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Chromatic Dispersion Characteristics of Single-mode Optical Fibers with Kerr-Type Nonlinearity

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Summary

Shooting method technique has been used to investigate the propagation and dispersion characteristics of nonlinear single-mode optical fibers for both step and quadratic profiles. Reasonable approximations are made to match the fiber cylindrical geometry. An error function is defined to estimate the discrepancy between the expected boundary conditions and that obtained by numerical integration of the wave equation through Runge-Kutta method. The secant method is then used to obtain the correct eigen values of the propagation constant and its normalized frequency derivatives through a two-path iterative manner. The effects of the injected optical power and the nonlinear Kerr coefficient on the effective index and the dispersion parameters are considered. The performance of the optical fiber under nonlinear conditions is found to be governed not only by the overall value of the normalized frequency but also by its constituent parameters. The obtained results are compared with the published ones giving a fair agreement.

1 Introduction

Several attempts are exerted to fully utilize the capabilities of optical fibers. To increase system margins, higher transmitter powers or lower fiber losses are required [1]. Nonlinear interactions between the information-bearing wave and the fiber material are noticed upon injecting high optical power. One of these interactions is Kerr nonlinearity that results from the dependence of the refractive index on the electric-field intensity [2].

The critical limitation in realizing the full bandwidth capability of optical transmission systems is pulse distortion due to dispersion. Using Kerr nonlinearity has been made to overcome this problem. Pulse broadening due to dispersion is opposed by the sharpness due to Kerr nonlinearity; the fact that may lead to a stable solitary solution of the optical pulse [3].

Numerous solution procedures have been proposed to calculate the chromatic dispersion parameters for linear optical fibers [3–7]. However, for fibers operating in the nonlinear regime, quite different propagation behavior does result due to the strong interplay between the chromatic dispersion and the material nonlinearity [4]. Several numerical methods have been developed for the analysis of one-dimensional optical waveguide problems. All these methods can be classified into two categories, namely; the shooting method [6] and the matrix eigensystem methods [4]. Li-Pen Yuan [6] proposed a numerical technique, based on the shooting method for determining the propagation characteristics of nonlinear planar optical waveguides with a nonlinear core cladded by two linear films. The matrix eigensystem methods were used by H. Y. Lin [4] and K. Okamoto [7] to analyze the propagation characteristics and chromatic dispersion of cylindrical fibers with Kerr-type nonlinearity.

In a previous work [8], we have used the shooting method to investigate the modal field distribution and the effective index of the cylindrical fibers with core nonlinearity. This method involves the numerical integration of the wave equation to yield the near field and its first derivative with respect to the radial direction. A comparison is made between the expected boundary condition and the calculated one at the core-cladding interface. With two errors being measured, the initial two guesses of the propagation constant are iteratively corrected till the error reduces to a specified order.

In the present paper, we rely on the same technique to analyzing the effect of Kerr nonlinearity on the dispersion parameters. The near-field distribution and the effective index are first obtained from applying the shooting method on the nonlinear wave equation. Then the same method is used twice more to obtain the first and the second derivatives of the normalized propagation constant with respect to the normalized frequency. The governing equations for these two derivatives are obtained from differentiating the original wave equation and the boundary condition once and twice, respectively. The effect of nonlinearity appearance, through eit-

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her large Kerr-coefficient value or large injected optical power, on the dispersion parameters is also represented.

2 Basic equations and numerical method

For circularly symmetrical nonlinear fibers, the overall refractive index of a silica core doped with GeO_2 may be expressed as:

$$n^2 = n_1^2(\lambda,\chi)[1-2\Delta f(r)] + \alpha_c|E|^2, \tag{1}$$

where n_1 is the refractive index at the core center given as a function of the wavelength λ and the doping ratio χ according to Sellmeier relation [9], Δ is the relative index difference, f(r) is the profile definition function, E is the modal field, and α_c is the nonlinear coefficient defined as [4]:

$$\alpha_c = \frac{n_2^2(\lambda)n_{NL}(r)}{\eta_o}, \tag{2}$$

where n_2 is the wavelength-dependent cladding refractive index, η_o is the free-space impedance, and n_{NL} is the nonlinear Kerr coefficient in m^2/W which is assumed to be constant along the core radius. The governing differential equation for the electric field radial component in both core and cladding regions for single mode fibers have the general forms, respectively [10]:

$$\frac{d^2E}{dr^2} + \frac{1}{r}\frac{dE}{dr} + \frac{V^2}{a^2}\left[1-\frac{\alpha_c E^2}{2n_1^2\Delta} - b - f(r)\right]E = 0, \tag{3a}$$

$$\frac{d^2E}{dr^2} + \frac{1}{r}\frac{dE}{dr} - \frac{V^2}{a^2}bE = 0, \tag{3b}$$

where a, b and V are the core radius, the normalized frequency, and the normalized propagation constant, respectively.

