

# Erbium Doped-Fiber Amplifier Dynamics: Effect of Pump Power and Number of Channels

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**Abstract** - This paper gives a numerical investigation of the ordinary differential equation (ODE) describing the Erbium doped fiber amplifier (EDFA) gain dynamics. Also, bringing into greater evidence the physical meaning of the amplification process, and greatly enhancing the utility of the ODE as an analysis and design tool. The effects on the amplifier dynamics by several parameters are studied. In addition, a comparative study between different EDFAs according to their dynamic behavior in order to judge their performance is presented.

## I. INTRODUCTION

Gain dynamics of rare earth doped fiber amplifiers (REDFAs) and specially EDFAs are of considerable interests in wave division multiplexing (WDM) networks, where network reconfiguration or network faults can lead to adding or dropping wavelength channels. EDFAs will be the key component of such networks, and the study of their gain dynamics is essential [1, 2].

Much research has been devoted to the solution of the set of coupled first-order nonlinear partial differential equations determining the wavelength-dependent, time-varying amplifier gain. For instance, Saleh et al. [3] eliminated the time dependence of the gain to arrive to a single transcendental equation for the steady state gain. Habbab et al. [4] arrived at a linear approximation for the small signal in single-channel amplifiers. Sun et al. [1] at Bell Laboratories have succeeded in reducing the system of coupled differential into a single ODE. More recently, Bononi et al. [5] have also succeeded in simplifying more the ODE. Furthermore, the analysis presented by Bononi et al. [5] neglects both excited state absorption (ESA) and saturation induced by the amplified spontaneous emission (ASE) produced inside the amplifier.

In this paper, a study of the EDFAs dynamics is presented concerning the effects of varying both the pumping power and the total number of channels in the amplifier. We note that this analysis is based on the assumptions of the model in [1, 3, and 5]. Also, it should be noted that Ko et al. [6] arrived at the same results as [5] even considering the effect of spontaneous emission noise indicating that saturation induced by the ASE is not an issue here. We further extended this study and introduced a novel method to compare between different doped-fiber amplifiers according to their dynamic behavior.

## II. THEORY

We start from the rate and photon equations used in [1], derived assuming a two-level system for the dopant ions. The doped ion distribution in the core is considered to be constant. Also, assumed a homogeneously broadened gain spectrum, no excited state absorption, no background loss, and no self-saturation by the ASE. The rate equation for the fraction of excited ions  $N_2$ , [5]  $0 \leq N_2 \leq 1$ , is

$$\frac{\partial N_2(z,t)}{\partial t} = -\frac{N_2(z,t)}{\tau} - \frac{1}{\rho A} \sum_{j=0}^N u_j \frac{\partial \Phi_j(z,t)}{\partial z}, \quad (1)$$

where the equations describing the propagation along  $z$  of the photon fluxes  $\Phi_j$  [photons/s] of channel  $j$ , with  $j = 0, \dots, N$ , are

$$\frac{\partial \Phi_j(z,t)}{\partial z} = \rho u_j \Gamma_j [\sigma_j^T N_2(z,t) - \sigma_j^A] \Phi_j(z,t), \quad (2)$$

where  $\tau$  (s) is the fluorescence time,  $\rho$  ( $m^{-3}$ ) is the ion density in the doped fiber of effective area  $A$  ( $m^2$ ) ( $A = \pi R^2$ ,  $R$  is the radius of dopant ion distribution);  $\Gamma_j$ ,  $\sigma_j^E$  ( $m^2$ ), and  $\sigma_j^A$  ( $m^2$ ) are the confinement factor, the emission and absorption cross sections of channel  $j$ , respectively, and  $\sigma_j^T = \sigma_j^E + \sigma_j^A$ . The length of the amplifier is  $L$  (m). The direction of propagation is defined by the parameter  $u_j$ . Channels entering at  $z = 0$  have  $u_j = 1$  while those entering at  $z = L$  have  $u_j = -1$ . Also, it should be noted that the value  $j = 0$  represents the pumping channel.

