Performance Characteristics of Rare-Earth Doped Optical Amplifiers Pumped at 0.98 μm, 1.015 μm and 1.48 μm in Unsaturated Gain Regime

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***Abstract*-This paper gives a numerical investigation for the performance of optical amplifiers in the unsaturated gain regime. Erbium doped optical amplifiers (EDFAs) pumped at 0.98 μm and 1.48 μm are studied in alumino-germanosilicate glass hosts. The performance of praseodymium doped optical amplifier (PDFA) in a ZBLAN host is also investigated at a pump wavelength of 1.015 μm. For both EDFA and PDFA the noise figure and the unsaturated gain are calculated for different input signal wavelengths and used to judge the amplifier's performance.**

I. INTRODUCTION

Rare-earth doped fiber amplifiers play a significant role in the progressive development of optical communication systems due to their high gain, low noise figure, long transmission distances and increased channel capacity [1-4]. In the low-loss wavelength region for silica optical fibres, at 1.55 μm, erbium-doped fiber amplifiers (EDFAs) provide for more efficient operation at pump wavelengths of 0.98 μm and 1.48 μm than Raman amplifiers, Brillouin amplifiers and travelling-wave semiconductor laser amplifiers [5, 6].

Another rare earth element of interest is praseodymium as it is capable of providing gain at around 1.31 μm, another low loss window in silica fibres and also the wavelength corresponding theoretically to zero dispersion [7-9]. However, praseodymium-doped fiber amplifiers (PDFAs) have a very short upper-state lifetime due to nonradiative decay processes. To overcome this problem special types of glass hosts are generally used.

The gain associated with rare-earth doped amplifiers depends on the pump power, signal wavelength, fiber length, and doping concentration. These factors can all be tailored according to the intended application. The effect of the host material on the optical noise figure has recently been established [10]. The material controls the performance through its absorption and emission cross sections. The study of different values of the pump wavelength has showed that, in the high gain limit, optimum results are achieved at 1.48 μm for EDFA and 1.015 μm for PDFA.

In this paper we continue investigating both types of optical amplifiers through the calculation of the gain and the noise figure in the unsaturated gain regime. EDFAs are studied in alumino-germanosilicate glass hosts for two different pump schemes, at 0.98 μm and 1.48 μm. PDFAs in a ZBLANfiber are investigated when pumped at 1.015 μm.

II. THEORY

A. The Rate Equation

An optical amplifier uses a pumping signal of wavelength λ_p and a power level P_p that is usually a function of the transmitted coordinate z. Such a signal amplifies the transmitted signal λ_s that is characterized by a power level P_s. The pumping is either achieved in forward or backward mechanisms. In forward pumping (+) the pump power decreases in the transmission direction while in the backward case (−), the pumping signal is launched from the output end of the fiber. In terms of these variables and the saturation levels of the optical powers P_{sat} ($\lambda_{p,s}$), the normalized pump power q (z) and the normalized signal power p (z) are

$$q^{\pm}(z) = P_p^{\pm}(z) \,/\, P_{sat}(\lambda_p), \tag{1a}$$

$$p^{\pm}(z) = P_s^{\pm}(z) \,/\, P_{sat}(\lambda_s), \tag{1b}$$

Equation (1) is valid only in the case of confined doping in which the dopant has a constant concentration ρ_0 over a radial distance a_0 within the fiber. In such a case the rate equations for the optical powers are [1]

$$\pm\frac{dq^{\pm}}{dz} = \left(\gamma_{ep} - \gamma_{ap}\right) q^{\pm}, \tag{2}$$

$$\pm\frac{dp_k^{\pm}}{dz} = \left(\gamma_{ek} - \gamma_{ak}\right) p_k^{\pm} + 2\gamma_{ek}\, p_{0k}, \tag{3}$$

where p_k is the normalized sum of the signal power and the noise power introduced by amplified spontaneous emission (ASE), p_{0k} is the normalized input power and γ_{ij} corresponds to the emission (i=e) and absorption (i=a) coefficients.

