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Near Field and Propagation Constant of Nonlinear Single-mode Optical Fibers: A Simple Numerical Technique

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Summary

Near field distribrution and propagation constant of nonlinear single-mode optical fibers have been investigated. The shooting method technique is used and implemented into a computer code for the case of step index and graded index fibers. Some reasonable and accurate approximations were made to match the cylindrical geometry. An error function is defined to estimate the discrepancy between the expected boundary values and the computed ones obtained by numerical integration through the use of Runge-Kutta method. More accurate values are then depicted using the secant method. The accuracy and convergence of the results have been considered by performing these methods in a two-path iterative manner. The effects of various guiding parameters and the nonlinear Kerr coefficient on the modal field distribution and on the effective index are studied. Typical b-V curves for both linear and nonlinear fibers are presented. The cutoff value of the normalized frequency is investigated and approximate relations with the Kerr coefficient are deduced for both fiber types.

1 Introduction

The attractiveness of lightwave communications is the ability of silica-optical fibers to carry large amounts of information over long repeaterless spans. To utilize the available bandwidth, numerous channels at different wavelengths can be multiplexed on the same fiber and higher transmitter powers are required to increase system margins. All these attempts to fully utilize the capabilities of silica fibers will ultimately be limited by nonlinear interactions between the information-bearing light waves and the transmission medium. These optical nonlinearities may lead to interference, distortion, and excess attenuation of the optical signals, resulting in system degradation [1]. Extremely low-loss fibers and narrow-band lasers all move toward the regime where nonlinear effects become important [2].

During the last two decades, fibers operating in the nonlinear regime have stimulated much research interest since the interplay between both the chromatic dispersion and the propagation characteristics and the material nonlinearity has been found to result in quite different propagation behavior [3–6].

Several numerical methods have been developed for the analysis of planar optical waveguides. Nearly all these methods can be classified into two categories, namely the shooting method [7] and the matrix eigen system methods [3] and [8].

In the following study, the shooting method is used to investigate nonlinear cylindrical optical fibers with nonlinear Kerr-type core. This method involves integrating the wave equation numerically to yield the near field and its first derivative with respect to the corresponding unit directional vector measuring the expected error and correcting the initial guesses iteratively until the error reduces to a specified order. Most of the field power is confined in the core region while in the cladding region the electric field has a decaying form resulting in a really reduced probability of the presence of Kerr-type nonlinearity in the cladding region. So, cladding nonlinearity is excluded without tears.

Although the initial work concerned with nonlinear optical effects used relatively large core multimode fibers, more recently such phenomena have become very important within the development of low-loss, single-mode fibers [9]. Single-mode fibers are preferred for long-haul transmission and high bandwidth applications. The small core diameters, together with the long propagation distances that may be obtained with these fibers, has enabled the observation of certain nonlinear phenomena at power levels of a few milliwatts which are well within the capability of semiconductor lasers.

In the present work, both step index and graded index single-mode fibers are considered where Kerr-type nonlinearity is clearly pronounced with higher degree than multimode fibers which have large core diameters: thus reduced power intensity. Furthermore, in multimode fibers, optical power is shared among several modes with the result that each mode will have no power enough for

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nonlinearity excitation. The effects of various guiding parameters and the nonlinear Kerr coefficient on the propagation characteristics and the modal field distribution are studied.

2 Theory and numerical method

The refractive index of a medium results from the applied optical field perturbing the atoms or molecules of the medium to induce an oscillatory polarization, which then radiates, producing an overall perturbed field. At low intensities, the polarization is a linear function of the applied field and hence the resulting perturbation of the field can be realistically described by a constant refractive index. However, at higher optical intensities, the perturbations do not remain linear functions of the applied field and Kerr nonlinearity may be observed [9]. Since the electric field amplitude is a direct measure of the optical power intensity, the nonlinear refractive index of the fiber may be represented as the sum of constant refractive index term and intensity-dependent term [7]:

$$n^2(r) = n_o^2(r) + \alpha_e |E|^2, \tag{1}$$

where $n_o(r)$ is the zero-field refractive index, and α_e is the nonlinear coefficient which is positive for self-focusing media and negative for self-defocusing media. Both cases will be considered in the following analysis. The physical meaning of either positive or negative nonlinear coefficient could be explained from the point of view of the nature of the perturbation response of the dielectric material to the applied electric field. The presence of doping materials in the dielectric material could play the main role which governs the sign of the nonlinear coefficient.

