INFLUENCE OF TEMPERATURE AND PRESSURE ON DISPERSION PROPERTIES OF NONLINEAR SINGLE MODE OPTICAL FIBERS

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Abstract: Near field distribution, propagation constant and dispersion characteristics of nonlinear single-mode optical fibers have been investigated. Shooting-method technique is used and implemented into a computer code for both profiles of step-index and graded-index fibers. An error function is defined to estimate the discrepancy between the expected electric-field radial derivative at the core-cladding interface and that obtained by numerically integrating the wave equation through the use of Runge-Kutta method. All of the above calculations done under the ocean depth in which the depth will affect the refractive index that have a direct effect on all the optical fiber parameters.

Key words: *Nonlinear refractive index, Normalized propagation constant, Mode delay factor, Material dispersion, Waveguide dispersion.*

1. INTRODUCTION

The attractiveness of the lightwave communications is the ability of the silica-optical fibers to carry large amount of information over long repeaterless spans. To utilize the available bandwidth, numerous channels at different wavelengths can be multiplexed on the same fiber and higher transmitter powers are required to increase system margins. All these attempts to fully utilize the capabilities of silica fibers will ultimately be limited by nonlinear interactions between the information-bearing light waves and the transmission medium. These optical nonlinearities can lead to interference, distortion and excess attenuation of the optical signals, resulting in the systems degradation [1].
Refractive index as a function of the pressure is very important for all the fiber parameters like the electric field, the normalized propagation constant and all dispersion types. So, it plays a vital role in the optical fiber communication system in particular for the undersea submarine optical fiber cables which are used to transmit optical signals from one continent to another. Up to now, most of the researches are focused on the waveguide structure mentioned above. Boardman and Egan [4] developed a general dispersion equation for nonlinear optical waveguides of the second kind with a Kerr-type nonlinearity. This equation has been used in the investigation of guided

characteristics for a saturable case.[3] The dispersion equation has been modified to exclude possible spurious modes by dividing the two types of dispersion equations. The results are considered to be exact when compared with results obtained numerically in the present work. Yuan performed calculations above the sea level without taking into account the influence of pressure and temperature on the refractive index for planner waveguide [5].

Throughout this paper, the second-order differential equation for nonlinear optical fibers has been solved by the Runge-Kutta numerical integration and the secant method, using the concept of the shooting method. All effects of the various fiber parameters on the modal field distribution, the propagation constant and the dispersion characteristics have been studied for both step and graded index fibers. Calculations were done after taking into account the refractive index as a function of pressure and temperature.

2. BASIC MODEL AND ANALYSIS

The nonlinear refractive index of the fiber, n, can be represented as a sum of two terms:

$$n = n_0 + n_{II} I \tag{1}$$

where n_0 is the ordinary refractive index associated with the material, n_{II} is the intensity dependent refractive index and I is the optical intensity. Equations used for determining the exact value for no due to pressure dependence have the form:

$$n^2(\lambda, p) = A + \frac{B\lambda^2}{\lambda^2 - C} + \frac{D\lambda^2}{\lambda^2 - E} \quad (2),$$

where λ is the wavelength,

$$\begin{aligned} A &= 1.29552 + 9.86385\times10^{-6} P + 0.544763\times10^{-5} P^2 \\ B &= 0.809872 + 42.0899\times10^{-6} P - 1.71823\times10^{-8} P^2 \\ C &= 1..07945\times10^{-2} - 0.56693\times10^{-3} P + 0.894313\times10^{-10} P^2 \\ D &= 0.917151 + 38.7911\times10^{-6} P - 1.13552\times10^{-8} P^2 \\ E &= 100 \end{aligned} \quad (3),$$

and P is the pressure in MN/m^2. The effect of temperature on the refractive index can also be calculated from:

$$n_o^2(\lambda, T) = 1 + \sum_{i=1}^{3} \frac{A_i(T)\lambda^2}{\lambda^2 - \lambda_i^2(T)} \quad (4),$$