Two restrictions govern the solution of the above differential equations. First, the electric field at the core center has a maximum value (initially assumed) and zero radial derivative. This value depends on the injected optical power through the fiber cross section according to the proper integration [4]:

$$P = \frac{\pi n_2}{\eta_o}\int_0^R E^2(r)\,r\,dr, \tag{4}$$

where R represents the extension of integration which, in this paper, is taken to be ten times the core radius. Second, a matching condition is necessary to achieve the continuity at the core-cladding interface. This occurs only for certain b value which is given as a guess at the beginning of the numerical procedure. For continuous solution we must have:

$$\left.\frac{1}{E}\frac{dE}{dr}\right|_{r=a} = -\frac{W}{a}\frac{K_1(W)}{K_0(W)}, \tag{5}$$

where K_0 and K_1 are the modified Bessel functions of zero and first order, respectively, and W is the cladding decay parameter.

Before numerically integrating the wave equation, Gaussian approximation has been used to estimate the electric field and its first radial derivative at the first step. The used Gaussian function has the form:

$$E = E_0 \exp\left(-r^2/\omega_o^2\right) \tag{6}$$

where ω_o is the spot size of the fundamental mode [10] which is given for step- and quadratic-index profiles, respectively, by:

$$\frac{\omega_o}{a} = 0.65 + 1.619V^{-1.5} + 2.879V^{-6}, \tag{7a}$$

$$\frac{\omega_o}{a} = \sqrt{\frac{2}{V}}. \tag{7b}$$

Proceeding with the integration till reaching the core cladding interface, the deviation of dE/dr from the expected value is evaluated using an error function defined as:

$$ERR = -\frac{a}{W}\frac{dE}{dr} - \frac{K_1(W)}{K_0(W)}E. \tag{8}$$

The whole process described above is performed twice for two initial guesses of the effective index, β_e. Being the error function estimated twice, the secant method is used to decide a new guess of β_e:

$$\beta_{new} = \beta_2 - \frac{ERR2(\beta_2-\beta_1)}{(ERR2-ERR1)}, \tag{9}$$

where β_1 and β_2 are the initial guesses of the effective index while ERR1 and ERR2 are the corresponding error values.

The correct value of β_e is achieved by performing the above procedure in a two-path iterative manner (to avoid guesses-dependence results) where each time β_{new} is considered as a new guess. After that, the modal field distribution along the core and cladding regions is obtained by integrating (3). The total power is then determined by performing the integration of (6) and comparing it with the known injected power. The preassumed value of the electric field at the core center is then replaced by the correct one according to its dependence on the square root of P. The whole procedure is repeated to finally obtain the correct field distribution with the correct b value.

Obtaining the normalized propagation constant, b, and the modal field distribution, the chromatic dispersion parameters are now to be estimated. To obtain the first derivative of b with respect to V, we repeat the same procedure used to get b and the modal field distribution while using another set of equations. First (3) is differentiated with respect to V to get:

$$\frac{d^2\dot{E}}{dr^2}+\frac{1}{r}\frac{d\dot{E}}{dr}+F_1(E)E+F_2(E)\dot{E}=0, \tag{10}$$

where the dot denotes the differentiation with respect to V,

$$F_1(E)=\frac{1}{a^2}\left\{2V\left[1+\frac{\alpha_c}{2n_1^2\Delta}E^2-b-f(r)\right]-V^2\dot{b}\right\}, \tag{11}$$

and

$$F_2(E)=\frac{V^2}{a^2}\left[1+\frac{3\alpha_c}{2n_1^2\Delta}E^2-b-f(r)\right]. \tag{12}$$