In cylindrical waveguides, the core nonlinear coefficient may be expressed as [3]:

$$\alpha_e = \frac{n_2^2 n_{NL}(r)}{\eta_0}, \tag{2}$$

where n_2 is the cladding refractive index, $n_{NL}(r)$ is the distribution of the nonlinear Kerr coefficient in units of m^2/W and η_0 is the free space impedance. In the following analysis, we will assume that the nonlinear Kerr coefficient is constant along the core radius. Although, n_{NL} has a very small value ($\cong 10^{-18} - 10^{-8}$ [10]), the long interaction length in optical fibers magnify this effect.

Under weakly guiding conditions, the wave equation for the radial electric field component, in both core and cladding, for single-mode cylindrical waveguides has the general form [9]:

$$\frac{d^2E}{dr^2} + \frac{1}{r}\frac{dE}{dr} + k_0^2\left\{n^2(r) - \beta_e^2\right\}E = 0, \tag{3}$$

where k_0 is the free space wave number, β_e ($= \beta/k_0$) is the effective propagation constant and $n(r)$ is the refractive index at the radial coordinate r given by (1) in the core and has a constant value, n_2, in the cladding.

In nonlinear fibers, (3) has not a typical form and thus should be solved numerically. Since it is of the second order, two boundary conditions are needed. In single-mode fibers, the LP_{01} mode has a maximum value for electric field at the core axis. This value, which is assumed in our work to be 10^4 V/m [7], acts as the first boundary condition. The second boundary condition to be taken into account is that the tangential derivative of the electric field is zero at the core center. (3) has two solutions; one in the core and another in the clad and therefore, matching the electric field and its first derivative is necessary at their interface.

Since we have assumed that nonlinearity does not appear in the cladding, the fundamental mode in the cladding region will have the form of the zero order modified Bessel function [9]. Thus, the matching condition may be represented by:

$$\frac{1}{E}\frac{dE}{dr}\Big|_{r=a} = -\frac{W}{a}\frac{K_1(W)}{K_0(W)}, \tag{4}$$

where K_1 is the first order modified Bessel function and W is the cladding decay parameter having the form:

$$W = ak_0\left(\beta_e^2 - n_2^2\right)^{1/2}. \tag{5}$$

Using the shooting method, two initial guesses of the effective index are used separately. Beginning with the assumed electric field and its zero first derivative at the core center, the core differential equation is integrated step by step using the fourth-order Runge-Kutta method [11]. The electric field and its first derivative are estimated each step till reaching the core-cladding interface. The number of segments, N, into which the core radius is divided, is chosen to improve the results accuracy, running time, and solution convergence. The Gaussian approximation has been used to estimate both the electric field and its first derivative at the radial coordinate corresponding to the first step of integration. These are then used as initial values in the following numerical integration process.

Performing the integration process till reaching the core-cladding interface, an error function is used to evaluate the mismatch between the exact value of dE/dr and that obtained by numerical integration. This error function is defined as:

$$ERR = \frac{-a}{W}\frac{dE}{dr} - \frac{K_1(W)}{K_0(W)}E. \tag{6}$$

This function is used twice, one for each of the two used initial guesses of β_e. Using the secant method [11], the two obtained error values are used to estimate a correction term for a new guess of the effective index, β_{new}, through the formula:

$$\beta_{new} = \beta_2 - \frac{ERR2(\beta_2 - \beta_1)}{(ERR1 - ERR2)}, \tag{7}$$

where β_1 and β_2 are the two initial guesses of the effective index, and ERR1 and ERR2 are the two error values obtained. This new guess together with one of the two initial guesses which gives smaller value of ERR are used again in an iterative manner to obtain the correct value of the effective index β_e, thus obtaining an eigen mode. Since the Runge-Kutta method determines the values of the unknown field and its first derivative during the integration process, the propagation characteristics and the modal field distribution can be obtained simultaneously.

However, the results obtained were found to depend on the two initial guesses. Thus, a second iteration process had to be used. This had been operated by defining a variable which was equated to one of the two introduced initial guesses and comparing the obtained effective index value with it. With a difference being existing, as expected, the obtained value of β is introduced as a guess and the process is performed iteratively till the difference being smaller than a preassigned value (10^{-5}, which corresponds to a negligible percentage error). This procedure has been found to give results that are independent of the initial input guesses in a convergent manner. Having obtained the correct effective index value, Runge-Kutta-method is used again to obtain the electric field distribution in both core and cladding by integrating (3) with the more accurate obtained value of β.

