

# Erbium Doped Fiber Amplifier in WDM Systems Using Different Glass Hosts

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## Abstract

Multichannel erbium-doped fiber amplifiers (EDFAs) are analyzed based on the rate equations with an analytical approximation model. Forward pumping is considered taking both amplified spontaneous emissions (ASE) scattering loss into account. The output signal power is calculated for ten multiplexed channels along the fiber. The optimum amplifier length which corresponds to the maximum output signal power is then studied showing how it is affected by pump power and input signal power. Finally, the amplifier gain at the optimum length is calculated for each channel. This procedure is repeated for eight different glass hosts, namely; almino-germanosilicate, bismuth, LiNbO<sub>3</sub>, tellurite, sodium niobium phosphate, oxyfluoride silicate, Al<sub>2</sub>O<sub>3</sub> and fluoride phosphate glasses.

## I. INTRODUCTION

With rapid development of computer networks and other data-transmitting services, the demand for the increase of transmission capacity of wavelength division multiplexing (WDM) system is urgent. Erbium-doped fiber amplifiers (EDFAs) are key devices for optical systems in the 1.55  $\mu\text{m}$  wavelength region [1, 2]. The use of EDFA poses some problems [3, 4]. The reason is that the gain spectrum of an EDFA is not flat over the entire 1550 nm window. Although EDFAs can simultaneously amplify many signals, the gain is dependent on wavelength, and therefore each signal will experience different optical gain. So, EDFA should provide a wide and flat gain spectrum in order to accommodate as many WDM channels as possible.

In the present paper, the multichannel EDFA is analyzed for ten channels at 1 nm spacing. Approximate analytical solutions of steady state rate equations have been used to investigate the behavior of different host materials. In the described model, the signal power for the different channels is calculated along the fiber. The optimum amplifier length that provides a maximum gain for all channels, maximum power and maximum gain of each channel are studied for different host materials. Eight different glass hosts are considered to select the best one which can be used in WDM systems.

In the next section, the mathematical model including the approximate analytical solution is described. Section III presents the obtained results discussion. Finally, some general conclusions are given in section IV.

## II. MATHEMATICAL MODEL

### II.1- Rate Equations

In three-level system, the time dependent rate equations are given by [5]:

$$\begin{aligned} \frac{\partial N_2(z,t)}{\partial t} = & \left(\frac{\lambda_p \Gamma_p}{hcA}\right) \{ \sigma_{ap} [N - N_2(z,t)] - \sigma_{ep} N_2(z,t) \} \times [P_p^+(z,t) + P_p^-(z,t)] - \frac{N_2(z,t)}{\tau} \\ & - \left(\frac{\Gamma_s}{hcA}\right) N_2(z,t) \int \sigma_e(\lambda) \times [P^+(z,t,\lambda) + P^-(z,t,\lambda)] \lambda d\lambda \\ & + \left(\frac{\Gamma_s}{hcA}\right) [N - N_2(z,t)] \int \sigma_a(\lambda) \times [P^+(z,t,\lambda) + P^-(z,t,\lambda)] \lambda d\lambda \quad , \end{aligned} \quad (1)$$

$$\begin{aligned} \pm \frac{dP_s^\pm(z,t,\lambda)}{dz} = & \Gamma_s \{ \sigma_e(\lambda) N_2(z,t) - \sigma_a [N - N_2(z,t)] \} \\ & \times P_s^\pm(z,t,\lambda) + \Gamma_s \sigma_e(\lambda) N_2(z,t) P_0(\lambda) - \alpha(\lambda) P_s^\pm(z,t,\lambda) \quad , \end{aligned} \quad (2)$$

$$\pm \frac{dP_p^\pm(z,t)}{dz} = \Gamma_p \{ [\sigma_{ep} - \sigma_{24}(\lambda_p)] N_2(z,t) - \sigma_{ap} [N - N_2(z,t)] \} \times P_p^\pm(z,t) - \alpha(\lambda_p) P_p^\pm(z,t) . \quad (3)$$

The dopant concentration is given by  $N$ , whereas  $N_2$  is the upper-level population and  $A$  is the fiber core cross sectional area. The forward and backward propagating pump powers at a wavelength  $\lambda_p$  are given by the power densities  $P_p^\pm(z,t,\lambda)$ . The functions  $\sigma_e(\lambda)$  and  $\sigma_a(\lambda)$  are the emission and absorption cross sections, and  $\sigma_{ep}(\lambda) \equiv \sigma_e(\lambda_p)$ ,  $\sigma_{ap}(\lambda) \equiv \sigma_a(\lambda_p)$ . The positive coefficient  $\alpha(\lambda)$  represents scattering loss at  $\lambda$ .