The logarithmic gain is defined by

$$G_j(t) = B_j r(t) - A_j, \quad (3)$$

or

$$G_j(t) = \int_0^L \frac{u_j}{\Phi_j} \partial \Phi_j = \ln \left[ \frac{\Phi_j^{out}(t)}{\Phi_j^{in}(t)} \right], \quad (4)$$

where

$$A_j = \rho \Gamma_j \sigma_j^A L, \quad (5)$$

$$B_j = \frac{\Gamma_j \sigma_j^T}{A}. \quad (6)$$

The constants  $A_j$  and  $B_j$  are non-dimensional parameters;  $\Phi_j^{in}(t)$  and  $\Phi_j^{out}(t)$  are the input and output photon fluxes of channel  $j$ . The reservoir  $r(t)$  is a number between 0 and  $r_{max}$  ( $r_{max} = \rho AL$ ), which represents the number of available ions to be converted into signal photons and is defined by [5]

$$r(t) = \rho A \int_0^L N_2(z, t) dz \quad (7)$$

Using the above equations, one arrives at a first order ODE describing the dynamic behavior of the system state  $r(t)$  and is given by [5]

$$\dot{r}(t) = \frac{-r(t)}{\tau} + \sum_{j=0}^N \Phi_j^{in}(t) \left[ 1 - e^{B_j r(t) - A_j} \right], \quad (8)$$

with an initial condition given by [1]

$$r(0) = \tau \sum_{j=0}^N \Phi_j^{in}(0^+) \left[ 1 - e^{B_j r(0) - A_j} \right]. \quad (9)$$

The initial condition  $r(0)$  could be any number in the range  $[0, r_{max}]$ , although the range spanned by a real amplifier is narrower and have an upper value  $r_{upper}$  and is given by [5]

$$r_{upper} = \left[ \frac{\sigma_p^a}{\sigma_p^e + \sigma_p^a} \right] r_{max} \quad (10)$$

where  $\sigma_p^e$  ( $m^2$ ), and  $\sigma_p^a$  ( $m^2$ ) are the emission and absorption cross sections of the pumping channel.

In this analysis, we introduced a technique to study the gain dynamics of REDFAs and specially EDFAs with a comparative study between different types through different fiber compositions (host) [5, 7] concerning several issues. These are the effect of changing the total number of channels  $N$  propagating through the fiber, the effect of changing the pumping power  $\Phi_0$ , the effect of changing the length of the doped-fiber amplifier  $L$ , and the effect of changing the concentration of the dopant ions  $\rho$  in the fiber. It should be noted that these effects were studied individually keeping the other parameters fixed. The following analysis is carried out numerically by solving Eq.(8) and then finding the steady state value of the reservoir  $r(t)$ , output power excursion of a certain channel, defined as  $10 \log[\Phi_j^{out}(t)/\Phi_j^{in}(0^+)]$ , and the settling time.

The settling time introduced here by the given model, is the time defined by the reservoir  $r(t)$  to settle down to 98% of its steady state value from the time at which the pump was turned on (i.e.  $t = 0$ ).

### III. RESULTS

Figure 1 shows a numerical investigation of doped fiber amplifiers at different number of input channels. The reservoir dynamics and output power excursion of one the propagating channels (for example, channel no.1) for the exact solution of Eq.(8) is shown in Fig.1. The data used here are taken from [3, 5]. The amplifier input channels are in the C-band around 1550 nm; at a 6.25 GHz spacing [8], with initial input powers of  $-2$  dBm/channel. The amplifier is pumped at 980 nm, with a pump power of 18.4 dBm, and has  $L = 35$  m, fluorescence time of 10.5 ms. The absorption coefficients are 0.257 and 0.145  $m^{-1}$  and the intrinsic saturation powers are 0.440 and 0.197 mW at the pumping channel and input channels, respectively.

From Fig.1, it could be seen that as the total number of input channels propagating through the fiber increases, both the steady state value of the reservoir and the time taken by the reservoir to settle down to its steady state value decreases. The steady state values decreases because the reservoir is limited by an upper value  $r_{upper}$  [5] or in the ideal case  $r_{max}$ . The decrease of the settling time is explained as the number of channels increases, the amplification process which is done by the stimulation increase of excited atoms and hence, the reservoir discharges faster.