3 Numerical results and discussion

According to the algorithm presented in the previous discussion, a computer program has been developed to investigate the performance of the cylindrical optical fibers that exhibit nonlinear behavior. This computer program has been used to solve for the waveguide characteristics and to study the effect of all parameters that governs the modal field distribution and the propagation characteristics. These parameters include the core radius a, the cladding refractive index n_2, the relative refractive index difference Δ, the wavelength λ, and the nonlinear Kerr coefficient n_{NL}. The effect of the number of segments N, into which the core region is divided, on the obtained value of the effective index is also considered.

Step-index fibers will be considered first. In Fig. 1, the modal field distribution is displayed against the radial distance for different values of core radius, a, at the defined set of other parameters. From Fig. 1, it is noted that an increase of core radius results in raising the electric-field curve in the core region and lowering it in the cladding region. This reflects the fact of increasing the confinement of the optical power in core region. In a similar manner, the other affecting parameters were studied and it was found that more confinement is obtained at higher values of Δ and lower values of λ.

Concerning the cladding refractive index, n_2, it was found that decreasing the value of n_2 results in lowering the electric field curve in both core and cladding regions. Another important fact to be considered is that with decreasing n_2 value, which has positive effect on increasing the modal field confinement, another negative effect may appear simultaneously for self-defocusing fibers (positive n_{NL}, (2)). However, this negative effect is practically negligible due to the small values of n_{NL}. Thus, the effect of decreasing the cladding refractive index on increasing V value is practically larger than that on decreasing α_c value.

The effect of various nonlinear Kerr coefficients, n_{NL} on the modal field distribution is shown in Fig. 2. From Fig. 2, it is noted that positive values of n_{NL} have a focusing effect on the modal field distribution while negative n_{NL} have a defocusing effect. This could be explained easily by noting that positive nonlinear coefficient increases the overall core refractive index.

The variation of the normalized propagation constant, b, with the effecting parameters (combined in the V number) is studied at different values of n_{NL}, and the obtained results are shown in Fig. 3.

The normalized propagation constant depends only on the normalized frequency at a constant nonlinear coefficient. It is noted from Fig. 3 that for each curve, the most right end point represents the cutoff value, V_c, after which the fiber operates as a multimode. For zero n_{NL},

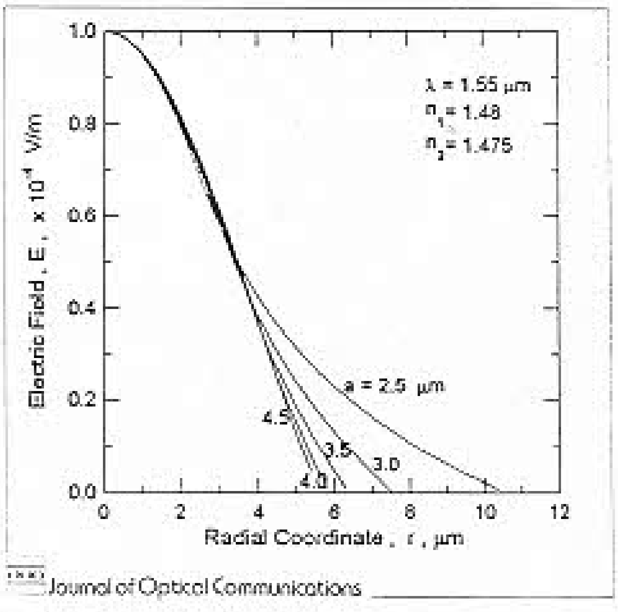

Fig. 1: Field distribution in the fiber core: effect of core radius (step index fiber)