The function  $P_0(\lambda)$  represents the contribution of the spontaneous emission into the mode and is given by  $P_0(\lambda) = 2hc^2/\lambda^3$  [5].  $\Gamma_s$  and  $\Gamma_p$  are the power filling factors for the signal and pump. In this model, the steady state solution ( $\partial/\partial t = 0$ ) is considered. Denote  $\lambda_s^i$  the signal wavelength for the  $i$ th channel with an input signal power  $P_s^i (i = 1 \dots I)$ . The signal power at  $\lambda = \lambda_s^i$  is  $P_s^{\pm}(z) = P^\pm(z, \lambda_s^i) \Delta \lambda_s^i$ .

### II.2. Approximate Equations

To get the approximate equations, we have many assumptions. First, ignoring the reverse pumping, one has  $P_s^i(z) \equiv P_s^{i+}(z)$  and  $P_p(z) \equiv P_p^+(z)$ . Second, the spontaneous emission and amplified spontaneous emission (ASE) can be neglected, compared to the pump and signal powers at  $\lambda = \lambda_s^i (i = 1 \dots I)$ . Third, we assume strong pumping resulting in a small value of the term  $(hcA)/(\hbar \Gamma_p \sigma_p \lambda_p P_p(0))$  if compared with both  $\sum_{i=1}^I \sigma_{es}^i / \sum_{i=1}^I \sigma_{as}^i$  and  $B$ , where  $\sigma_{es} = \sigma_e(\lambda_s)$  and  $\sigma_{as} = \sigma_a(\lambda_s)$ ,  $B$  is defined in the appendix. Fourth, the spectral distance should be less than 4 nm [6]. Consequently, the approximate solutions of the rate equations are:

$$N_2(z) = N \frac{Q_p P_p(z) + \sum_{i=1}^I Q_{as}^i P_s^i(z)}{Q_p P_p(z) + \sum_{i=1}^I (Q_{as}^i + Q_{es}^i) P_s^i(z)} , \quad (4)$$

$$\frac{1}{P_p(z)} \cdot \frac{dP_p(z)}{dz} = U_3 N_2(z) - U_4, \quad (5)$$

$$\frac{1}{P_p(z)} \cdot \frac{dP_p(z)}{dz} = U_3 N_2(z) - U_4, \quad (6)$$

where  $Q_{es}^i, Q_{as}^i, U_1^i, U_2^i, Q_p, U_3$  and  $U_4$  are defined in the appendix.

Solving Eqs. (4)-(6), one can get:

$$P_s^i(z) = P_s^i(0) \exp(v_i z) \phi_i(z), \quad (7)$$

where  $v_i, q_i,$  and  $R_i$  are defined in the appendix and  $\phi_i(z)$  is a dimensionless function defined by:

$$\phi_i(z) = \phi_1(z)^{\frac{q_i}{q_i}} \exp\left(\frac{v_1 - v_i}{q_i} z\right) \quad \forall_i = 2 \dots I \quad (8)$$

Provided that  $i=1$  is chosen to be the signal channel with smallest  $q_i$  (for all  $i$ ).

The function  $\phi_1(z)$  is derived from:

$$\left| \phi_1^{q_1-1} \right|^M \left| \frac{B_8 \phi_1^{q_1-1} + B_9 \exp(Tz)}{(B_8 + B_9) \exp(Tz)} \right|^{\frac{K}{B_8}} = \exp(z(M+1)T), \quad (9)$$

where  $B_8, B_9, M, K,$  and  $T$  are defined in the appendix. The amplification of any given channels depends on the number of channels, the input signal power in each channel, and the cross sections in each channel.