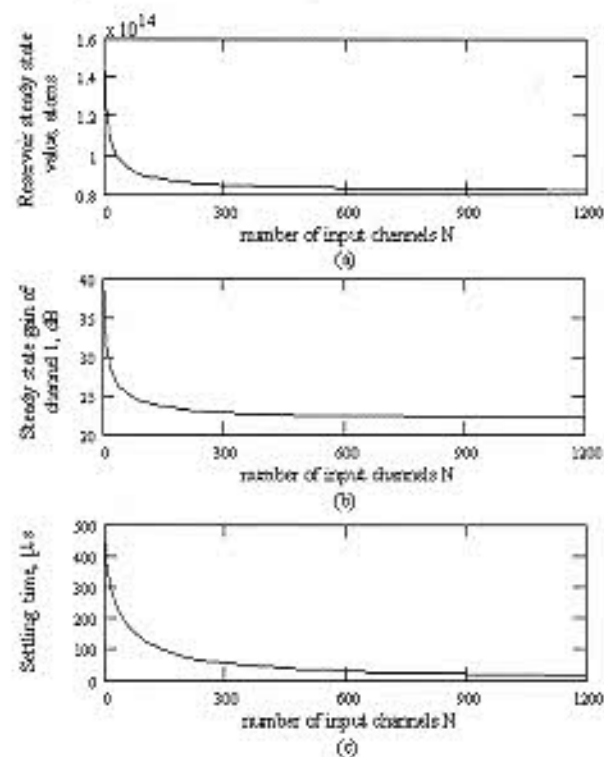


Figure 1

Reservoir dynamics with the variation of the number of channels in the amplifier, (a) reservoir steady state value, (b) steady state gain of a single channel (channel no.1), and (c) settling time.

The same analysis carried above is repeated but with varying the pumping power at a certain number of channels (8-channels), with the same parameters used in the previous example. From Fig.2, it is clear that as the pumping power increases, the reservoir increases and the time taken by the reservoir to settle down to its steady state value decreases. This is because as the pumping power increases the rate of stimulation is increased and hence more atoms are excited and the reservoir dynamics will be faster, as shown in Fig.2. Also, as the pumping power becomes relatively large (i.e.  $P_0 \rightarrow \infty$ ), the reservoir settles to a constant value confirming with that the reservoir has a maximum limit  $r_{upper}$  that could not be exceeded.

In addition, the concentration of the dopant ions has a direct effect on the gain dynamics of the amplifier. From Fig.3, it is seen that the settling time of the amplifier

increases with the dopant density,  $\rho$ . This is due to that as  $\rho$  increases, there are more ions to be excited in the reservoir and hence more time is needed by the amplifier to settle down to its steady state values. Also, the number of channels propagating through the fiber has less effect on the dynamics of the amplifier for lower values of dopant concentrations. The standard data used is taken from reference [1]. It should be noted that, it is not possible to increase the dopant density above a certain level, due to the detrimental effect of cooperative energy transfer, which results in a drastic reduction in the net gain achieved [9].

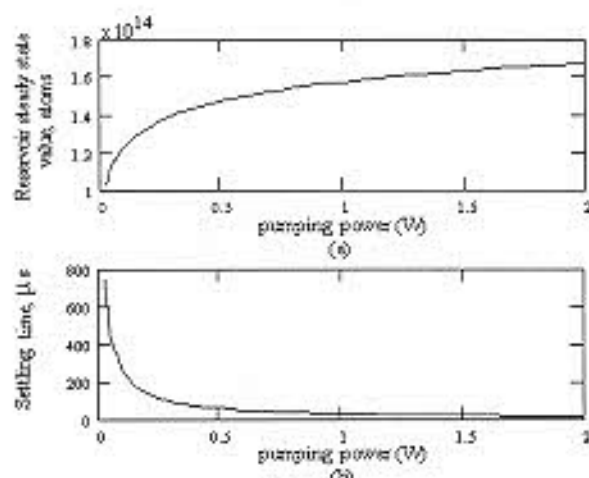


Figure 2

Reservoir dynamics with the variation of the pumping power in the amplifier, (a) reservoir steady state value, and (b) settling time.