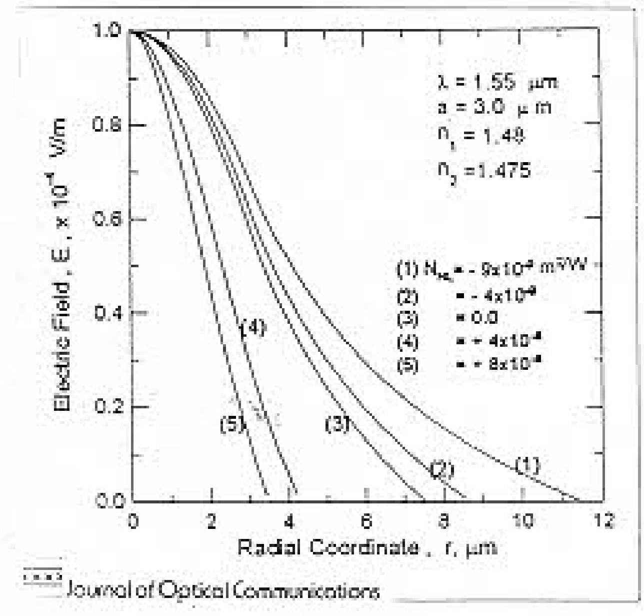

Fig. 2: Field distribution in the fiber core: effect of Kerr coefficient (step index fiber)

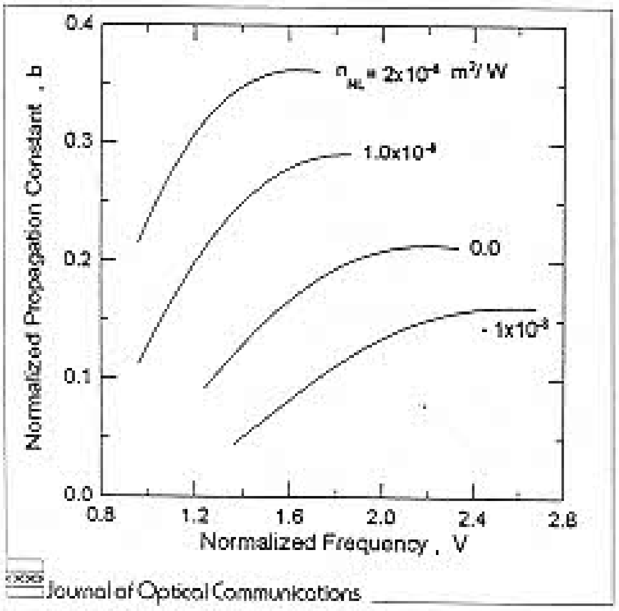

Fig. 3: Variation of the normalized propagation constant with the normalized frequency, (step index fiber)

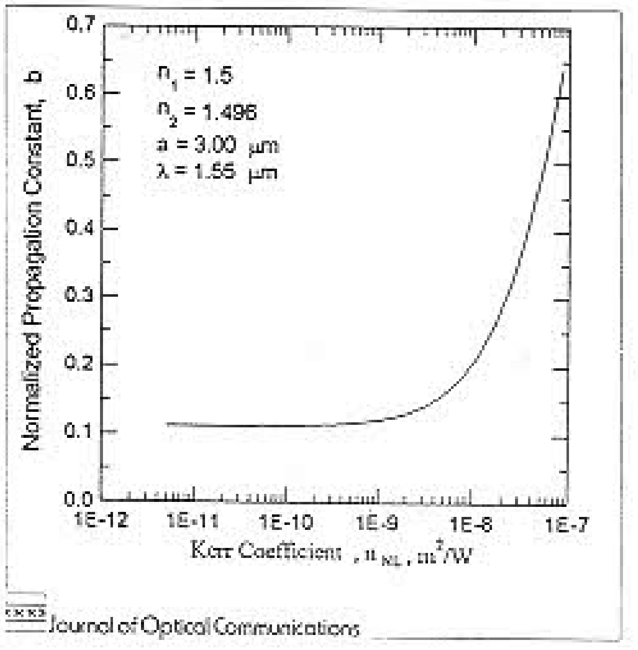

Fig. 4: Variation of the normalized propagation constant with Kerr coefficient

V_c is nearly equal to the typical value of 2.405 in consistence with the well known value [9]. Positive values of n_{NL} result in a decreased V_c value while negative ones increases it. A simple formula can be approximated for the relation between V_c and n_{NL} for step index fibers as:

$$V_c = 2.244 - 2.49 \cdot 10^7 n_{NL} + 1.843 \cdot 10^{14} n_{NL}^2, \quad -1 \cdot 10^{-8} \le n_{NL} \le +1 \cdot 10^{-8}. \tag{8}$$

It is concluded from Fig. 3 that nonlinearity increases field confinement and thus the normalized propagation constant, b. Thus, for a certain value of b, the V-number for nonlinear case is less than for the linear one. Since the cutoff condition is restricted to a certain value b, so we expect that the cutoff value for the normalized frequency V_c for nonlinear waveguides will be less than that for linear ones. This expectation has been tested and proved to be right. Of course, increased nonlinearity results in further decreases in the cutoff value. Decreased V_c value for nonlinear operation reflects the ability of the nonlinear optical waveguides to operate under higher wavelengths while keeping single mode operation. This may encourage the use of optical sources operating in the infrared and far infrared regions.