The first step of integration is obtained by differentiating (6) with respect to V and then with r. The matching condition at the core-cladding interface is got by differentiating (5) with respect to V:

$$\left.\frac{d\dot{E}}{dr}\right|_{r=a}=-W\left[Z\dot{E}+E\dot{W}\frac{dZ}{dW}\right]_{r=a}, \tag{13}$$

where

$$Z=-W\frac{K_1(W)}{K_o(W)}, \tag{14}$$

$$\frac{dZ}{dW}=W\left(1-\frac{Z^2}{W^2}\right), \tag{15}$$

and

$$\dot{W}=\frac{W}{V}+\frac{W\dot{b}}{2b}. \tag{16}$$

Similarly, to obtain the second derivative of b, (10) and (13) are differentiated with respect to V:

$$\frac{d^2\ddot{E}}{dr^2}+\frac{1}{r}\frac{d\ddot{E}}{dr}+F_3(E)E+F_4(E)\dot{E}+$$
$$+F_5(E)\dot{E}^2+F_2(E)\ddot{E}=0, \tag{17}$$

where

$$F_3(E)=\frac{2F_2(E)}{V^2}-\frac{4V\dot{b}+V^2\ddot{b}}{a^2}, \tag{18}$$

$$F_4(E)=\frac{4}{V}F_2(E)-\frac{2V^2\dot{b}}{a^2}, \tag{19}$$

and

$$F_5(E)=\frac{3V^2\alpha_c}{a^2n_1^2\Delta}. \tag{20}$$

The matching condition at the core-cladding interface is obtained by differentiating (14) again with respect to V to get:

$$\left.\frac{d\ddot{E}}{dr}\right|_{r=a}=\frac{1}{a}\cdot\left[Z\ddot{E}+2\dot{W}\frac{dZ}{dW}\dot{E}+\left(\ddot{W}\frac{dZ}{dW}+\dot{W}^2\frac{d^2Z}{dW^2}\right)E\right]_{r=a}, \tag{21}$$

where

$$\frac{d^2Z}{dW^2}=1-2Z+\frac{Z^2+2Z^3}{W^2}, \tag{22}$$

and

$$\ddot{W}=\frac{-W}{V^2}+\frac{\dot{W}}{V}+\frac{b\left(W\ddot{b}+\dot{W}\dot{b}\right)-W\dot{b}^2}{2b^2}. \tag{23}$$

Obtaining the correct values of the normalized propagation constant and its first and second derivatives, one can easily estimate the dispersion parameters; the mode delay factor, d and the normalized waveguide dispersion coefficient, g represented, respectively, by [4]:

$$d=\frac{d(Vb)}{dV}=V\dot{b}+b, \tag{24}$$

$$g=V\frac{d^2(Vb)}{dV^2}=V^2\ddot{b}+2V\dot{b}. \tag{25}$$

The chromatic dispersion parameter of a single mode fiber can be calculated from the formula [10]:

$$D_T=\frac{d\tau_g}{d\lambda}=-\frac{\lambda}{c}\frac{d^2\beta_e}{d\lambda^2}. \tag{26}$$

The second derivative of the effective index with respect to the wavelength has a complex form and represents the various dispersion mechanisms including the profile dispersion.

3 Numerical results and discussion

According to the algorithm represented above, a computer program is implemented to investigate the chromatic dispersion characteristics of the cylindrical optical fibers that exhibit nonlinear behaviour. Both step and quadratic profiles are considered. The number of segments, into which the core is divided, is chosen as one hundred which is found to give convergent results.

From (1) and (2) the nonlinear behaviour appears when either the injected optical power, the nonlinear coefficient, or both is large enough for affecting the nonlinear term of the core refractive-index. The effect of this is to increase the power confined in the fiber as shown in Fig. 1 by about 3.7 %. The nonlinearity effect, however, loses its significance while approaching the core-cladding interface while E decreases gradually.

The power normalized electric field E_n represented by the vertical axis of Fig. 1 reflects the power-confinement efficiency of the optical fiber. It is defined as [4]:

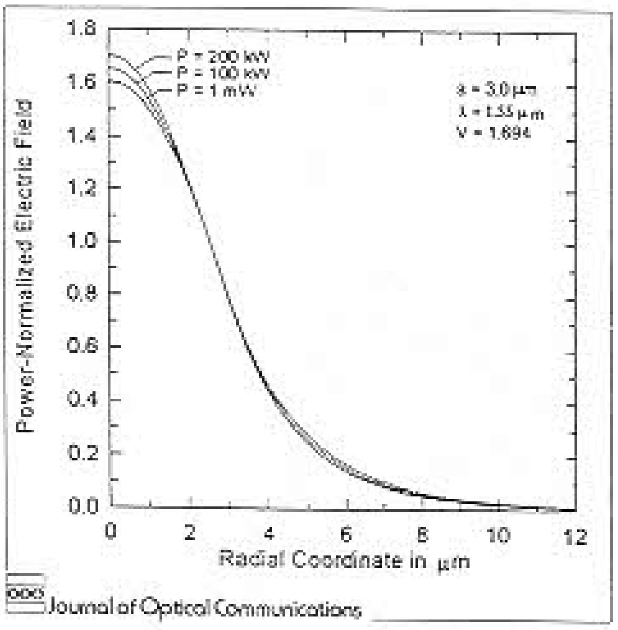

Fig. 1: Modal field distribution as a function of the radial coordinate for several input powers

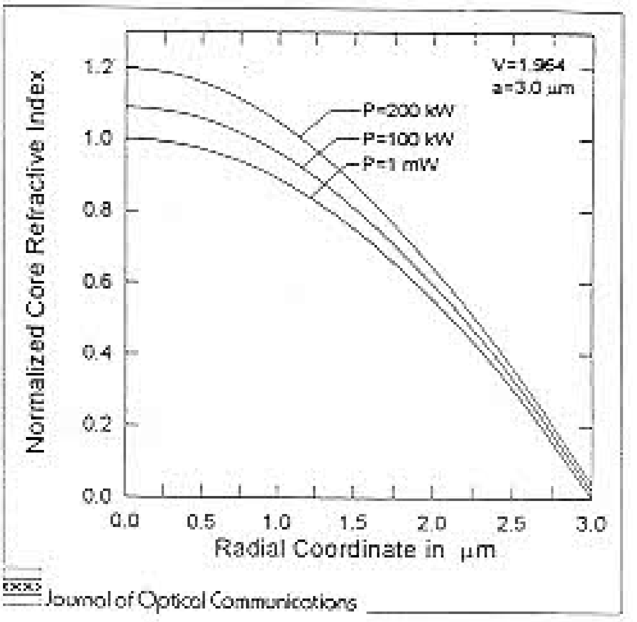

Fig. 3: The normalized core refractive index for the quadratic profile as a function of the radial coordinate for linear (1 mW) and nonlinear (100, 200 kW) operation

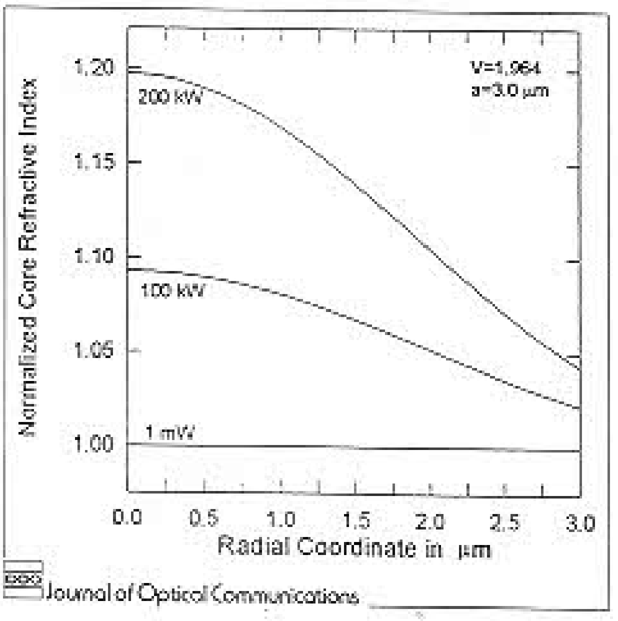

Fig. 2: The normalized core refractive index for step index profile as a function of the radial coordinate for linear (1 mW) and nonlinear (100, 200 kW) operation

