Optimization of Self-Trapping and Thermal Effects in W-Shaped Optical Fibers

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ABSTRACT

In the present paper, both self-trapping (paraxial propagation) and thermal effects (caustic zones and maximum axial temperature) in W-shaped refractive index fibers are parametrically investigated to minimize the thermal effects and to maximize the self-trapping. Ray optics is employed with a closed form solution for the ray trajectory. Conditions for maximum self-trapping are parametrically investigated. A figure of merit is designed and tested for good performance.

I. INTRODUCTION

Single mode (SM) optical fibers are most suitable for high data-rate transmission systems because of the absence of modal dispersion. However, some drawbacks arise due to difficulty in excitation and splicing because of the small core radius.

Single-mode graded-core W-fibers are proposed to overcome these problems [1] and the structure of the phase devices [2]. According to the rapid development in recent all-optical networks or all-optical signal processing technology [3], optical fibers or waveguide based optical devices [4] have been investigated extensively.

El-Halafawy et. al. [5,6 and 7] had investigated the soliton transmission in nonlinear inhomogeneous biquadratic graded (W-shaped) index fibers. They have tailored the W-shaped refractive index as (Fig. 1):

$$n^2 = n_o^2(1-\alpha\rho^2+\beta\rho^4), \qquad (1)$$

where n_o is the axial refractive index, both α and β are tailoring parameters and ρ is the radial position, ($\rho = r/R$). Both α and β control the position and the value of the minimum refractive index in the W-shaped structure.

In the present paper, two basic problems are investigated; namely, the control of both self-trapping and thermal effects in the W-shaped optical fiber carrying high power as in optical amplifiers, sensors, and medical applications. Section I is a concise review, Section II handles the basic model and analysis, while Section III processes the obtained results with a general discussion and Section IV summarizes the major conclusions.

II. BASIC MODEL AND ANALYSIS

The object of the above investigations starts by solving the ray equation [8,9]:

$$\frac{\partial^2\rho}{\partial\eta^2} = \frac{1}{2n_o^2 N_o^2}\frac{\partial n^2}{\partial\rho}, \qquad (2)$$

where $\eta = z/a$ (a is the fiber radius) is the normalized axial distance and N_o is the directional cosine of the incident ray.

The use of $\rho_n = \rho\sqrt{2\beta/\alpha}$ and $\eta_n = \eta\sqrt{\alpha}/N_o$ in Eq.(2) yields:

$$\frac{\partial^2\rho_n}{\partial\eta_n^2} + \rho_n - \rho_n^3 = 0.0. \qquad (3)$$

El-Halafawy et. al, [5] and El-Badawy et. al. [9] employed the power series technique to solve Eq.(3) while El-Halafawy et. al.[5] processed the phenomena under a closed form. The launch conditions (initial conditions) are:

$$\rho_{on} = \rho_n\Big|_{z=0} = A_o = \frac{r_o}{a}\sqrt{\frac{2\beta}{\alpha}} = \rho_o\sqrt{\frac{2\beta}{\alpha}}, \quad (4)$$

and

$$\rho'_{on} = \frac{d\rho_n}{d\eta_n}\Big|_{z=0} = A_1 = \frac{dr}{dz}\Big|_{z=0} N_o\sqrt{\frac{2\beta}{\alpha^2}} = \rho'_o N_o\sqrt{\frac{2\beta}{\alpha^2}} \qquad (5)$$

Integrating Eq.(3), we get:

$$\rho_n'^2 + \rho_n^2 - 0.5\rho_n^4 = A_1^2 + A_o^2 - 0.5A_o^4 = C \text{, or}$$

$$\frac{d\rho_n}{d\eta_n} = \sqrt{0.5\rho_n^4 - \rho_n^2 + C}\,,$$

where

$$C = A_1^2 + A_o^2 - 0.5A_o^4. \quad (6)$$

and finally

$$\int_{\rho_o}^{\rho_n} \frac{d\rho_n}{\sqrt{(\rho_n^2 - A^2)(\rho_n^2 - B^2)}} = \frac{1}{\sqrt{2}}\eta_n\,, \quad (7)$$