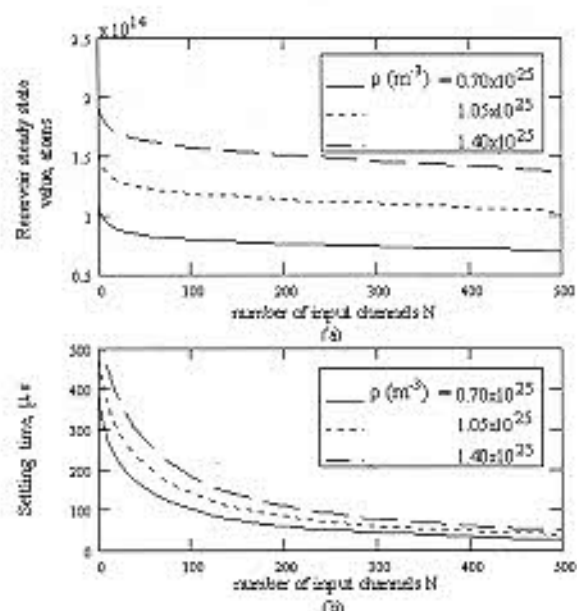


Figure 3

Reservoir dynamics of  $Al_2O_3-SiO_2$  EDFA at several dopant density values with the variation of the number of channels in the amplifier, (a) reservoir steady state value, and (b) settling time.

The same analysis is repeated but with varying the pumping power at a certain number of channels

(8-channels) at different dopant concentrations, with the same parameters used in the previous example.

From Fig.4 as the dopant density increases, the steady state reservoir and gain are increased with a small increase in the settling time. Also, the settling time behaviors concerning the pumping power at different dopant densities are very close in their values. This confirms that the increase in the pumping power affects more the settling time than the increase in the number of input channels in the fiber, as mentioned previously.

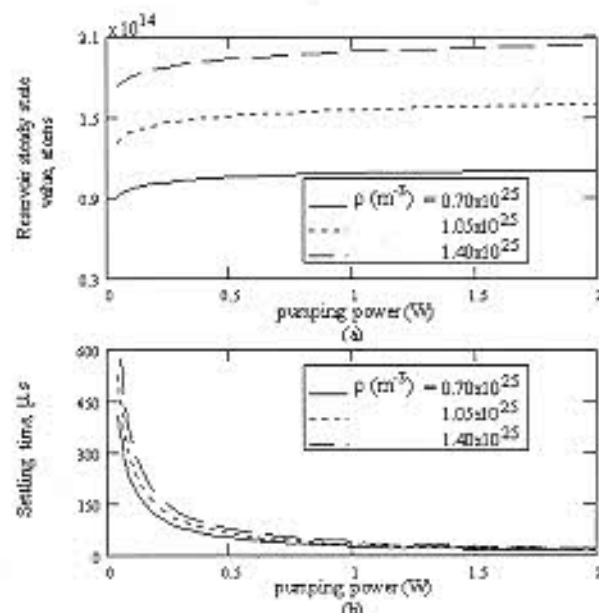


Figure 4

Reservoir dynamics at several dopant density values with the variation of the pumping power, (a) reservoir steady state value, and (b) settling time.

Beside of the number of channels in the fiber, pumping power and dopant density, the fiber length has a direct influence. Figure 5 and 6 show that the gain dynamics of the amplifier have an approximately linear relation with the fiber length. It should be noted that it is not possible to increase the fiber length indefinitely, because the effect of background loss from the silica glass material would affect pump power and signal powers as well as the effect of the ESA and ASE [1, 9].

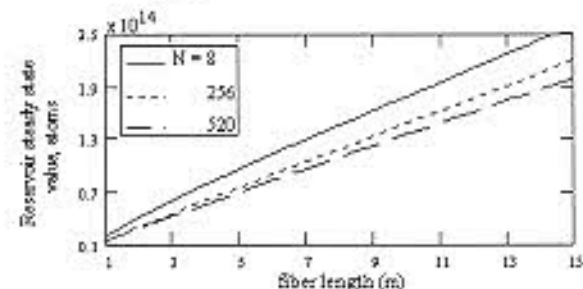


Figure 5

Reservoir steady state values of  $Al_2O_3-SiO_2$  EDFA at several fiber length values with the variation of the amplifier length at different number of input channels.