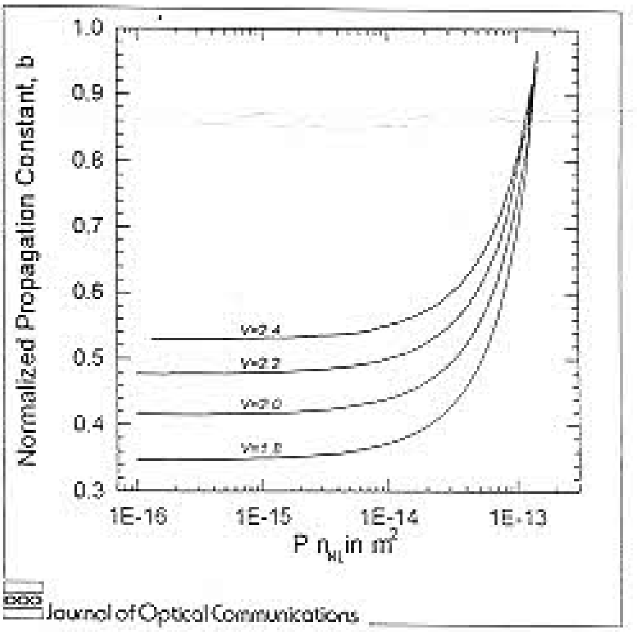

Fig. 4: The normalized propagation constant b as a function of $n_{NL}P$ product at different V-values

$$E_n = a\sqrt{\frac{n_2\pi}{P\eta_0}}\,E, \tag{27}$$

The nonlinear Kerr coefficient is chosen to be $3\cdot10^{-20}$ m²/W [4]. The 200 kW input power may appear impractical value. However, Kerr nonlinearity depends on both values of P and n_{NL}, so much less power could be effective when a material of large n_{NL} value is used.

The effect of the input power on the overall core refractive index is illustrated in Figs. 2 and 3 for step and quadratic index fibers, respectively. The normalized core refractive index which labels the vertical axis is defined as [7]:

$$q(r) = 1 - f(r) + \frac{n_1^2 n_{NL} E^2}{n_1^2 - n_2^2}. \tag{28}$$

For both profiles, injecting 200 kW optical power increases the normalized refractive index at the core center above the linear value of unity when only 1 mW is injected. Also for step index fiber, it is noted that Kerr nonlinearity has the same effect as self-grading the core refractive index. This effect appears also in quadratic index fibers but its own grading technique has a stron-