The pump power along the fiber is given in terms of the signal power as:

$$\left| \phi_1^{q_1-1} \right|^M \left| \frac{B_8 \phi_1^{q_1-1} + B_9 \exp(Tz)}{(B_8 + B_9) \exp(Tz)} \right|^{\frac{K}{B_8}} = \exp(z(M+1)T). \quad (10)$$

The optimum amplifier length, for this particular signal wavelength, is obtained when the propagating signal power reaches its maximum, providing a maximum gain. This satisfies  $((dP_s^i/dz) \downarrow_{z=L_{opt}} = 0)$ . This length can be obtained by employing (7) and (9) for

$P_s^i$ , as:

$$L_{opt}^{\max} = \frac{\ln\left((B_{10})^M \left| \frac{B_8 B_{10} + B_9}{B_8 + B_9} \right|^{\frac{K}{B_8}}\right)}{T}. \quad (11)$$

This yields the maximum available output signal power as:

$$P_{\max}^1(L_{opt}^{\max}) = P_s^1(0) B_{10}^{q_1-1} \exp\left(\left(v_1 + \frac{T}{q_1-1}\right) L_{opt}^{\max}\right). \quad (12)$$

Again, all symbols used are defined in the appendix.

The numerical values used for have been changed appropriately for the host materials. However, some of the numerical constants required for the convergence of the model which are known to be nearly constant for different host materials are listed in Table 1.

Three level parameters		Fiber parameters	
$\lambda_{\text{pump}}$	980 nm	A	$1.96 \times 10^{-11} \text{ m}^2$
$\lambda_{\text{signal}}$	1550 nm	N	1.5
T	10.8 ms	N.A	0.18
$\sigma_{\text{ap}}$	$2.0616 \times 10^{-25} \text{ m}^2$	$A(\lambda_p)$	$6.45 \times 10^{-3} \text{ m}^{-1}$
$\sigma_e(\lambda)$	Refs. [7, 8 and 10]	$A(\lambda_s)$	$3.22 \times 10^{-3} \text{ m}^{-1}$
$\sigma_a(\lambda)$	Refs. [7, 8 and 10]	$\Gamma_p$	0.889
$\sigma_{24}$	0	$\Gamma_s$	0.694

Table 1 Parameters used in calculations.

### III. RESULTS AND DISCUSSION

This section introduces a set of examples for multichannel EDFAs with different host materials near the central frequency of each material. The signal power for ten channels with a channel spacing 1nm is introduced along the fiber. Optimum length and maximum output power corresponding to maximum gain, its change with the pump and signal power at the input and the gain for each channel for different host materials are studied.

The host materials used in the present study includes: alumino- germanosilicate, bismuth-based, fluoride phosphate, lithium niobate, oxyfluoride silicate, aluminum oxide, tellurite and sodium niobium phosphate glasses. Through calculations, the erbium concentration, N, is set at  $1 \times 10^{25} \text{ m}^{-3}$ .

The first host is chosen as alumino-germanosilicate. The signal power at the optimum length for ten channels (from 1535 to 1544 nm) and the maximum power output at that length are displayed in Fig.1 at a central wavelength  $\lambda_0=1540 \text{ nm}$ .

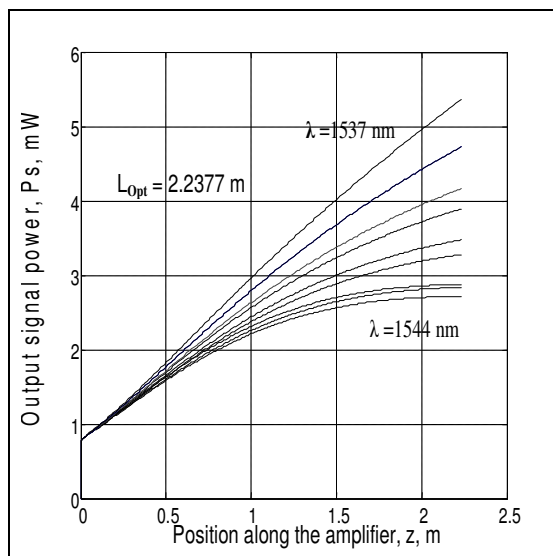


Fig.1. Signal power for ten channels, pump power = 60 mW and signal power = 0.8 mW at the optimum length for alumino-germanosilicate