This integration yields the elliptic functions [10], where:

$$A^2 + B^2 = 2 \text{, and} \quad (8\text{-a})$$

$$A^2B^2 = 2C\,, \quad (8\text{-b})$$

which yields

$$A^2 = 1 + \sqrt{1-2C} \text{, and} \quad (9\text{-a})$$

$$B^2 = 1 - \sqrt{1-2C}\,. \quad (9\text{-b})$$

Using $\rho_n = Bu_n$ in Eq.(7), one can get:

$$\int_{u_o}^{u_n} \frac{Bdu_n}{\sqrt{(B^2u_n^2 - A^2)(B^2u_n^2 - B^2)}} = \int_0^{\eta_n} \frac{1}{\sqrt{2}}d\eta_n \text{, or}$$

$$\int_{u_o}^{u_n} \frac{Bdu_n}{\sqrt{(1-u_n^2)(1-mu_n^2)}} = \omega\eta_n\,, \quad (10\text{-a})$$

where $m = B^2/A^2 < 1.0$ and $\omega = A/\sqrt{2}$.

This is the standard form of the "Jacobian" elliptic periodic sine (sn)of period T_e, i.e., we get:

$$sn^{-1}u_n - sn^{-1}u_o = \omega\eta_n\,,$$

or finally:

$$\rho_n = B\,sn(\omega\eta_n + \theta)\,. \quad (10\text{-b})$$

where $\theta = sn^{-1}u_o$.

The period of this kind of function is T_e where:

$$T_e = 2\pi\left[1 + \left(\frac{1}{2}\right)^2 m + \left(\frac{1}{2}\cdot\frac{3}{4}\right)^2 m^2 + \left(\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\right)^2 m^3 + \ldots\right] \quad (11)$$

The amplitude of this elliptic sine (the maximum distance away-of-the axis) ρ_m is given by:

$$\rho_m = B = \sqrt{1-\sqrt{1-2C}} \text{, or} \quad (12)$$

$$\frac{r_m}{a} = B\sqrt{\frac{\alpha}{2\beta}}\,. \quad (13)$$

The radial position of minimum refractive index is:

$$\rho_{min} = \sqrt{\frac{\alpha}{2\beta}}\,. \quad (14)$$

Thus, good confinement (paraxial propagation) requires:

$$\frac{r_m}{r_{min}} \le 1.0 \text{, or} \quad (15\text{-a})$$

$$B \le 1.0\,, \quad (15\text{-b})$$

i.e.,

$$\sqrt{1-2C} \le 0.0\,,$$

and finally

$$0.0 \le \sqrt{1-2C} \le 1.0 \quad (15\text{-c})$$

Thermal effects in fibers doped with rare earth or other absorbers play an increasingly important role as in fiber amplifiers, laser sources, attenuators, and all optical fiber switches [11].

A theoretical analysis of the temperature rise under two pumping regimes (short pump pulse or continuous wave pumping) has been presented [11], but we will employ the simple (but accurate) relation [12]:

$$T_m \cong T_o + 3.54(P_R + P_{TS}) = T_o + 3.54P_t\,; \quad (16)$$

where T_m is the caustic temperature, T_o is the ambient temperature, and P_R is the Raman injected power in watts (in Raman amplifiers) or the injected power in industrial applications, and P_{TS} is the total power of the multiplexed signals in the fiber. T_m is caused at different periodic lengthes L_{pe} given by:

$$L_{pe} = \frac{0.5T_e}{\omega} = \frac{0.5\sqrt{2}T_e}{A} = \frac{0.5\sqrt{2}T_e}{\sqrt{1+\sqrt{1-2C}}} \quad (17)$$

Reduction of thermal effects requires the optimization of both T_m and L_{pe}, i.e., the minimization of T_m and the maximization of L_{pe} as well as satisfying the condition of self-trapping given by Eq.(15).

The impact of thermal effects arises in different applications in both fields of industry and communications. The local caustic zone results in a temperature rise which increases the heat transfer coefficient [11] and consequently fasters the cooling in the caustic zones yielding different stresses and stains along the fiber [13].

The temperature dependence of erbium doped fiber amplifiers (EDFA's) characteristics is of a great importance as the temperature rise reduces the amplifier gain [14,15], as systems evolve towards more wavelengths (WDM and UWWDM), higher bit-rate and greater distances, such temperature dependence are no longer acceptable.

Pumping-induced thermal effects in doped fibers due to nonradiative processes are detrimental in most doped fiber devices including high power lasers and amplifiers.

Also, the fiber dispersion characteristics and the spectral losses [16,17,18] undergo severe changes due to the temperature rise which increases both losses and dispersion, thus reducing the transmitted bit-rate, and the repeater spacing.