where

$$\begin{aligned} &A_1(T) = 0.6961663\times\alpha_T,\ A_2(T) = 0.4079426\times\alpha_T,\ A_3(T) = 0.8974749\times\alpha_T \\ &\lambda_1(T) = 0.0684043\times\beta_T,\ \lambda_2(T) = 0.1162414\times\beta_T,\ \lambda_3(T) = 9.896161\times\beta_T \end{aligned} \quad (5),$$

$$\alpha_T = 0.93721 + 2.0857\times10^{-4}\times T,\ \beta_T = \frac{T_O}{T} \quad (6),$$

in which $T_o = 27\ ^{\circ}C$ and T is the ambient temperature. The pressure and temperature can be calculated as a term of the ocean depth (D in km) from the relation:

$$P = 9.91D - 0.23744\times10^{-1}D^2 + 0.19232\times10^{-3}D^3 \qquad D \leq 10\text{ km}$$
$$T = 27.7 - 0.62671\times10^2\times D + 0.53838\times10^2\times D^2 - 0.20674\times10^2\times D^3 + 3.5994\times D^4 - 0.23156\times D^5 \qquad (7)$$

The controlling differential equations for the radial field component in both core and cladding are respectively written as [5]:

$$\frac{d^2E}{dr^2} + \frac{1}{r}\frac{dE}{dr} + \frac{V^2}{a^2}\left[1 - \frac{\alpha_e E^2}{2n_1^2} - b - f(r)\right]E = 0 \qquad (8),$$

$$\frac{d^2E}{dr^2} + \frac{1}{r}\frac{dE}{dr} - \frac{V^2}{a^2}bE = 0 \qquad (9),$$

where a is the core radius, b is the normalized propagation constant, V is the normalized frequency, α_e is the nonlinearity term and $f(r)$ is an assumed function that defines whether the fiber under consideration is step or graded index. For step index $f(r) = 0$ and for graded index $f(r) = (r/a)^2$. So, using the shooting method, two initial guesses, β_1 and β_2 for the propagation constant are used.

The core differential equation is integrated step-by-step using the fourth order Runge-Kutta method. The electric field and its derivatives are calculated each step till reaching the core-cladding interface.

A certain problem appeared in the numerical integration process due to the cylindrical geometry. An approximation must be used to overcome this problem. The Gaussian approximation has been used to estimate both the electric field and its derivative at the radial coordinate as an initial condition. The used Gaussian function has the form:

$$E = E_o \exp(-r^2/\omega_o^2) \qquad (10),$$

where E_o is the axial value of the electric field and ω_o is the spot size of the fundamental mode. Both E and dE/dr obtained from Gaussian approximation are used as initial values in the numerical integration process. Performing the integration, a matching condition may be represented as the formula in:

$$\frac{1}{E}\frac{dE}{dr} = \frac{W}{a}\frac{K_1(W)}{K_0(W)} \qquad (11),$$

where $K_1(W)$ is the modified Bessel function of first order and $W^2 = a^2(n_1^2k^2 - \beta^2)$ with k the wave number and β is the propagation constant.

An error function is used to evaluate the mismatch between the predicted result of the first derivative of the electric field and that obtained by the numerical integration. This error function is defined as:

$$ERR = \frac{a}{W}\frac{dE}{dr}\frac{K_1(W)}{K_0(W)}E \quad (12),$$

The error function is estimated twice, once for each of the two initial guesses for β, using the secant method. The two obtained error values are used to estimate a correction term for a new guess of the effective index, β_{new}, through the formula:

$$\beta_{new} = \beta_2 - \frac{ERR2(\beta_2 - \beta_1)}{(ERR1 - ERR2)} \quad (13)$$

where ERR1 and ERR2 are corresponding error values obtained. These new guesses together with one of the initial guesses, will give smaller error. Also, a second iteration process is necessary.

To obtain the first derivative of the normalized propagation constant b, Eq.(8) is differentiated with respect to V and to obtain the second derivative of b with respect to V, the wave equation differentiated twice, by obtaining the exact value for b, db/dv and also d^2b/dv^2, we can easily calculate the power normalized electric field, propagation constant and the dispersion properties.