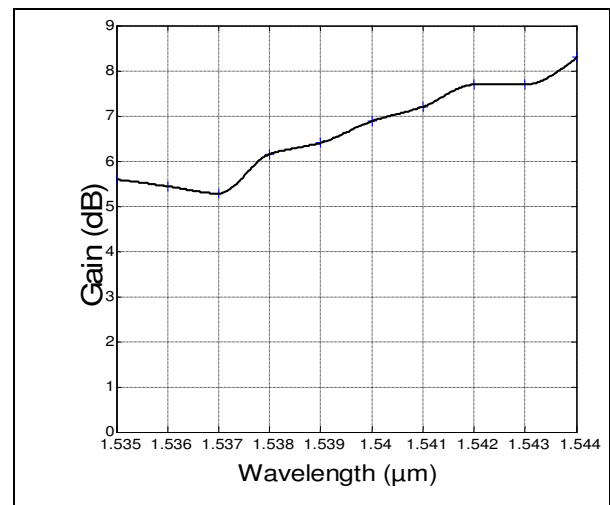


Fig.2. Amplifier gain for alumino-germano-silicate.

From Fig.1, the optimum amplifier length which corresponds to the maximum signal output power for ten channels (1535 - 1544 nm) is found to be 2.2377 m. It is important to note that the absorption and emission cross sections for  $\lambda_0=1540 \mu\text{m}$  are collected from Ref. [7]. Figure 2 clearly indicates the flatness of gain between maximum and minimum values which means a good response in WDM systems.

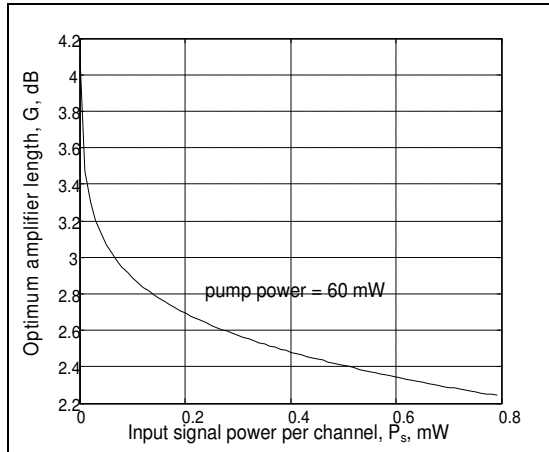


Fig.3 Optimum amplifier length for ten channels with 1nm spacing and 60 mW pump power for alumino-germanosilicate.

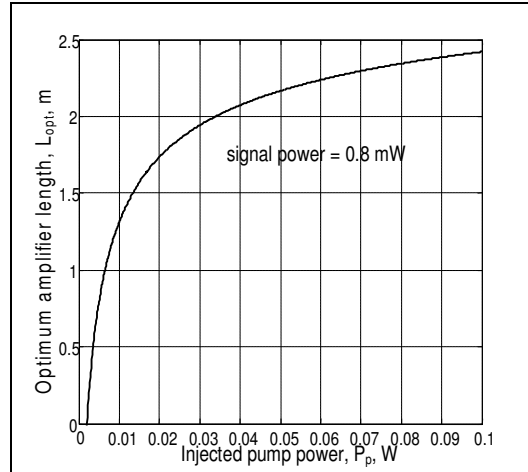


Fig.4 Optimum amplifier length for ten channels with 1nm spacing and 0.8 mW signal power of alumino-germanosilicate.

Figures 3 and 4 display the dependence of the optimum amplifier length on the input signal power and the injected pump power. As shown, as the input signal power increases the optimum length decreases until  $L_{\text{opt}}=2.2377$  m at 0.8 mW signal power which is a logic result. Also, the optimum length increases with the injected pump power for higher number of excitation electrons (population inversion) until  $L_{\text{opt}}=2.2377$  m for a pump power of 60 mW.

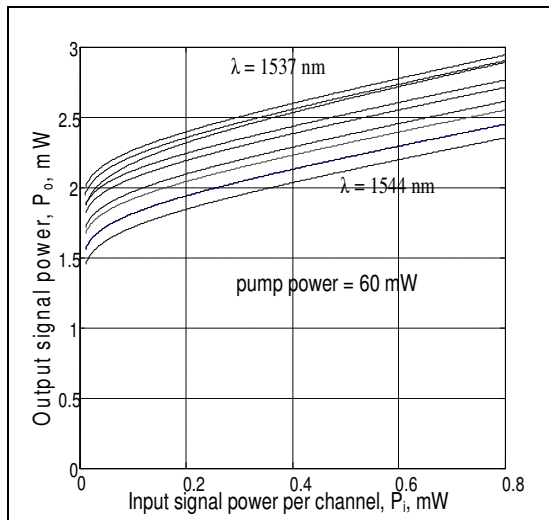


Fig.5 Output signal power as a function of input signal power (per channel) for ten channels.