For a 2-level pumping scheme, these coefficients are

$$\gamma_{ek} = \alpha_k \frac{\sum_j \frac{\eta_k}{1+\eta_j}\left(p_j^+ + p_j^-\right)}{1+\sum_j \left(p_j^+ + p_j^-\right)}, \tag{4}$$

$$\gamma_{ak} = \alpha_k \frac{1+\sum_j \frac{\eta_j}{1+\eta_j}\left(p_j^+ + p_j^-\right)}{1+\sum_j \left(p_j^+ + p_j^-\right)}, \tag{5}$$

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where α_k and η_k are the dopant absorption coefficient and the ratio between emission and absorption cross-sections for the transmitted signal, respectively, and the summations are taken over all possible electronic transitions.

In the case of 3-level pumping scheme (4) and (5) become

$$\gamma_{ep} = \alpha_p \frac{1+\sum_j \frac{\eta_j}{1+\eta_j}\left(p_j^+ + p_j^-\right)}{1+q^+ + q^- + \sum_j \left(p_j^+ + p_j^-\right)}; \quad \gamma_{ap} = 0\,, \tag{6}$$

$$\gamma_{ek} = \alpha_k \frac{\eta_k\left(q^+ + q^-\right) + \sum_j \frac{\eta_k}{1+\eta_j}\left(p_j^+ + p_j^-\right)}{1+q^+ + q^- + \sum_j \left(p_j^+ + p_j^-\right)}, \tag{7}$$

$$\gamma_{ak} = \alpha_k \frac{1+\sum_j \frac{\eta_j}{1+\eta_j}\left(p_j^+ + p_j^-\right)}{1+q^+ + q^- + \sum_j \left(p_j^+ + p_j^-\right)}, \tag{8}$$

where α_p is the dopant absorption coefficient at λ_p.

The absorption coefficients are defined in terms of $\sigma_{ap,\,ak}$, the absorption cross-sections at $\lambda_{p,\,k}$, by

$$\alpha_p = \rho_0\, \sigma_{ap}\, \Gamma_p\,, \tag{9}$$

$$\alpha_k = \rho_0\, \sigma_{ak}\, \Gamma_s\,, \tag{10}$$

with $\Gamma_{p,\,s}$ the overlap factors at $\lambda_{p,\,s}$ and ρ_0 the dopant density.

B. *Unsaturated Gain Regime*

Our calculations are made for the case of the unsaturated gain regime, in which p_k at any point of the fiber length is much smaller than the total normalized pump power $(q^+ + q^-)$. It may also be considered as the case in which the signal gain is independent of the input signal power. In this case, the unsaturated absorption and emission coefficients $\bar{\gamma}_{ap}$, $\bar{\gamma}_{ek}$ and $\bar{\gamma}_{ak}$, based on (4) - (8), are

$$\bar{\gamma}_{ep} - \bar{\gamma}_{ap} = -\alpha_p \frac{1}{1+q^+ + q^-}, \tag{11}$$

for both 2-level and 3-level schemes, and

$$\bar{\gamma}_{ek} = \alpha_k \frac{\frac{\eta_k}{1+\eta_p}\left(q^+ + q^-\right)}{1+q^+ + q^-}, \tag{12}$$

$$\bar{\gamma}_{ak} = \alpha_k \frac{\frac{\eta_p}{1+\eta_p}\left(q^+ + q^-\right)}{1+q^+ + q^-}, \tag{13}$$

only for the 2-level system.

From (2), (3) and (11) - (13), one can get the rate equations for the unsaturated gain regime with confined doping as

$$\pm\frac{dq^\pm}{dz} = -\alpha_p \frac{1}{1+q^+ + q^-} q^\pm - \alpha'_p\, q^\pm\,, \tag{14}$$

$$\pm\frac{dp_k^\pm}{dz} = \alpha_k \frac{1}{1+q^+ + q^-}\left[\left\{\frac{\eta_k - \eta_p}{1+\eta_p}\left(q^+ + q^-\right) - 1\right\} p_k^\pm + \frac{\eta_k}{1+\eta_p}\left(q^+ + q^-\right)\left(2\,p_{ok}\right)\right] - \alpha'_s\, p_k^\pm\,, \tag{15}$$

for both 2-level and 3-level systems, with $\alpha'_{p,\,s}$ the background absorption coefficient at $\lambda_{p,\,s}$.