3. RESULTS

3.1 Power normalized electric field

We discuss the effect of the depth, D, on the normalized electric field in Fig. 1. The normalized electric field is represented by the following equation:

$$E_n = a\sqrt{\frac{n_2\pi}{P\eta_0}}E \quad (14),$$

where n_2 is the cladding refractive index, η_0 is the impedance of free space and P is the input power. Changing the depth D from 0 to 5 km, we found an obvious change on the values of the normalized electric field.

In Fig. 2, the change of core radius at 3 km depth, increasing the core radius (which increases the V-number in a direct manner) gives an obvious increase in the model field distribution. This is achieved by taking the power equal to 1-mW. The effect of the depth appears also for the core radius of 4 and 5 μm by a shift up. The values of the normalized electric field behind the core appear closer to the sea level. Far from the core center and closer to the clad, there will be a wider change in the modal field at 3 km rather than that at the sea level.

3.2 Normalized propagation constant

For the step index fibers the variation of the normalized propagation constant as a function of the normalized frequency for both linear and nonlinear operation at 3 km depth (n_2 = 1.44) is shown in Fig. 3. It is shown that as the core radius or its refractive index is increased, the effect of nonlinearity begins to increase and then the nonlinearity effect slowly loses its strength. Increasing the core refractive index, that behavior is a normal behavior because of the increase of the refractive index due to the depth increase. For graded index fibers, similar to the case of step index fibers, the normalized propagation constant for a certain *V*-value under nonlinear operation is dependent on the overall *V*-value and also its constituent parameters. Figure 4 represents the normalized propagation constant as a function of the normalized frequency, and the dotted curve represents the propagation constant at 3 km depth.

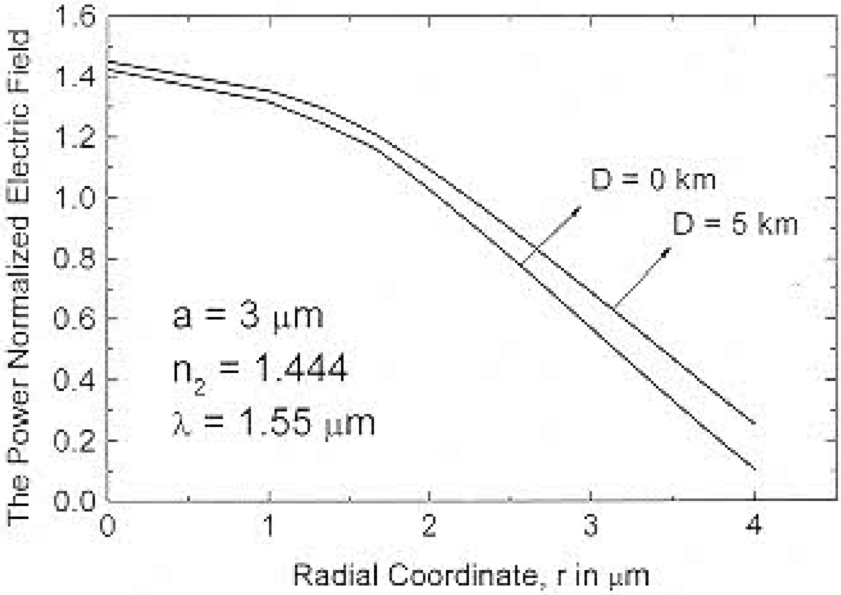

Fig. 1: The effect of the ocean depth from 0 to 5 km on the model field distribution along the core radial coordinate