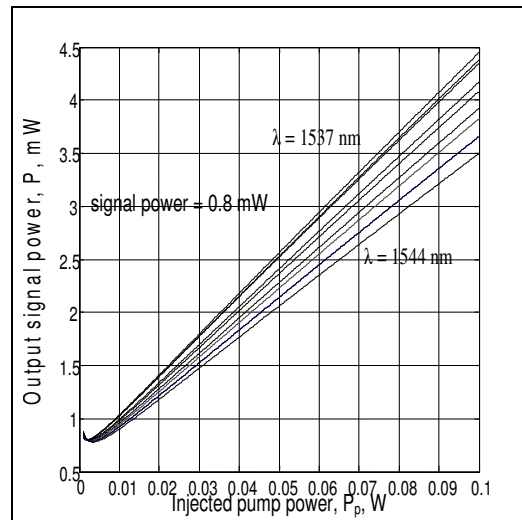


Fig.6 Output signal power as a function of injected pump power for ten injected signals.

The output power versus input signal power is plotted in Fig. 5 at an injected pump power of 60 mW for signal wavelengths in the range 1535-1544 nm. One can note that as the output signal power increases with the input power for a specified optimum length as shown in the Fig 3. In Fig. 6, injected pump power versus output signal

power is shown. The output signal power increases for higher injected pump which gives a good response to the amplification process.

Tanabe et al. [7] reported some kinds of bismuth-based glasses, which present good broadband properties, thermal stability and flat gain which are important when bismuth-based glass is used as a host material. At the optimum amplifier length for ten channels, the output power and amplifier gain are shown in Figs. 7 and 8, for bismuth at  $\lambda_0=1550$  nm.

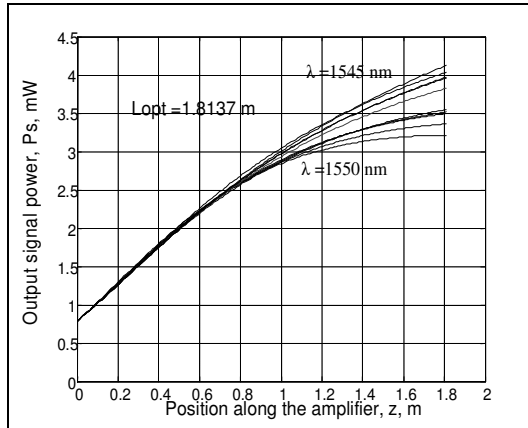


Fig.7 Output signal power for ten channels at a pump power 60 mW and signal power 0.8 mW for bismuth.

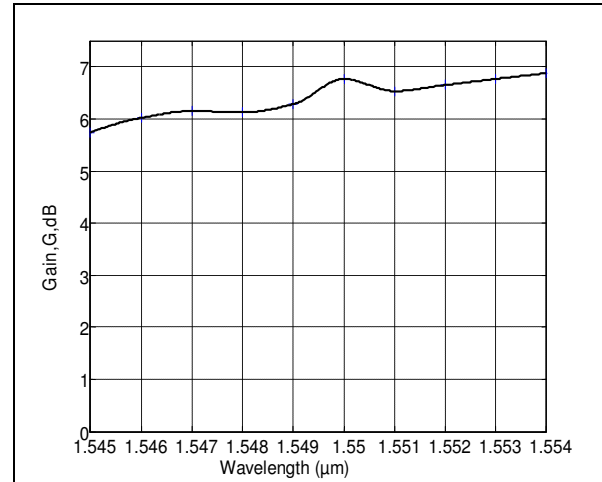


Fig. 8 Amplifier gain for bismuth.

The absorption and emission cross sections at a central wavelength  $\lambda_0=1550$  nm for ten wavelengths in the range 1545-1554 nm are collected from Ref.[8]. As shown in Fig.7, the optimum amplifier length is 1.8137 m which gives the maximum signal output power for ten channels. A flat gain is obtained, Fig.8, which gives a very good response in WDM systems.