Solving (14) and (15) we obtain an expression for the relation between the input pump powers (q_0^+ and q_L^-) and the output pump powers (q_L^+ and q_0^-) as a function of the fiber gain (G_k) at the signal wavelength, as

$$q_L^+ = q_0^+ \exp\left(-A_{pk}\, L\right), \tag{16}$$

$$q_0^- = q_L^- \exp\left(-A_{pk}\, L\right), \tag{17}$$

where A_{pk} is the gain dependant pump absorption coefficient that is defined through [1]

$$A_{pk} = \alpha_p \frac{1+\eta_p}{1+\eta_k}\left\{\left(1+\varepsilon_k\right)\left(1-bC\right)\varepsilon_p - \frac{\log G_k}{\alpha_k L}\right\}, \tag{18}$$

$$C = \frac{\frac{\eta_p - \eta_k}{1+\eta_p} + \varepsilon_k}{1+\varepsilon_k}, \tag{18a}$$

with $b = (1+\varepsilon_p)/\varepsilon_p$ and $\varepsilon_{p,\,k} = \alpha'_{p,\,k}/\alpha_{p,\,k}$.

C. *Unsaturated Gain and the Optical Noise Figure*

We consider the general case, in which the pump power decreases along the amplifier length. This is done for both cases of pumping configurations.

From the rate equation for the unsaturated gain regime with confined doping, (14) - (18), we calculate the gain G_k and the ASE noise photon numbers in the forward and backward directions, $N^+(\lambda_k, q_L)$ and $N^-(\lambda_k, q_0)$, respectively, as [1]

$$G_k = G_k^+(q_L) = G_k^-(q_0) = \exp\left\{\frac{\alpha_k}{\alpha_p}\left[\frac{\eta_k - \eta_p}{1+\eta_p}(q_0 - q_L) - \log\left(\frac{q_0}{q_L}\right)\right]\right\}, \tag{19}$$

$$N^{+}(\lambda_k, q_L) = \left(\frac{\alpha_k}{\alpha_p}\right)\left(\frac{\eta_k}{1+\eta_p} I^{+}(q_0, q_L)\right) \times \exp\left[\left(\frac{\alpha_k}{\alpha_p}\right)\left(\log(q_L) - \frac{\eta_k - \eta_p}{1+\eta_p} q_L\right)\right], \tag{20}$$

$$N^{-}(\lambda_k, q_o) = \left(\frac{\alpha_k}{\alpha_p}\right)\left(\frac{\eta_k}{1+\eta_p} I^{-}(q_0, q_L)\right) \times \exp\left[\left(-\frac{\alpha_k}{\alpha_p}\right)\left(\log(q_0) - \frac{\eta_k - \eta_p}{1+\eta_p} q_0\right)\right], \tag{21}$$

with

$$I^{\pm}(q_0, q_L) = \int_{q_L}^{q_0} \frac{\exp\left(\pm\frac{\alpha_k}{\alpha_p}\frac{\eta_k - \eta_p}{1+\eta_p} x\right)}{x^{(\pm\alpha_k/\alpha_p)}} dx. \tag{22}$$

In the case of neglected background losses, i.e. $\alpha_{p,k} \to 0$, (16) and (18) are reduced to

$$q_L = q_0 \exp(-A_{pk} L), \tag{23}$$

$$A_{pk} = \alpha_p\left(\frac{1+\eta_p}{1+\eta_k}\right)\left(\frac{\eta_k - \eta_p}{1+\eta_p} - \frac{\log G_k}{\alpha_k L}\right). \tag{24}$$