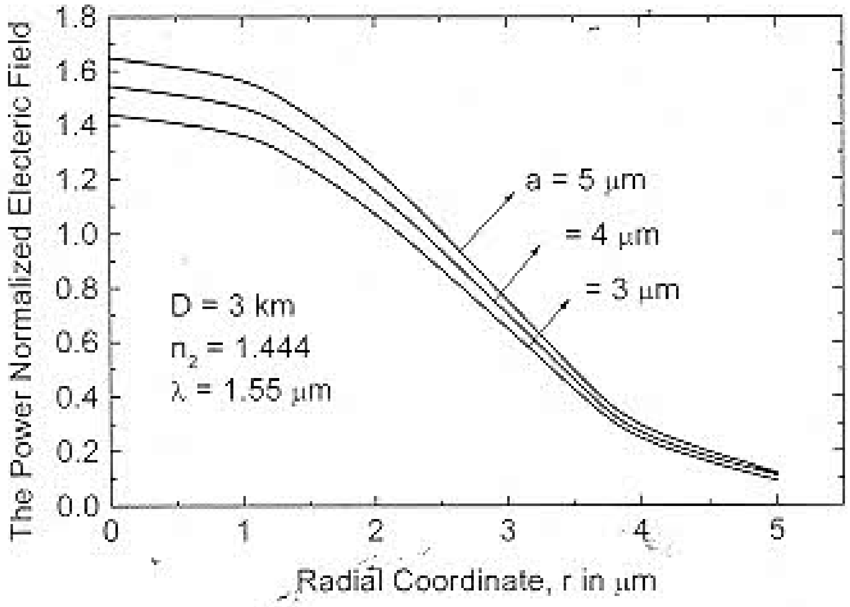

Fig. 2: The effect of variation of the core radius on the model electric along the core radial coordinates

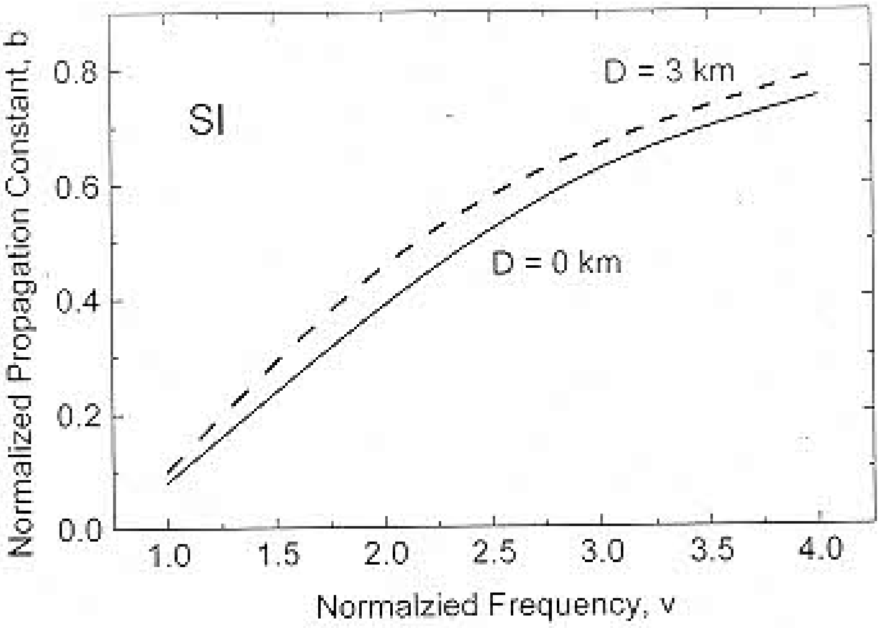

Fig. 3: The variation of the propagation constant with the normalized frequency at 0 and 3 km depths for the step index fibers

