخدام المكبرات البصرية (نون المرور على المرحلة الكهربائية). ويعد التكبير عن طريق مكبر رمان أحد طرق التكبير البصري مما يؤدي إلى زيادة عدد المسافات بين المعيدات ونقل كمية كبيرة من المعلومات وأيضاً زيادة سعة نظم الاتصالات. وتقوم فكرة مكبر الرمان على تكبير الموجات الضوئية المضمحلة باستخدام خاصية الانبعاث المستحث، وهي نفس الظاهرة التي يعتمد عليها عمل جهاز الليزر، وذلك دون الحاجة إلى عملية التحويل من وإلى النظام الكهربى.

وهذا البحث دراسة لمساحة مقطع مكبر رمان وقدرة التكبير ومعامل تكبير رمان مع الطول الموجى في المدى من 1.5 إلى 1.6 ميكرومتر. وقد تمت الدراسة لأربعة أنواع مختلفة من الزجاج وهي أكسيد سيلكون (SiO<sub>2</sub>) وأكسيد الجرمانيوم (GeO<sub>2</sub>) وأكسيد الباريوم (B<sub>2</sub>O<sub>3</sub>) وأكسيد الفوسفور (P<sub>2</sub>O<sub>5</sub>).

وقد وجد أن مكبرات أكسيد السيلكون ومكبر أكسيد الجرمانيوم تعطى أكبر معامل للتكبير ولكنها تعطي أقل نطاق من الطول الموجى، ولذا ينصح باستخدامها كمكبرات قدرة لموجة واحدة (أو عدد قليل جداً من الموجات ذات الطول الموجى المتقارب). بينما وجدت مكبرات أكسيد الباريوم وأكسيد الفوسفور تعطي تكبيراً أقل ونطاقاً أكبر من الطول الموجى، ولذلك ينصح استخدامها في نظم الاتصالات متعددة الأطوال الموجية (WDM).