Eliminating G_k from (19), (23) and (24), one can get the following relation between the pump's input and output normalized powers

$$q_0 - q_L = \alpha_p L - \log\left(\frac{q_0}{q_L}\right). \tag{25}$$

In terms of G_k and the ASE noise photon numbers, the equivalent input noise factors for the forward and backward pumping schemes, $n_{eq}^{\pm}$, are defined by [1]

$$n_{eq}^{\pm}(\lambda_k) = \frac{N^{\pm}(\lambda_k)}{G_k}, \tag{26}$$

and the optical noise figures $F_0^{\pm}$ are

$$F_0^{\pm}(\lambda_k) = \frac{1 + 2N^{\pm}(\lambda_k)}{G_k}. \tag{27}$$

The equivalent noise factors and noise figures are then obtained from (19) - (21) and (26), (27) in the forms

$$n_{eq}^{+}(\lambda_k) = \left(\frac{\alpha_k}{\alpha_p}\right)\left(\frac{\eta_k}{1+\eta_p} I^{+}(q_0, q_L)\right) \times \exp\left[\left(\frac{\alpha_k}{\alpha_p}\right)\left(\log(q_0) - \frac{\eta_k - \eta_p}{1+\eta_p} q_0\right)\right], \tag{28}$$

$$n_{eq}^{-}(\lambda_k) = \left(\frac{\alpha_k}{\alpha_p}\right)\left(\frac{\eta_k}{1+\eta_p} I^{-}(q_0, q_L)\right) \times \exp\left[-\left(\frac{\alpha_k}{\alpha_p}\right)\left(\log(q_L) - \frac{\eta_k - \eta_p}{1+\eta_p} q_L\right)\right], \tag{29}$$

$$F_o^{\pm}(\lambda_k) = 2 n_{eq}^{\pm}(\lambda_k) + \frac{1}{G_k}. \tag{30}$$

It is clear from the above equations that the equivalent input noise factors and the optical noise figures depend strongly on the input and output pump powers as well as the parameters relating the dopant to the glass host material ($\alpha_{p,k}$ and $\eta_{p,k}$).

The input pump power is then expressed as a function of the gain G_k by eliminating q_L from (23) - (25) as

$$q_0 = \alpha_p L \frac{Q_k}{1 - \exp(-\alpha_p L(1 - Q_K))}, \tag{31}$$

with

$$Q_k = \frac{1+\eta_p}{1+\eta_k}\left(1 + \frac{\log(G_k)}{\alpha_k L}\right). \tag{32}$$

The output pump power is then written, using (23) and (31), in terms of the function Q_k as

$$q_L = q_0 \exp(-\alpha_p L(1 - Q_k)). \tag{33}$$

For the forward pumping case the pump power level at the fiber's output is always smaller than its value at the launching end and hence, we conclude from (33) that Q_k must be smaller than one.

Finally, from (32), we can get the following condition for the peak gain

$$G_k < \exp\left(\frac{\eta_k - \eta_p}{1+\eta_p}\alpha_k L\right). \tag{34}$$

Equation (34) sets the upper limit for the gain, G_{max}, through which the amplifier's length and the rest of the performance parameters can be determined.

III. Results

A. Er^{3+} Doped Fiber Amplifier Pumped at 0.98 μm

In the following calculations, we assumed that the required maximum gain is not more than 25 dB. From (34), with η_p = 0, i.e., operating as a 3-level laser system, η_s (λ_s = 1.53 μm) = 0.92, the pump absorption coefficient α_p = 0.5 dB/m and the peak signal absorption coefficient α_s (λ_s) = 1 dB/m, we got the required length of the fiber amplifier (L = 54 m). The peak gain G_{max} is assumed to vary between -24 dB and 24 dB.

Carrying out the integration (22) numerically, the equivalent input noise factor is calculated and the obtained values are used with the values obtained for the gain in calculating the noise figure from (30) for backward and forward pumping directions.

In Fig. 1, the gain spectrum for alumino-germanosilicate amplifiers, G (λ_s), is plotted for different values of G_{max}. It is obvious that the gain has its maximum value at a signal wavelength of 1.53 μm, as adjusted, and its shape depends on the characteristics of the emission and absorption cross sections.