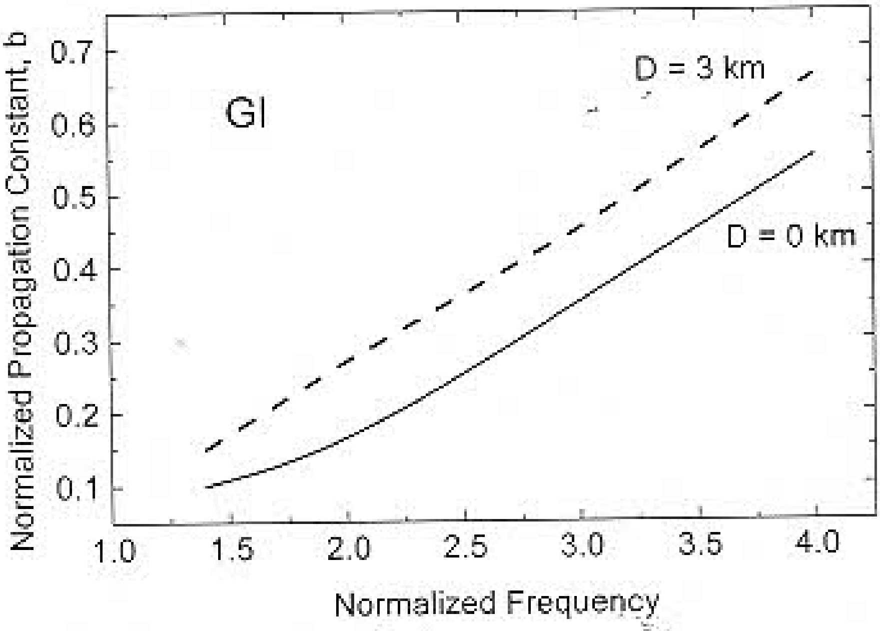

Fig. 4: The variation of the normalized propagation constant with normalized frequency at 0 and 3 km depths for the graded index fibers

3.3 Mode delay factor

The mode delay factor, d, is a universal dispersion parameter which plays a major role in the theory of single mode fibers. It is defined through the formula:

$$d = \frac{d(Vb)}{dV} = V\frac{db}{dV} + b \tag{15}$$

The mode delay factor affects the group delay of the guided mode fibers as it described the change in-group delay caused by the changes in power distribution between the fiber core and cladding. The variation of the mode delay factor for the step index fibers as a function of the normalized frequency is illustrated in Fig. 5 under both cases above the sea level and at 3 km depth. It is clear from the graph that the effect of the ocean depth leads to the refractive index increase that gives an increase in the normalized propagation constant and also its derivative with respect to V ended with the increase in the mode delay factor. Calculations in this part will be done in two different ways: one of them uses the dependence of the refractive index on the pressure only which is represented in the graph by the solid line, and second uses the dependence of the refractive index on both the pressure and temperature represented by the dashed line.
The dependence of the mode delay factor on the normalized frequency for the graded index fibers is illustrated in Fig. 6 for a depth change from 0 to 3 km taking into account both cases mentioned above for the pressure and temperature dependence. V number is increased by increasing the radius or the refractive index of the fiber core, $d(V)$ begins to increase slowly at the small values of V, then increases till reaching a certain value of V, the $d(V)$ values begin to be nearly constant.
This behavior could be explained physically by considering the relation between the mode delay factor and the sharing of the optical power between the core and cladding. As either the core radius or the refractive index is increased from small values, the percentage of the power confined in the core region starts to rise up quickly and the normalized propagation constant does in consequence.

3.4 Waveguide dispersion coefficient

The third parameter, which governs the dispersion behavior of the optical fiber, is the waveguide dispersion coefficient, which is defined as:

$$g = V\frac{d^2(Vb)}{dV^2} = V\left(\frac{db}{dV} + 2\frac{db}{dV}\right) \tag{16}$$

For step index fibers, the variation of the normalized waveguide parameter, as a function of the normalized frequency is shown in Fig. 7. For larger values of V, the values of the waveguide coefficient begin to be much closer for both cases. It is noted that, for the single mode operation, where the normalized frequency is less than 2.404, the waveguide dispersion is always positive which means that the waveguide dispersion parameter is always negative.

Figure 8 represents the relation between the waveguide parameter and the V-number for the graded index fibers. Calculations are performed two times as mentioned in the step index discussion. One can deduce from the two figures that, for any V value, the step index has a greater value of the waveguide parameter than the graded index under any chosen depth.