Using these results, the dependence of the optimum fiber length on the input signal power and the injected pump power for a ten channel amplifier of bismuth is given in Figs. 9 and 10.

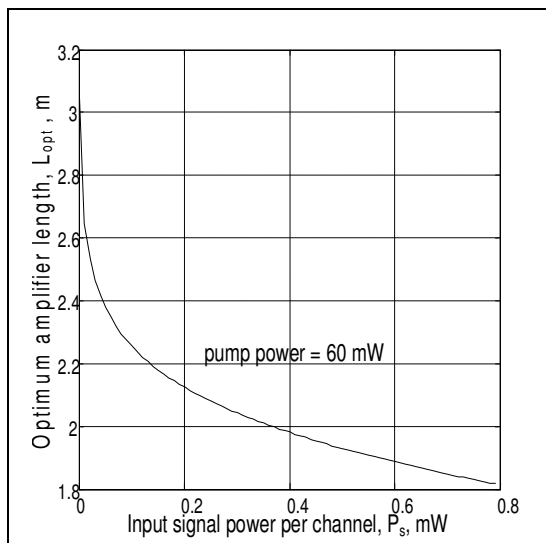


Fig.9 Optimum amplifier length for ten channels with 1nm spacing and 60 mW pump power of bismuth.

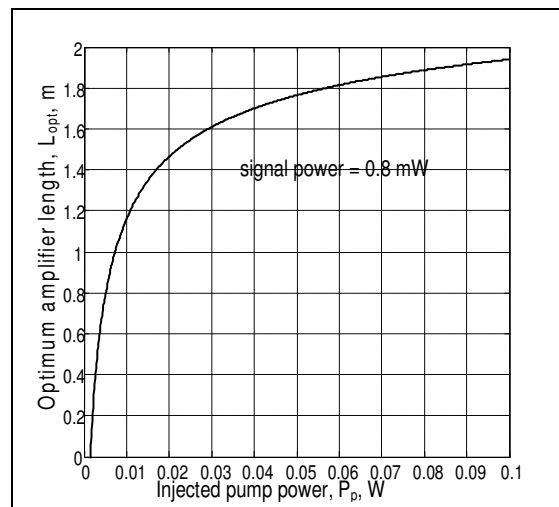


Fig.10 Optimum amplifier length for ten channels with 1nm spacing and 0.8 mW signal power of bismuth.

The output power versus input signal power is plotted in Fig.11 at an injected pump power of 60 mW and signal wavelengths in the range 1545-1554 nm. One can notice the closeness of the output power for a variation in the input signal power and hence, more flattened gain can be obtained.

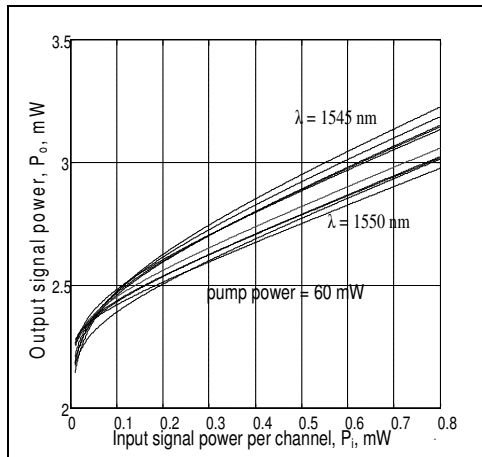


Fig.11 Output signal power as a function of input signal power (per channel) for ten channels.

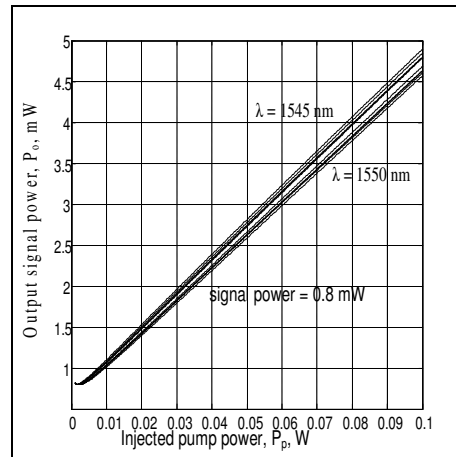


Fig. 12 Output signal power as a function of injected pump power for ten injected signals.