Figs. 2 and 3 show the dependence of the equivalent noise factor on both input normalized pump power and signal wavelength for forward and backward pumping. We notice that, for high gain values, the equivalent input noise factor becomes independent of the signal wavelength and for the case of complete population inversion it reaches unity. We conclude that the equivalent input noise factor for the case of forward pumping is smaller than that for the backward pumping. The difference between the values of the equivalent input noise factor for the forward and backward pumping mechanisms decreases with the gain and that difference vanishes for the case of complete inversion.

The noise figure for both pumping configurations is plotted in Fig. 4 for different values of the maximum gain.

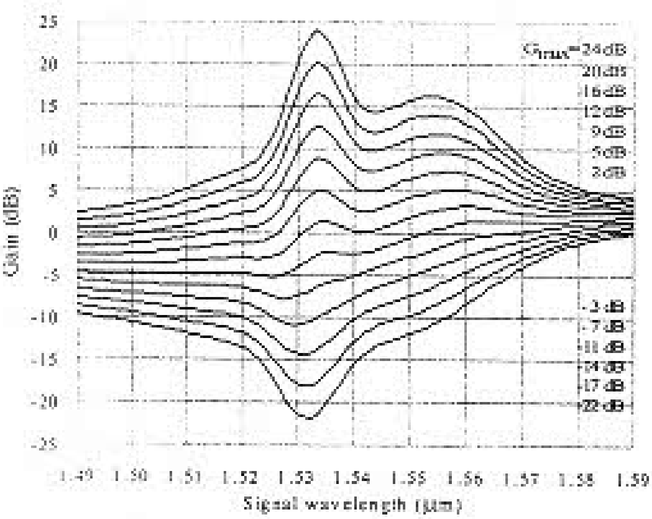

Fig. 1 Gain spectrum of Er^{3+} doped alumino-germanosilicate amplifier pumped at 0.98 μm for different values of the peak gain.

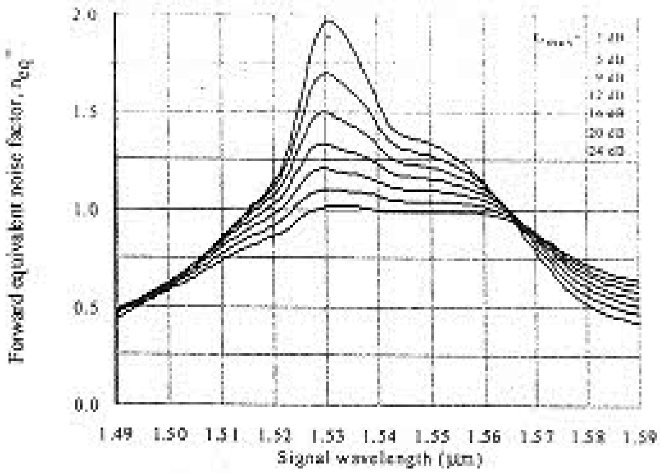

Fig. 2. Forward equivalent noise factor of Er^{3+} doped alumino-germanosilicate amplifier pumped at 0.98 μm.

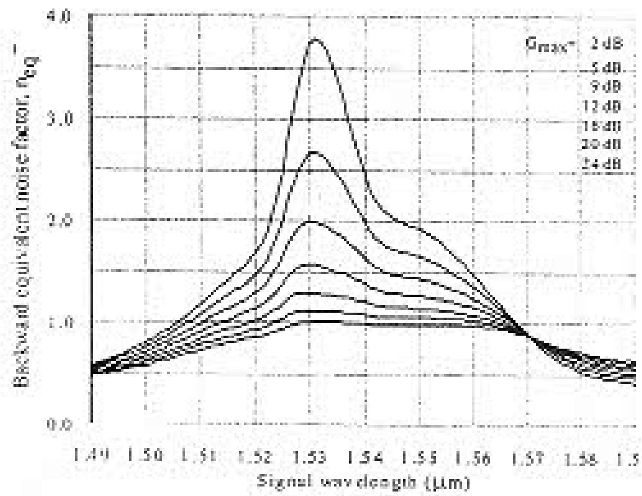

Fig. 3. Backward equivalent noise factor of Er^{3+} doped alumino-germanosilicate amplifier pumped at 0.98 μm.