It was found early [19] that the temperature dependence of the transient time delay shift, τ, thermal coefficient of the fiber stress σ_t, and the fiber strain ε are functions of the temperature rise ($40.0 < T°C \le 60.0$) and are phenomenologically derived, respectively, as:

$$\tau = \tau_o(T_n - 1) = 10.9(T_n - 1) \text{ , ps/km;} \quad (18)$$

$$\sigma_t = \sigma_{to}[1 + \sigma_1(T_n - 1) + \sigma_2(T_n - 1)^2]; \quad (19)$$

$$\sigma_t - \sigma_{to} = 493(T_n - 1) - 1009(T_n - 1)^2 \text{, MPa}$$

and

$$\varepsilon = \varepsilon_o[1 + \varepsilon_1(T_n - 1)] \quad (20)$$

$$\varepsilon \cong \varepsilon_o = 36.25\text{x}10^{-4}(T_n - 1)$$

where $T_n = T_m/T_o$, and τ_o, σ_{to}, σ_1, σ_2, ε_o, and ε_1 are constants.

In the following section, the investigated items (self-trapping and thermal effects) one parametrically processed.

III. RESULTS AND DISCUSSION

The software especially designed to parametrically process the present problem investigates the following optimized items:

i) the self-trapping, and
ii) the thermal effects.

The set of causes is $\{P, r_o, r_o', \alpha, \beta\}$ while the set of effects is $\{T_m, A, B, L_{pc}\}$.

Two important features of W-shaped optical fibers must be processed. The clad refractive index (n_2) and the location of minimum radial refractive index (n_{min} at ρ_{min}), where:

$$n_2 = n_o(1 - 0.5\alpha + 0.5\beta), \qquad (21)$$

$$\rho_{min} = \frac{\alpha}{2\beta}, \text{ and} \qquad (22)$$

$$n_{min} = n_o\left(1 - \frac{1}{8}\frac{\alpha^2}{\beta}\right). \qquad (23)$$

We will constraint both n_2 and n_{min} at the following numerical values:

$$n_2 = 0.95 n_o, \text{ and } n_{min} = 0.90 n_o. (24)$$

Thus, we have:

$$0.5(\alpha - \beta) = 0.05 \text{ or } \alpha - \beta = 0.1, \text{ and}$$

$$\frac{1}{8}\frac{\alpha^2}{\beta} = 0.1 \text{ or } \alpha^2 = 0.8\beta \qquad (25)$$

which yields:

$\alpha = 0.6828$ or 0.1172 and $\beta = 0.5828$ or 0.0172.

In general, the design assumptions:

$$n_2 = (1 - \Delta)n_o \text{ and } n_{min} = (1 - 2\Delta)n_o \ (26)$$

yield:

$$\alpha = 13.6568\Delta \text{ or } 2.3431\Delta, \text{ and}$$
$$\beta = 11.6568\Delta \text{ or } 0.3431\Delta$$

The second result is not reasonable and hence we employ only:

$$\alpha = 13.6568\Delta \text{ and } \qquad \beta = 11.6568\Delta$$

Thus, whatever the value of Δ, we have:

$$\rho_{min} = 0.7654$$

Thus, for self-trapping, ρ_o must be less than ρ_{min}, i.e., $\rho_o <$ 0.7654

Samples of the variations of the amplitude ρ_{max} of the "sn" function, where $\rho_{max} = B\sqrt{\alpha/2\beta} = 0.5858B$ and the length of the caustic zone L_{pe} against the variations of Δ, ρ_o, and ρ_o' are displayed in Figs. 2-4, while the thermal effects are displayed in Figs. 5-8. These figures assure the following facts:

i) The best set of launch conditions for good self-trapping, is the following:
   a) small ρ_o
   b) small ρ_o'
   c) large Δ

   Thus, we design an figure of merit, FM under the form:

$$f_m = \frac{\rho_o \rho_o'}{\Delta} \qquad (27)$$

   We feel that ρ_{max} and f_m possess positive correlation as in Fig. 9, also L_{Pe} and f_m are in positive correlation, Fig. 10. In fact as f_m decreases, ρ_{max} decreases. This effect is required for good self trapping. Also, as f_m decreases, L_{Pe} increases yielding less number of caustic zones and consequently decreases the harmful thermal effects.

ii) In general, the increase of the launch power yields higher temperature rise and consequently higher stresses, strains, and delay time.