The effect of the nonlinear Kerr coefficient on the normalized propagation constant, b, is displayed in Fig. 4 for wide range of values of n_{NL}. It is clear that nonlinearity has no significant effect on the value of b till a certain value of n_{NL} which has a significant contribution to the perturbed core refractive index. Upon reaching this value, the normalized propagation constant will increase strongly with the n_{NL}.

We noted that the obtained value of the effective index, β_e, depends on the number of segments into which the core cross section was divided. In contrast to the case of planar waveguides [7], where the effective index was

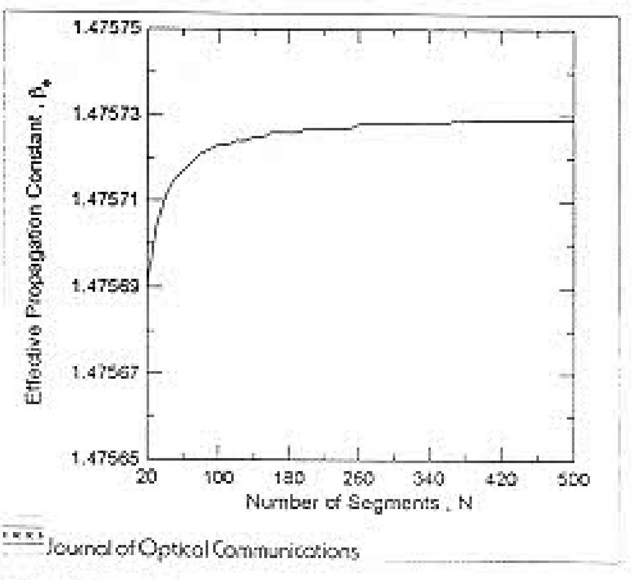

Fig. 5: Effect of the number of segments on the effective propagation constant

found to decrease, in a convergent manner, with the number of segments N. In our work in cylindrical waveguides, it has been found that β_e increases, also in a convergent manner, with N. However, it should be noted that increasing the number of segments is not necessarily an optimum choice to increase the accuracy of the obtained results because this may lead to an increase in build up error during the integration process. Also, it is clear that decreasing the number of segments affects the accuracy of the results. In the present work, to make a compromise, it has been found that N = 100 is a best choice. Figure 5 represents the relation between β_e and the number of segments N. The difference between the two obtained values of β_e with two values of N (≥ 100) could be neglected without significant error.

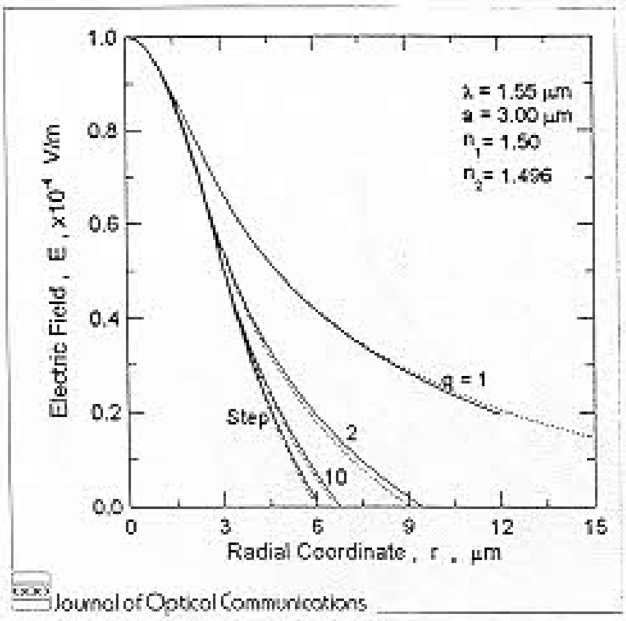

Fig. 6: Field distribution in the fiber core (graded index fiber; solid lines for linear and dashed lines for nonlinear fibers)