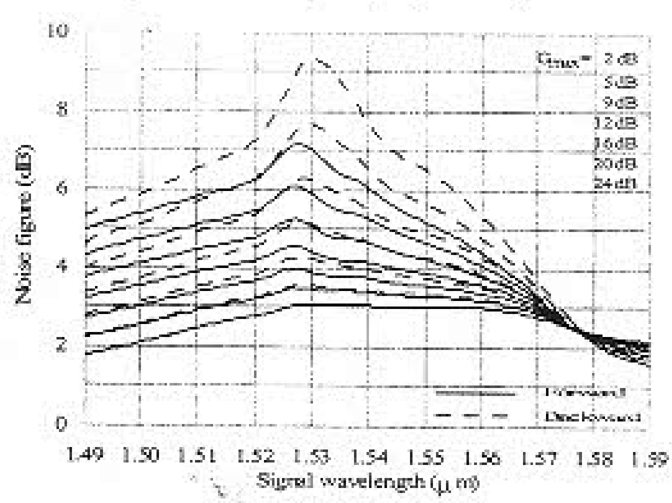

Fig. 4. Noise figure of Er^{3+} doped alumino-germanosilicate amplifier pumped at 0.98 μm.

From Fig. 4, it is clear that as the gain increases, the noise figure decreases for both forward and backward directions and that the forward pumping noise figure is less than the backward pumping one with that difference decreasing with the gain and vanishing for high gain. Fig. 4 shows that the noise figure reaches the lower quantum limit (3 dB) for high gain values.

B. Er^{3+} Doped Fiber Amplifier Pumped at 1.48 μm

We investigate the performance of EDFAs pumped at 1.48 μm, at which the amplifier operates as a two-level laser system. Again, we assume that the required maximum gain is not more than 25 dB. Similarly, with $\eta_p = 0.28$, η_s ($\lambda_s = 1.53$ μm) = 0.92, the pump absorption coefficient $\alpha_p = 0.44$ dB/m and the peak signal absorption coefficient α_s (λ_s) = 1 dB/m, we got the required length of the fiber amplifier in this case as (L = 100 m).

For erbium-doped alomino-germanosilicate optical amplifier, the gain is plotted as a function of the signal wavelength for different values of G_{max} in Fig. 5. The equivalent input noise factors are plotted for both forward and backward pumping schemes at different values of G_{max} in Figs. 6 and 7, respectively. The noise figures are plotted in Fig. 8.

From the plotted figures, one can see that the performance of the 2-level EDFAs at 1.48 μm differs from that of the 3-level EDFAs at 0.98 μm. Due to the stimulated emission at the pump wavelength, the equivalent input noise figures at 1.48 μm are always greater than that at 0.98 μm pump wavelength. Another difference also is in the lower values achieved at high gain for the noise figures that are always greater than the quantum limit of 3 dB.

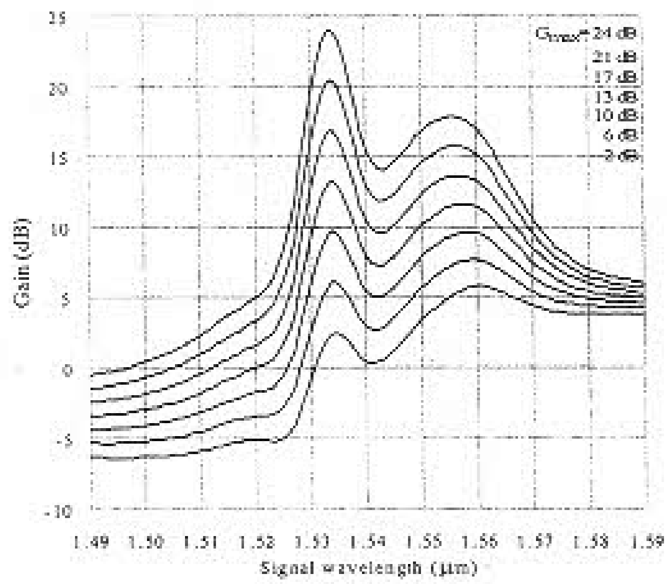

Fig. 5. Gain spectrum of Er^{3+} doped alumino-germanosilicate amplifier pumped at 1.48 μm for different values of the peak gain.