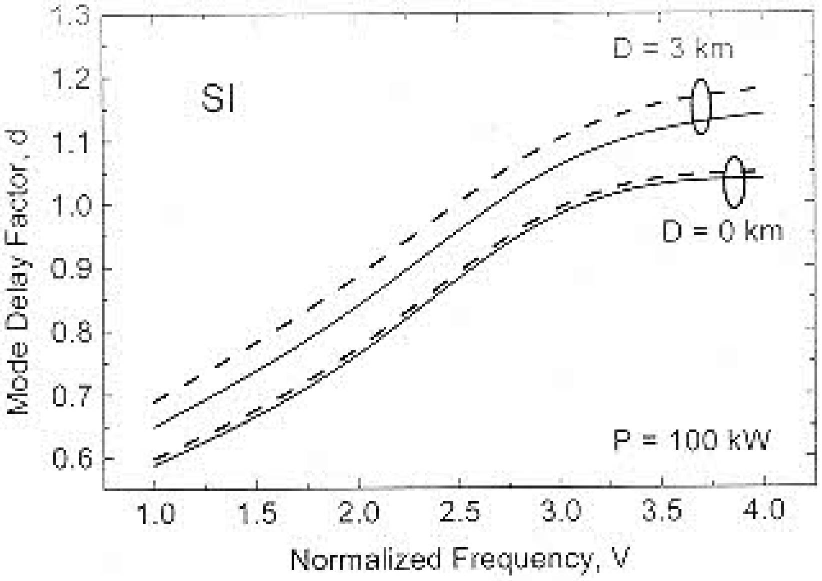

Fig. 5: The effect of the ocean depth from 0 to 3 km on the mode delay factor as a function of normalized frequency for the step index fibers, the dashed line is due to the dependence of the refractive index on pressure and temperature, the solid one is due to the pressure dependence only

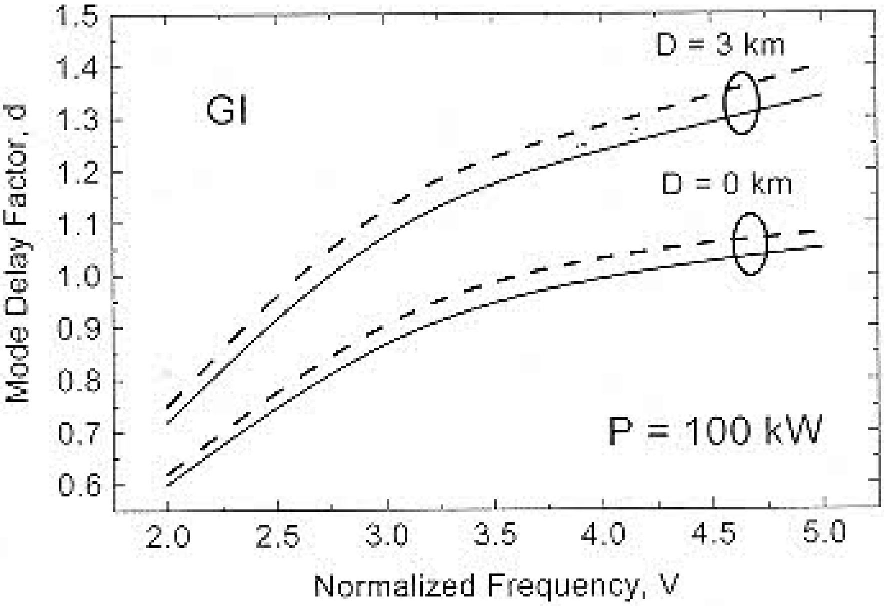

Fig. 6: The effect of the ocean depth from 0 to 3 km on the mode delay factor as a function of normalized frequency for the graded index fibers, the dashed line is due to the dependence of the refractive index on pressure and temperature, the solid one is due to the pressure dependence only.