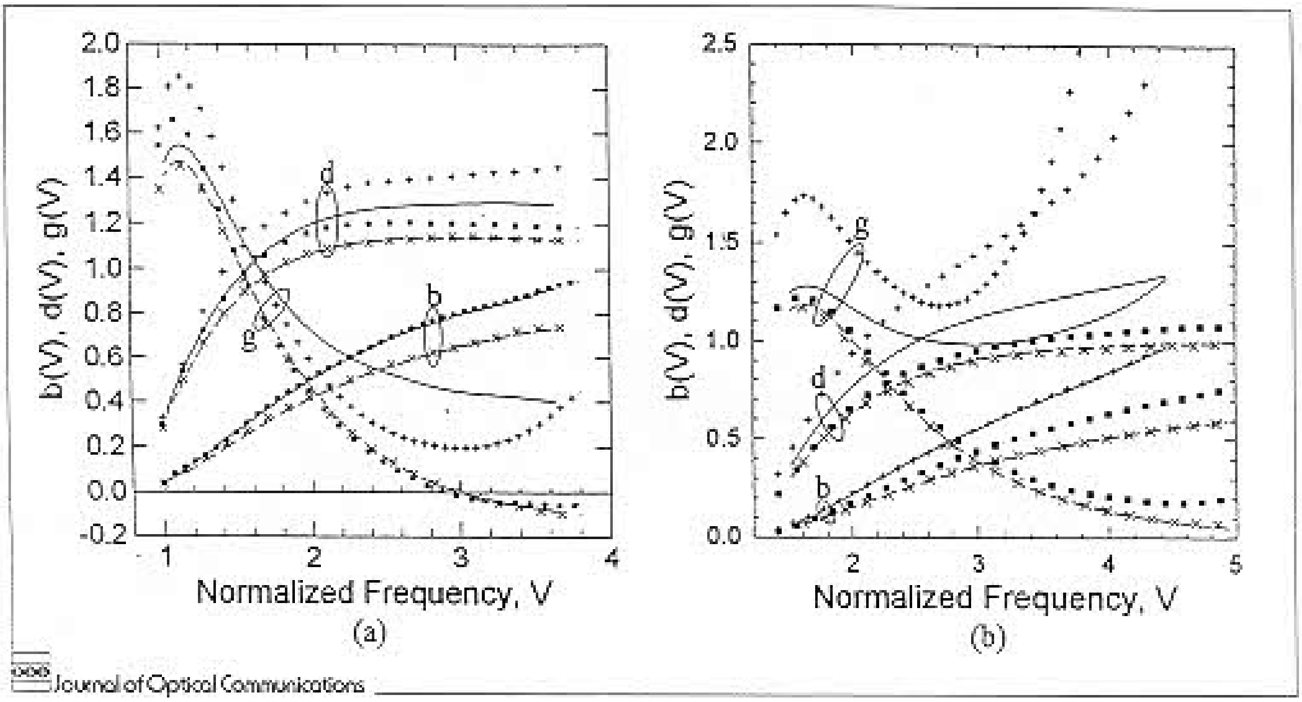

Fig. 5: The dispersion parameters b, d, and g as functions of V for (a) step and (b) quadratic index fibers the linear and nonlinear; the dashed curves are for P = 1 mW obtained using the present method while the crosses represent the same case obtained by the methods in [4] and [7]; the solid, the plus-sign, and the square curves are for P = 200 kW obtained using the present method, the method in [4], and the method in [9], respectively

ger effect; thus Kerr nonlinearity has somewhat weak effect on the quadratic index fibers except at large V-values when power confinement increases enough.

Since, as mentioned before, the nonlinear behavior of the optical fiber depends on both the input power and the Kerr nonlinear coefficient, one could unify this dependence by considering the effect of the product of both terms, i.e., Pn_{NL} on the normalized propagation constant b as illustrated in Fig. 4 for four V values. There is always a threshold value for this product at which the nonlinearity effects are seriously present. This threshold value is found to increase slowly with the normalized frequency.

The dispersion characteristics for both step and quadratic index fibers of core nonlinearity have been investigated. It is well known that the refractive index of any material has some kind of wavelength dependence. However, we will temporarily neglect this dependence to make a comparison with the works of both [4] and [7] who used $n_2 = 1.47$, $a\,(n_1^2 - n_2^2)^{1/2} = 0.22$ μm, and $n_{NL}P = 6.4 \cdot 10^{-15}$ m².

Figures 5a and 5b represent this comparison for both step and quadratic index fibers, respectively. The results corresponding to the linear case agree with that in both [4] and [7]. For nonlinear operation, the results of b for both step and quadratic index fibers agree with that of [4] while differ from that corresponding to quadratic index ones in [7]. The relation between b and b in [7] is actually incorrect. The results for d and g in [7] are also incorrect; the formula for d is wrong as pointed out in [11].

The performance of optical fibers under nonlinear operation is strongly affected by the wavelength dependence

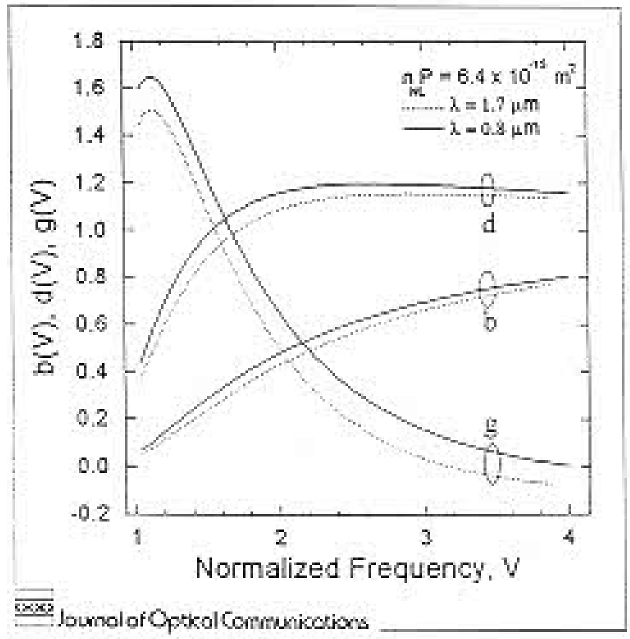

Fig. 6: The variation of the dispersion parameters b, d, and g with the normalized frequency for step index fibers with two λ values and with the same $n_{NL}P$ product value

of the refractive indices. The nonlinear term in the overall core refractive index depends on Kerr coefficient, optical power, and also the cladding refractive index which is wavelength dependent. This means that two completely different performances could be obtained at the same V value and at the same $n_{NL}P$ product too. This is illustrated in Fig. 6 which shows that two different dispersion parameters curves are obtained when opera-