IV. CONCLUSIONS

The refractive index of W-shaped single mode optical fibers is tailored with deep unambiguity to parametrically investigate two basic problems; namely, the self-trapping and the thermal effects (both are of interest in optical communication systems and industrial applications) to optimize these phenomena. The ray trajectory is derived in a closed from of "Jacobian " elliptic function. The designed dimensionless figure of merit is a good criterion to numerically measure both the self-trapping and the thermal effects phenomena.

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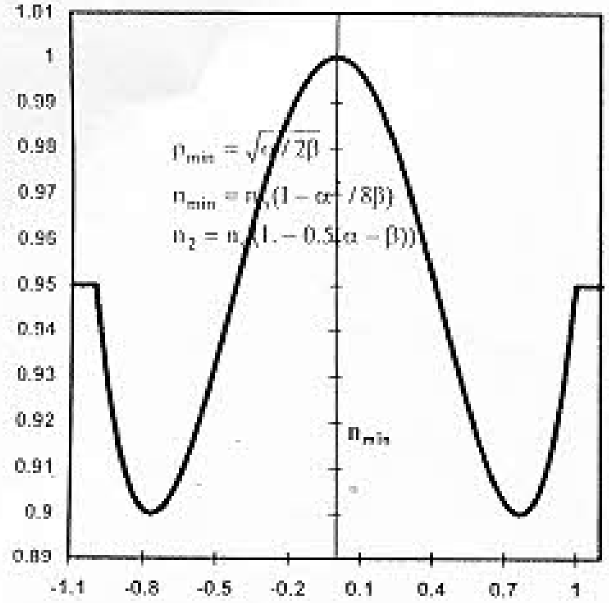

Fig. 1. Variations of the ratio (n/n_o) against variations of radial position, ρ for W-shaped refractive index SM fiber.

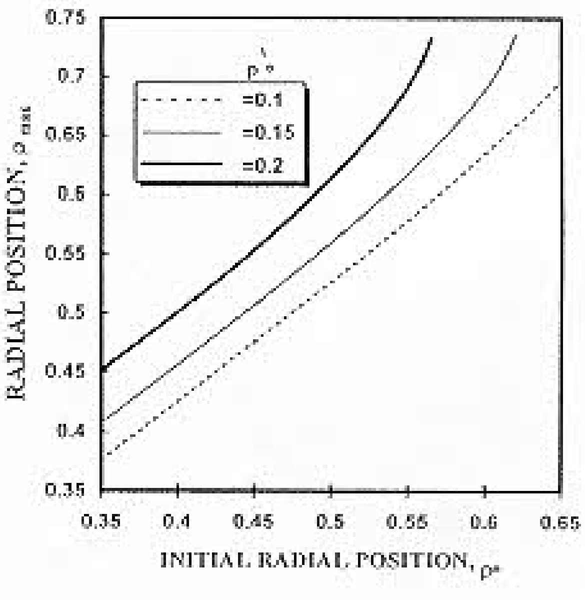

Fig. 2. Variations of ρ_{max} against variations of ρ_o at the assumed set of parameters

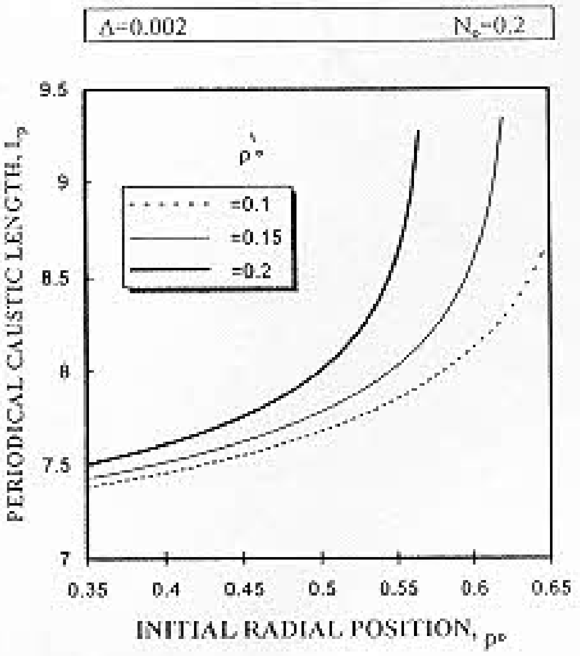

Fig. 3. Variations of L_p against variations of ρ_o at the assumed set of parameters.