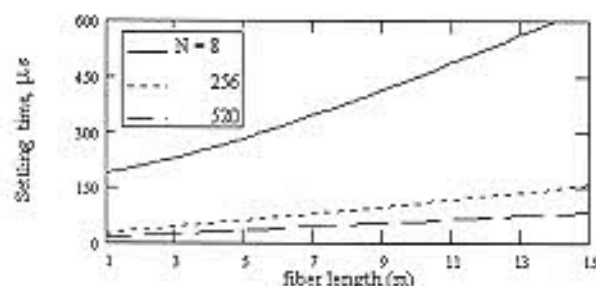


Figure 6

Settling time of  $\text{Al}_2\text{O}_3\text{-SiO}_2$  EDFA at several fiber length values with the variation of the amplifier length at different number of input channels.

In the following, we extend our study of the gain dynamics to compare between different EDFAs according to their behaviors when varying different parameters inside the amplifier, see Fig.7. The parameters used in the following examples are taken from references [1, 5, 6, 8, 9 and 10].

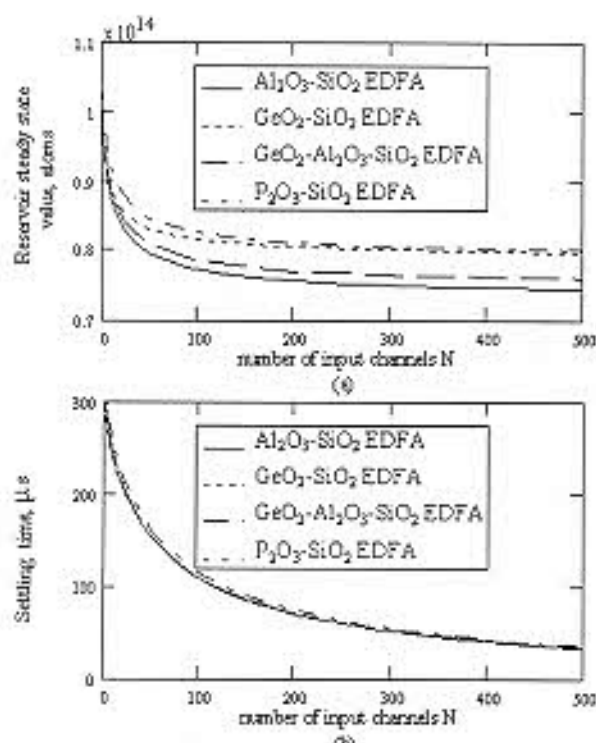


Figure 7

Reservoir dynamics of different EDFAs with the variation of the number of channels in the amplifier, (a) reservoir steady state value, and (b) settling time of the amplifier.

From this example one can see that if the gain is the important criteria in a WDM scenario, the  $\text{P}_2\text{O}_5\text{-SiO}_2$  EDFA is the best, see Fig.7(a). Concerning the settling time, all EDFA types have almost the same behavior; see Fig.7(b), especially as the number of input channels increases which means that the host material has no considerable effect on the amplifier dynamics. It should be noted that the previous analysis can be applied to compare between different doped fiber types according to their gain

dynamics as long as the amplifiers under analysis satisfy the model's assumptions.

#### IV. CONCLUSION

In this paper, we further study the ODE for the doped-fiber amplifier gain dynamics bringing into greater evidence the physical meaning of the amplification process, and greatly enhancing the utility of the ODE as an analysis and design tool. The effects on the amplifier dynamics by several parameters are studied. In addition, a comparative study between different EDFAs according to their dynamic behavior in order to judge their performance is presented.

An important consequence to the above and since a good amplifier design should achieve larger steady state values of the reservoir, since reservoir  $n(t)$  directly affect the gain, and shorter transients (i.e. smaller settling times), the design of the amplifier would encounter some contradictions. For example, one could increase the dopant ion density in the amplifier to achieve more gain, but the amplifier will have longer transients (i.e. larger settling time).

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