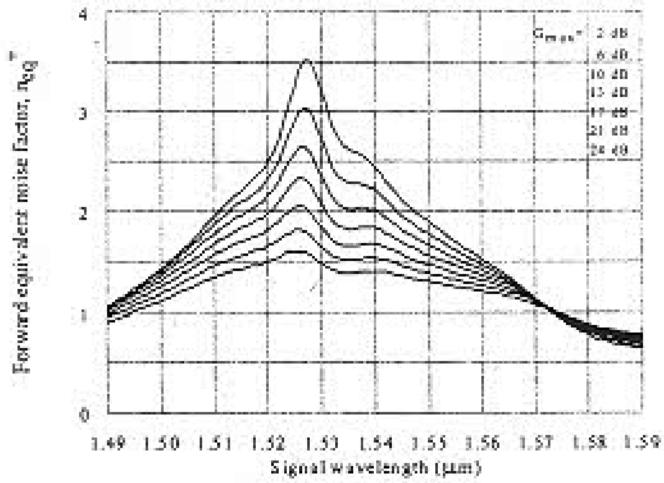

Fig. 6. Forward equivalent noise factor of Er^{3+} doped alumino-germanosilicate amplifier pumped at 1.48 μm.

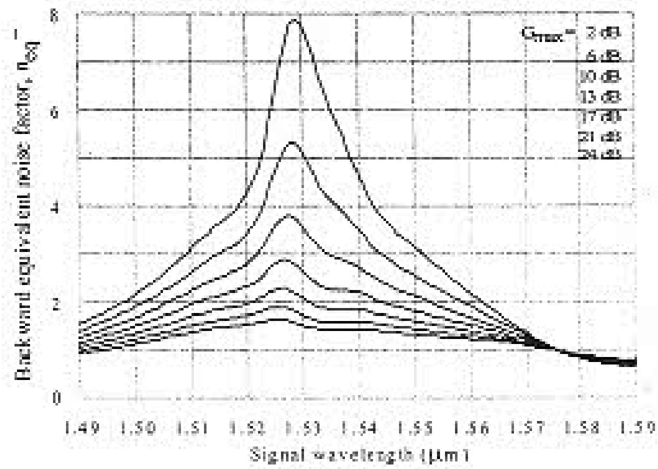

Fig. 7. Backward equivalent noise factor of Er^{3+} doped alumino-germanosilicate amplifier pumped at 1.48 μm.

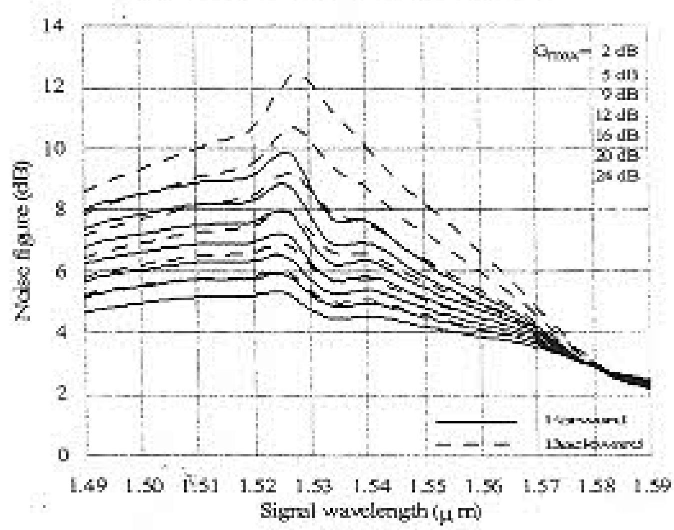

Fig. 8. Noise figure of Er^{3+} doped alumino-germanosilicate amplifier pumped at 1.48 μm.