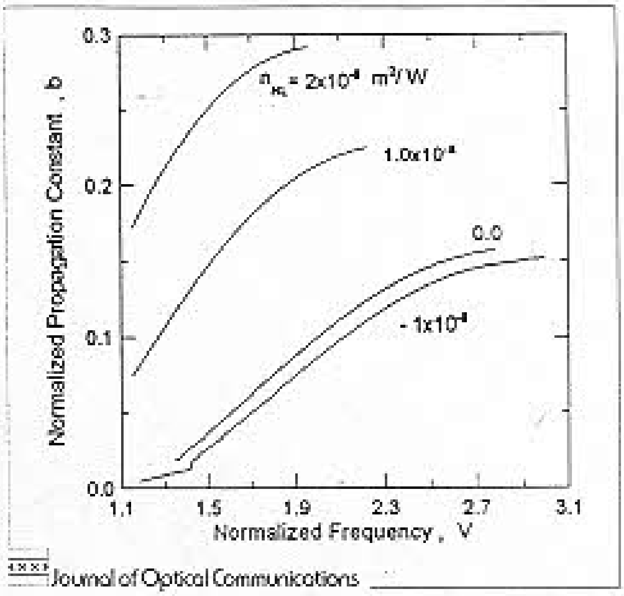

Fig. 7: Variation of the normalized propagation constant with the normalized frequency: graded index fibers

As already known, when nonlinearity does not exist, the modal electric field will have the form of Bessel functions of first kind and zero order, J_0 [9]. To ensure the accuracy of the method used, the nonlinear coefficient is set equal to zero and the obtained results are compared to that obtained by the well known solution of linear step index fibers. The effective indices obtained from the two techniques were found to differ only by 0.03%.

In graded index fibers with the nonlinearity presented, the overall core refractive index can be written as:

$$n^2(r) = n_1^2\left\{1 - 2\Delta (r/a)^q\right\} + \alpha_c |E|^2, \tag{9}$$

where q is the profile parameter which gives the characteristic refractive index profile of the fiber core [9].

Since for graded index fibers the numerical aperture *NA* is a function of the radial distance from the fiber axis and decreases with it; they therefore accept less light than the corresponding step index fibers ($q = \infty$) with the same relative refractive index difference. This is shown schematically in Fig. 6, where results of graded index fibers are displayed, from which it is noted that the modal field confinement decreases with decreasing q.

In (9), there are two competing effects on the overall core refractive index value at any radial coordinate, $r \le a$. The negative effect results from the grading technique which is achieved manufacturally. The positive effect is due to nonlinearity. Importance of the first effect varies inversely with the value of the profile parameter q. The positive effect depends on the square of the absolute value of the electric field. For $q \ge 2$, as shown in Fig. 6, nonlinear distributions shown in dashed curves reflect the confinement of the electric field. For triangular profile, nonlinearity results in an opposite effect. This is due to the faster decrease of the first term than the second one in the right hand side of (9). However, large nonlinearity may result in more confined electric field distribution. In a similar manner to the step index fibers, the normalized propagation constant, b, for graded (parabolic) index fibers is obtained and displayed with V in Fig. 7.

Also, similar to (8), the cutoff normalized frequency, V_c, for graded index fibers is approximated as:

$$V_c = 2.829 - 9.168 \cdot 10^7 n_{NL} + 3.068 \cdot 10^{15} n_{NL}^2,$$
$$-1 \cdot 10^{-8} \le n_{NL} \le +1 \cdot 10^{-8}. \tag{10}$$

4 Conclusion

The nonlinear wave equation is solved for single-mode fibers using the shooting technique. The effects of various waveguide parameters on the modal field distribution and the propagation constant are studied for both step and graded index fibers. From the parametric study, we can predict that for nonlinear waveguides the normalized propagation constant, b, depends totally on two instead of one as in the case of linear waveguides; parameters: the V number and the nonlinear Kerr coefficient n_{NL}. The efficiency of method used lies in the accuracy and convergence of the obtained results. It is found that nonlinearity has more effect on the propagation constant than that on the field distribution. Approximate relations for the dependence of the cutoff value, V_c, on n_{NL} are deduced for step and graded index fibers, (8) and (9). Results concerning nonlinear optical fibers encourage the use of optical sources operating in the infrared and far infrared regions.

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