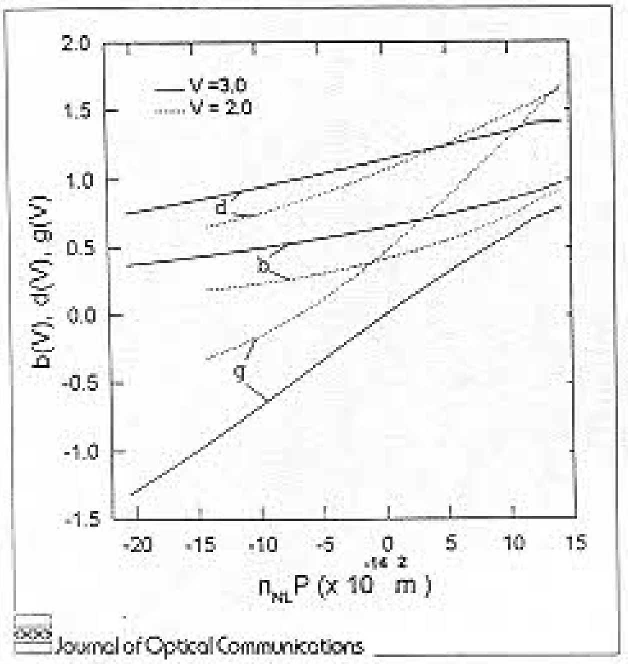

Fig. 7: The variation of the dispersion parameters b, d, g as functions of the product $n_{NL}P$ for two nonlinear step index fibers

ting at λ = 0.8 and 1.7 µm (the performance range of optical fibers).

When nonlinearity manifests itself, the characteristic curves of the dispersion parameters are not dependent alone on the normalized frequency (as in the case of linear operation). In addition, however, they depend on the nonlinear Kerr coefficient, injected optical power, and the cladding refractive index, which is inherently wavelength dependent.

The dependence of the dispersion parameters on the product $n_{NL}P$ is illustrated in Fig. 7 for two V values where n_2 is fixed at 1.444. The positive and negative regions of the horizontal axis correspond to fibers with self-focusing and self-defocusing effects, respectively. It is noted that the higher the normalized frequency, the stronger the nonlinearity effect.

The last point to be considered now is the effect of nonlinearity on the total dispersion for both step and quadratic index fibers. The total first order dispersion for both profiles is illustrated in Figs. 8a and 8b, respectively. The dotted curves represent the simple sum of the waveguide dispersion and the material dispersion while the solid curves correspond to (26) which includes the contribution of the profile dispersion. Positive nonlinearity shifts the zero dispersion point, ZDP to a longer wavelength while the reverse occurs for negative nonlinearity. Also, it is noted that ZDP for the quadratic profile occurs at a longer wavelength than for the step index one. At long wavelengths, the dispersion curves for both profiles tend to approach each other. In this region the material dispersion will be the dominant source of dispersion. Also the cladding refractive index becomes smaller and the nonlinear coefficient α_c does also; the fact that weakens the nonlinearity phenomena.

4 Conclusion

In this work, the second-order nonlinear wave equation and its normalized frequency derivatives has been solved for nonlinear optical fibers by Runge-Kutta numerical integration using the concept of the shooting method. The effect of Kerr nonlinearity on the modal field distribution, the propagation constant, and the dispersion parameters have been investigated for both step and quadratic index profiles. Kerr nonlinearity increases the optical power confinement. It is concluded that the nonlinear performance of the optical fiber does not only depend on the normalized frequency but