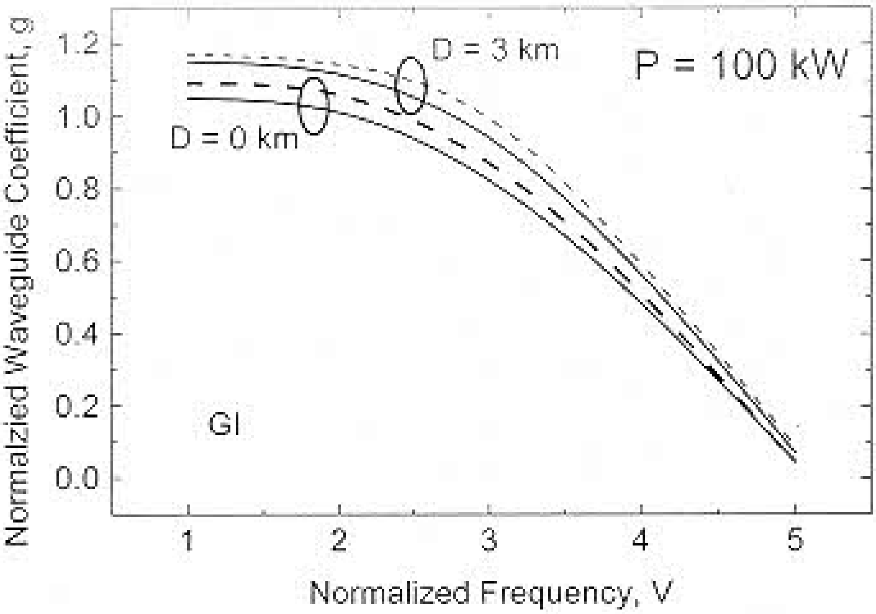

Fig. 7: The effect of the ocean depth from 0 to 3 km on the normalized waveguide coefficient as a function of normalized frequency for the step index fibers, the dashed line is due to the dependence of the refractive index on pressure and temperature, the solid one is due to the pressure dependence only

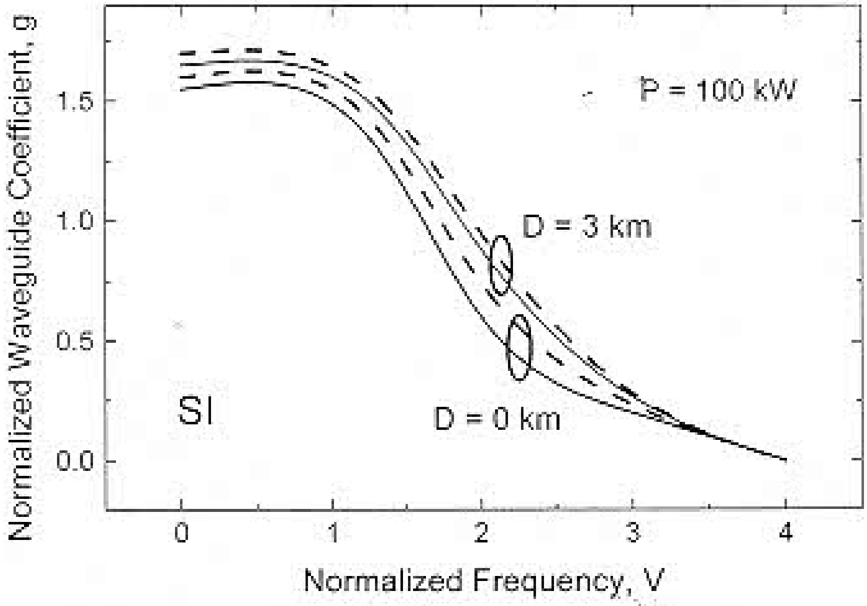

Fig. 8: The effect of the ocean depth from 0 to 3 km on the normalized waveguide coefficient as a function of normalized frequency for the graded index fibers, the dashed line is due to the dependence of the refractive index on pressure and temperature, the solid one is due to the pressure dependence only

3.5 Material dispersion

Material dispersion results from the variation of the velocity of light in a medium, and hence, its refractive index, with the optical wavelength, and is given by:

$$D_M = -\left(\frac{\lambda}{c}\right)\frac{d^2n}{d\lambda^2} \quad (17)$$

where n is the refractive index of the core or cladding.