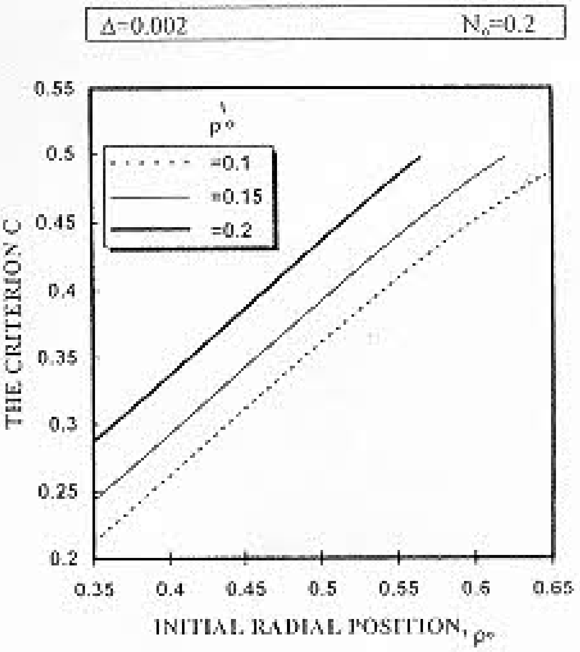

Fig. 4. Variations of C against variations of ρ_o at the assumed set of parameters.

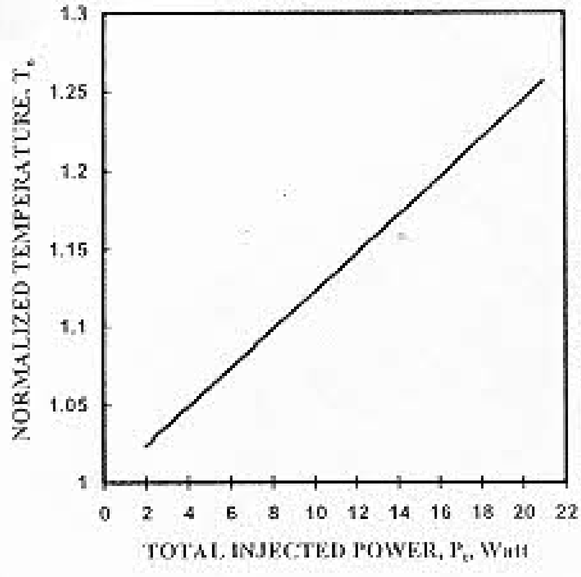

Fig. 5. Variations of T_o against variations of P_t at the assumed set of parameters.

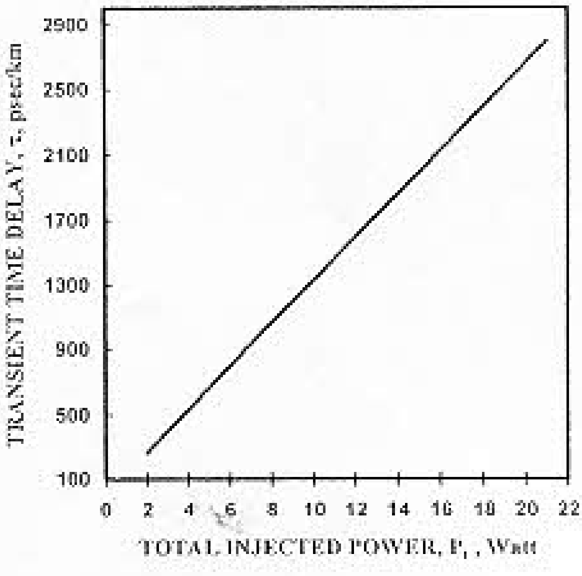

Fig. 6. Variations of τ against variations of P_t at the assumed set of parameters.

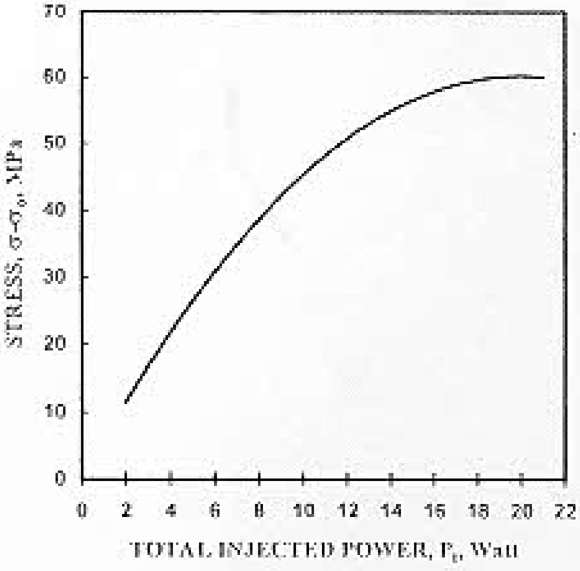

Fig. 7. Variations of $\sigma - \sigma_0$ against variations of P_t at the assumed set of parameters.