Figure 12 indicates that the output signal power increases at higher values of the injected pump power resulting in higher values of the amplifier gain. The closeness of the output power for a variation of pump power is noted and hence, better performance (flattened gain) can be is obtained.

The sodium-niobium phosphate glass host is then studied, Figs. 13 and 14. It gives a good quality optical waveguide by ion-exchange technique [9], which we believe is a good technological candidate thanks to its low cost and reproducibility for the production of such devices that have already been demonstrated in this type of glass. Though, this host shows a bad response for WDM as shown in Fig. 14. The absorption and emission cross sections for a central wavelength  $\lambda_0=1540$  nm and wavelengths for ten channels (1535-1544 nm) are obtained from Ref.[10]. Through calculations, it is found that the optimum amplifier length has an imaginary value!. Therefore, the output signal power is calculated at length 0.9 m which gives a moderate value of output signal power for the ten channels at  $\lambda_0= 1540$  nm.

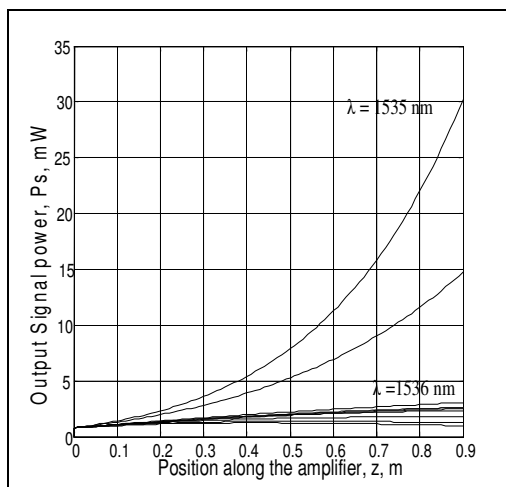


Fig.13 Signal power for ten channels at the optimum length for sodium-niobium phosphate. Pump power=60 mW and signal power=0.8 mW.

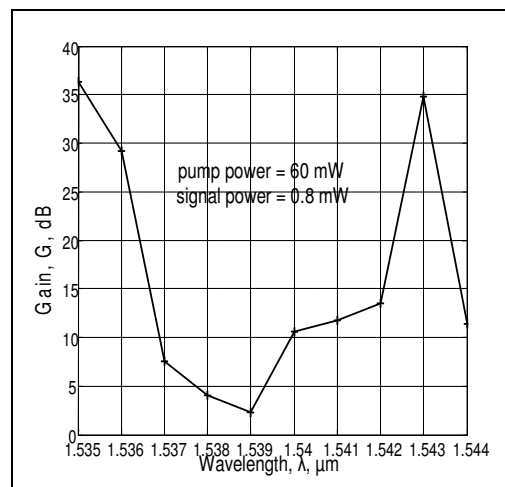


Fig.14 Amplifier gain for sodium-niobium phosphate.

The whole procedure is repeated for the other hosts; namely, LiNbO<sub>3</sub> [11], tellurite [12], oxyfluoride silicate [13], Al<sub>2</sub>O<sub>3</sub> [14] and fluoride phosphate [15]. Table 2 summarizes the obtained results for the different hosts of EDFA, including: central wavelength,  $\lambda_0$ , of each material, maximum and minimum gain,  $G_{\max}$  and  $G_{\min}$ , and the optimum amplifier length,  $L_{\text{opt}}$ .

Host	$L_{\text{opt}}$ (m)	$\lambda_0$ (nm)	$G_{\max}$ (dB)	$G_{\min}$ (dB)
Alumino-germanosilicate	2.237	1540	8.29	5.28
bismuth	1.181	1550	6.879	5.74
Sodium-Niobium Phosphate Glass	-	1540	15.78	0.92
LiNbO <sub>3</sub>	-	1553	16.9	-20.33
tellurite	2.766	1530	12.7	4.39
oxyfluoride silicate	2.004	1540	6.879	6.28
Al <sub>2</sub> O <sub>3</sub>	3.822	1530	13.54	3.27
fluoride phosphate	1.790	1530	9.22	3.52

Table 2 Amplifier parameters for different glass hosts.