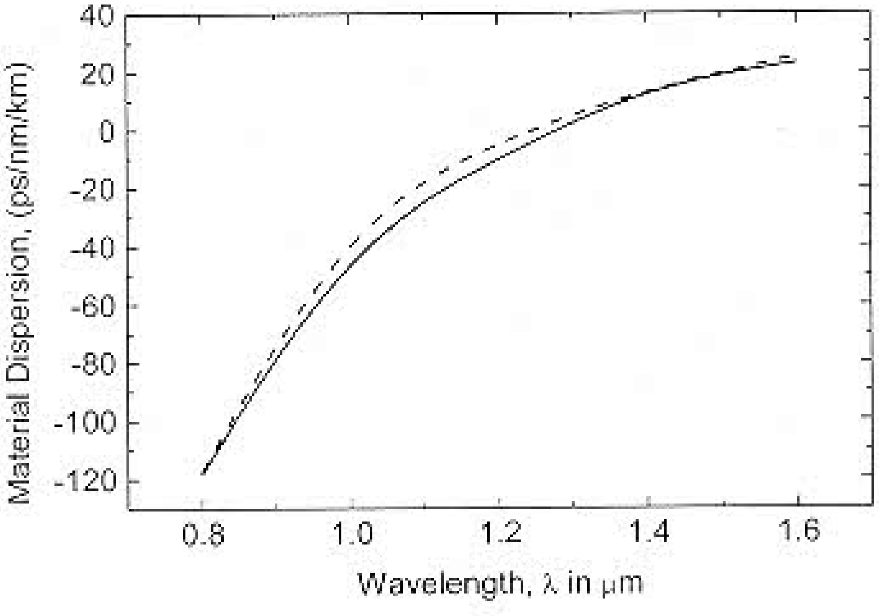

Fig. 9: The variation of the material dispersion as a function of the wavelength at 3 km depth (dashed line) and above the sea level (solid line)

3.6 Waveguide Dispersion

The waveguide dispersion coefficient, g, estimated before is used now to estimate the waveguide dispersion parameter D_W as:

$$D_W = -\left(\frac{n_1 - n_2}{\lambda c}\right)V\frac{d^2(Vb)}{dV^2} \quad (18)$$

The waveguide dispersion is illustrated as a function of the wavelength in Fig. 10 for two chosen depths, 0 and 3 km. From Fig. 10, the waveguide dispersion above the sea level has a greater shift up than at 3 km depth. This shift will decrease with the increasing of the wavelength.

The same behavior for the step and graded index fibers is expected because of the same change in the waveguide dispersion parameter which is the only parameter affecting the waveguide dispersion.

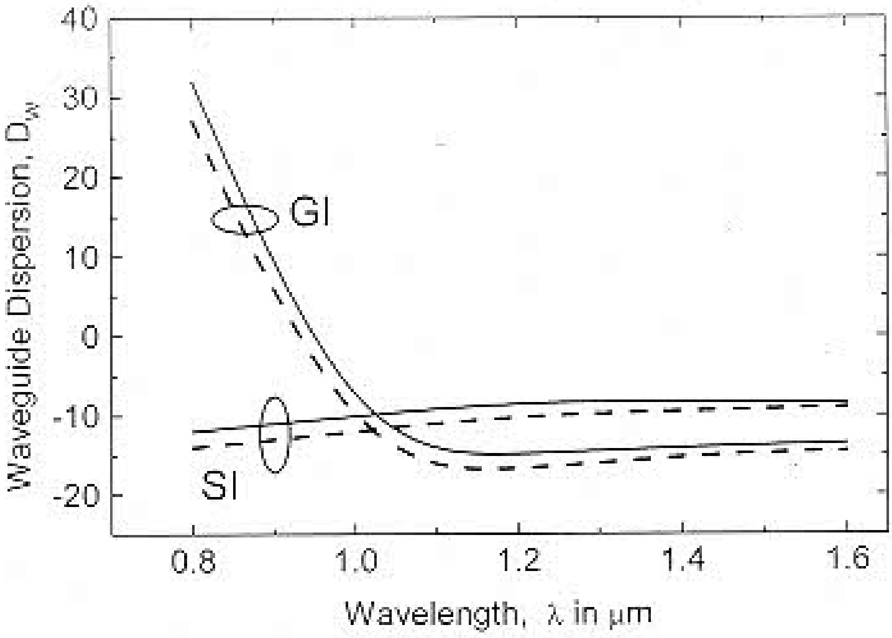

Fig. 10: Effect of the ocean depth (from 0 to 3 km) on the waveguide dispersion parameters as a function of the wavelength for the step index (SI) and graded index (GI) fibers

4. CONCLUSION

Below sealevel the refractive index increases, and this increase will affect all the parameters related to performance of the optical fiber including the power normalized electric field, propagation constant and especially the dispersion parameter. Also, calculations done above sealevel show that the effect of the refractive index increase due to temperature and pressure cannot be neglected.

In the nonlinear operation (in the core of the optical fiber), the characteristic curves of the normalized propagation constant, the mode delay factor, and the normalized waveguide dispersion coefficient are found to depend on the way by which the value of V is varied, the injected power, Kerr nonlinearity coefficient.

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