C. Pr^{3+} Doped Fiber Amplifier Pumped at 1.015 μm

To perform a comparison between EDFAs and PDFAs, Pr^{3+} doped ZBLAN fiber amplifier pumped at 1.015 μm is considered. At that wavelength, the amplifier operates as a 3-level system. The performance curves of PDFA are plotted in Figs. 9-11. The gain performance of Pr^{3+} doped fiber amplifier is similar to that obtained experimentally reported in [11].

IV. Conclusion

Erbium doped optical amplifiers pumped at 0.98 μm and 1.48 μm are studied for alumino-germanosilicate glass hosts. The performance of praseodymium doped ZBLAN optical amplifiers is also investigated at a pump wavelength of 1.015 μm. We found that the performances depend on the emission and absorption cross sections for the transmitted photons. We reported also that the equivalent input noise is greater when the propagation direction is opposite to the pump direction. For erbium-doped fiber amplifier pumped at 0.98 μm, the 3 dB noise figure limit could be achieved only for the high gain regions. The noise figure for EDFAs pumped at 0.98 μm is greater than that of EDFAs pumped at 1.48 μm. The maximum gain is achieved at a signal wavelength of 1.53 μm for EDFAs and 1.31 μm for PDFAs.

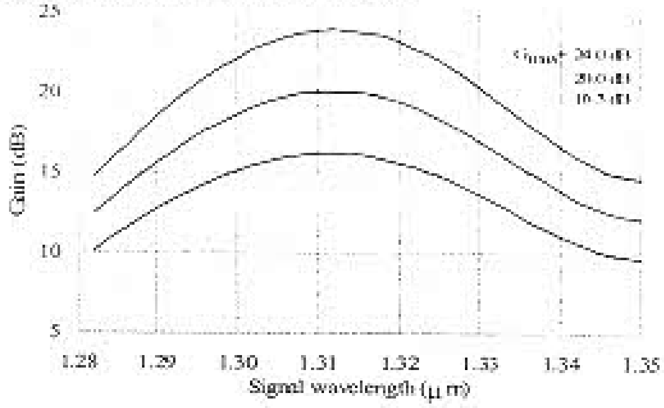

Fig. 9. Gain spectrum of Pr^{3+} doped ZBLAN amplifier.

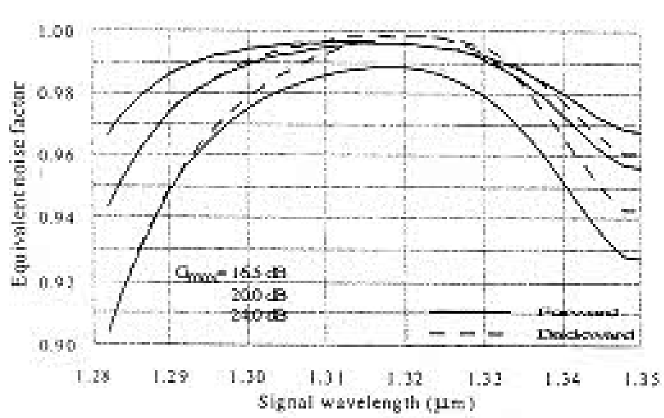

Fig. 10. Equivalent noise factor of Pr^{3+} doped ZBLAN amplifier.

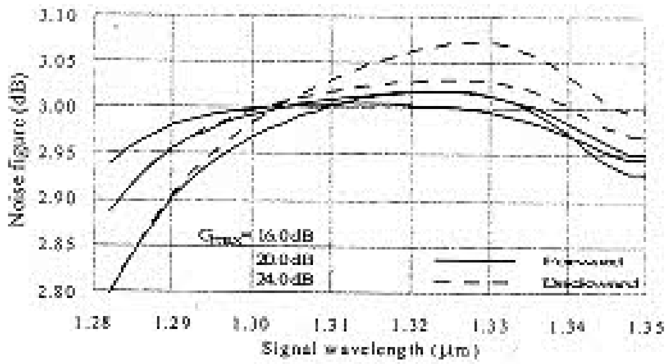

Fig. 11. Noise figure of Pr^{3+} doped ZBLAN amplifier.

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