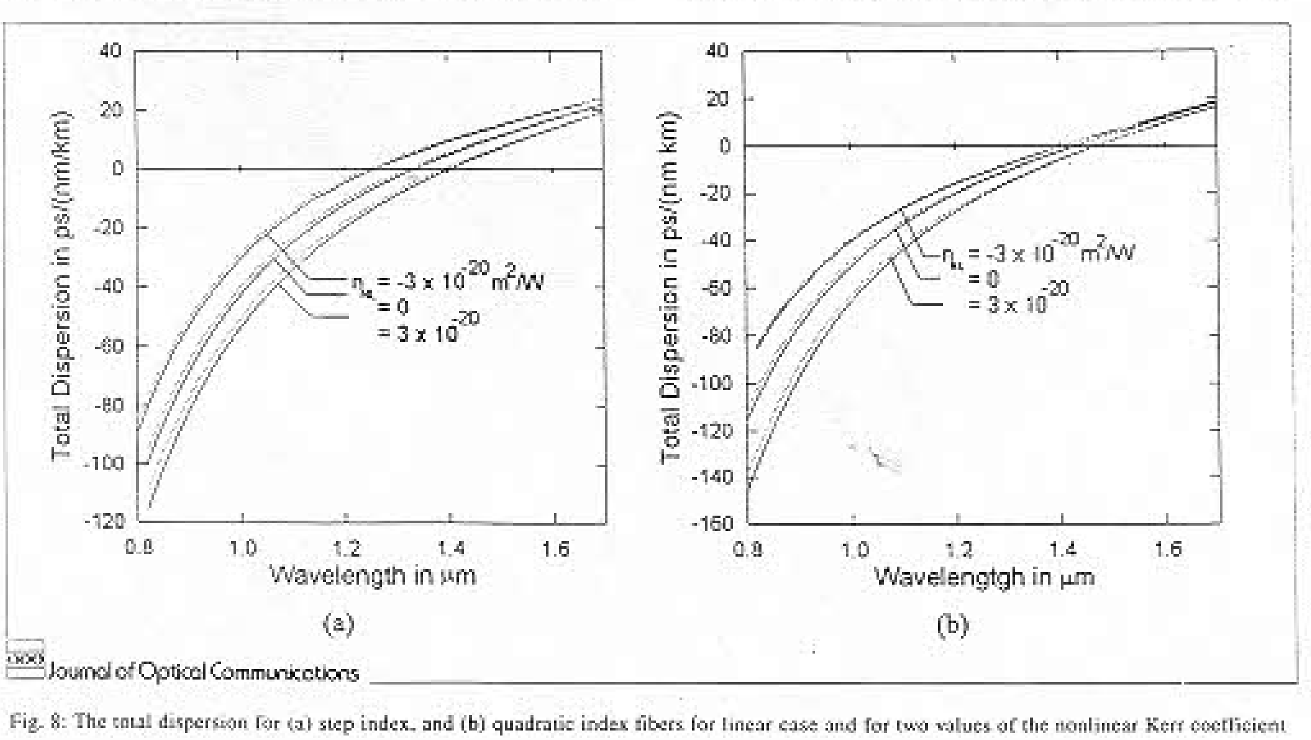

Fig. 8: The total dispersion for (a) step index, and (b) quadratic index fibers for linear case and for two values of the nonlinear Kerr coefficient

also on its constituent parameters. Complete dispersion behavior results as the step index profile becomes a graded index one according to the field-intensity profile. The efficiency of the used technique lies in the short computer time and the simplicity of the numerical manipulation in either linear or nonlinear regime.

References

[1] Andrew R. Chraplyvy: "Limitations on lightwave communications imposed by optical-fiber nonlinearities"; J. Lightwave Technol. 8 (1990), 1548

[2] Rogres H. Stolen: "Nonlinearity in fiber transmission"; Proc. of the IEEE 68 (1980), 1232

[3] Manoj Jain, Narkis Tzoar: "Propagation of nonlinear optical pulses in inhomogeneous media"; J. Appl. Phys. 49 (1978) 9, 4649

[4] Hoang Yan Lin, Hung-Chun Chang: "An efficient method for determining the chromatic dispersion characteristics of nonlinear single-mode optical fibers"; J. Lightwave Technol. 10 (1992), 1188

[5] J. Ariyasu, C. T. Seaton, G. I. Stegeman, J.V. Moloney: "New theoretical developments in nonlinear guided waves: Stability of TE_1 branches"; IEEE J. Quantum Electron QE-22 (1986) 6, 984–986

[6] Li-Pen Yuan: "A Numerical Technique for the Determination of the Propagation Characteristics of Nonlinear Planar Optical Waveguides"; IEEE J. Quantum Electron. QE-30 (1994), 134

[7] Katsunari Okamoto: "Chromatic dispersion characteristics of fibers with optical Kerr-effect nonlinearity"; J. Lightwave Technol. 7 (1989), 1988

[8] Moustafa H. Aly, M. Sami, A. Abouelwafa, Adel M. Wahba: "Near field and propagation constant of non-linear single-mode optical fibers: A simple numerical technique"; Alexandria Engineering Journal. 35 (1996) 4, D55

[9] J. W. Fleming: "Dispersion in step-index silicone-clad fibers"; Applied Optics. 18 (1979) 23, 4000

[10] John M. Senior: "Optical Fiber Communications; Principles and Practice"; London, Printice Hall, 1992

[11] R. A. Sammut, C. Pask: "Group velocity and dispersion in nonlinear optical fibers"; Opt. Lett. 16 (1991) 70–72