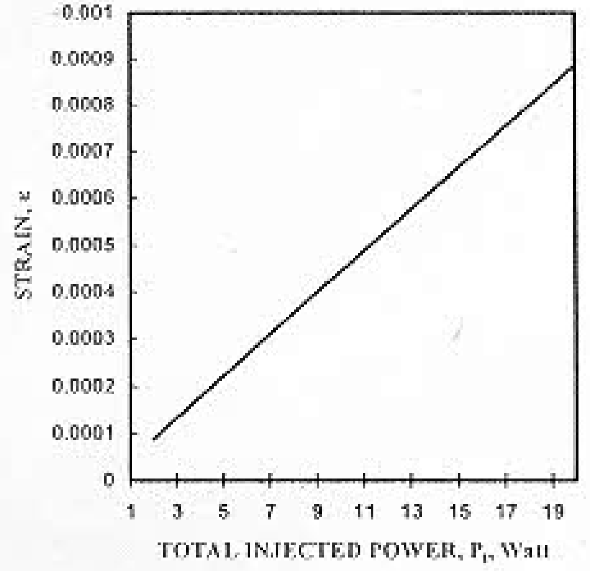

Fig. 8. Variations of ε against variations of P_t at the assumed set of parameters.

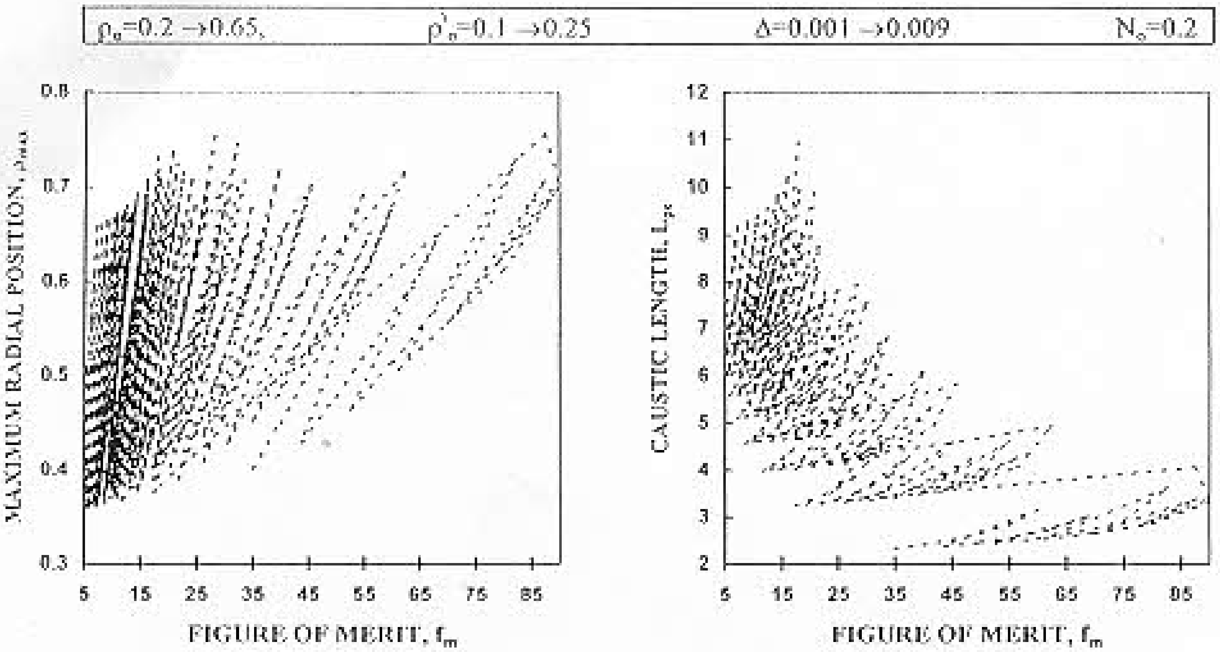

Fig. 9. Variations of ρ_{max} against variations of f_m at the assumed set of parameters

Fig. 10. Variations of L_{pe} against variations of f_m at the assumed set of parameters.