## IV. Conclusion

In this paper, the properties of multichannel EDFA are studied for eight different glass hosts. Assuming large pump and signal powers (compared to the ASE), simple approximate analytical expressions are used to calculate the amplifier gain and its optimum length. From Table 2 and the previously discussed graphs, one can conclude that:

1. The best flattened gain, shorter optimum length and wider bandwidth occur in the Bismuth glass host
2. Oxyfluoride-silicate has the most flattened gain which is useful for WDM systems.
3. Both Sodium-Niobium Phosphate Glass and LiNbO<sub>3</sub> have bad response for WDM systems.

## APPENDIX

### DEFINITION OF THE PARAMETERS

$$B = \left( \frac{\sum_{i=1}^I \Gamma_s^i \lambda_s^i (\sigma_{es}^i + \sigma_{as}^i) P_s^i(0)}{\lambda_p \Gamma_p \sigma_{ap} P_p(0)} + 1 \right) (1 + \sigma_{ap}). \quad (\text{A.1})$$

$$Q_p = \frac{\lambda_p \Gamma_p}{hcA} \sigma_{ap}. \quad (\text{A.2})$$

$$Q_{as}^i = \frac{\Gamma_s^i \lambda_s^i \sigma_{as}^i}{hcA}. \quad (\text{A.3})$$

$$Q_{es}^i = \frac{\Gamma_s^i \lambda_s^i \sigma_{es}^i}{hcA}. \quad (\text{A.4})$$

$$U_1^i = \Gamma_s^i (\sigma_{as}^i + \sigma_{es}^i). \quad (\text{A.5})$$



$$U_2^i = \Gamma_s^i \sigma_{as} N + \alpha(\lambda_s^i). \quad (\text{A.6})$$

$$U_3 = \Gamma_p \sigma_{ap}. \quad (\text{A.7})$$

$$U_4 = \alpha(\lambda_p) + \Gamma_p \sigma_{ap} N. \quad (\text{A.8})$$

$$v_i = \frac{R_i}{q_i - 1}. \quad (\text{A.9})$$

$$q_i = \frac{\Gamma_p (\sigma_{ap} + \sigma_{ep})}{\Gamma_s^i (\sigma_{as}^i + \sigma_{es}^i)}. \quad (\text{A.10})$$

$$R_i = \Gamma_p \sigma_{ap} N + \alpha(\lambda_p) - (\Gamma_s^i \sigma_{as}^i N + \alpha(\lambda_s^i)) q_i. \quad (\text{A.11})$$

$$B_8 = B_2 - \frac{B_1 Q_p P_p(0)}{P_s^1(0)}. \quad (\text{A.12})$$

$$B_9 = B_4 - B_1 B_7. \quad (\text{A.13}]$$

$$B_1 = \frac{1}{q_1 - 1} \left( \frac{R_1}{q_1 - 1} - \frac{1}{I} \sum_{i=1}^I \frac{R_i}{q_i - 1} \right) \times \left( \frac{I}{\sum_{i=1}^I q_i} - 1 \right). \quad (\text{A.14})$$

$$B_2 = \frac{Q_p P_p(0)}{P_s^1(0)} (\Gamma_s^1 \sigma_e^1 N - \alpha_s^1 - v_1). \quad (\text{A.15})$$

$$B_4 = \sum_{i=1}^I \frac{q_i (q_i - 1)}{q_i (q_i - 1)} \times [U_1^1 N Q_{as}^i - (U_2^1 + v_1)(Q_{as}^i + Q_{es}^i)]. \quad (\text{A.16})$$

$$B_4 = \sum_{i=1}^I \frac{q_i (q_i - 1)}{q_i (q_i - 1)} \times (Q_{as}^i + Q_{es}^i). \quad (\text{A.17})$$

$$T = \left( v_1 - \frac{\sum_{i=1}^I v_i}{I} \right) \left( \frac{I}{\sum_{i=1}^I q_i} - 1 \right). \quad (\text{A.18})$$

$$M = \frac{B_1 B_7}{B_9}. \quad (\text{A.19})$$

$$K = B_1 \frac{Q_p P_p(0)}{P_s^1(0)} - \frac{B_8 B_1 B_7}{B_9}. \quad (\text{A.20})$$

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