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Attenuation of Modes in Graded Index Multimode Fibers

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ABSTRACT

The electromagnetic tunneling loss of leaky modes is calculated in graded index biquadratic multimode fibers. The factors affecting the attenuation are investigated for the three dominant modes showing less attenuation in fibers with greater radii. The obtained results show also that the propagation of the fundamental mode (LP_{01}) is accompanied by a large attenuation in the other modes and so, it is suitable for mode filtering operation.

INTRODUCTION

The optical power in the optical fiber is guided in its core. The propagation of optical signal in the fiber suffers from two main kinds of losses: material and waveguide losses. The material losses are studied by many authors [1-5]. The waveguide loss appears as a tunneling of the optical signal from the fiber core to its cladding. In the present work we are interested in this type of loss which depends on the physical shape of the fiber, the core refractive index profile and the operating wavelength.

The propagation constant, β, has a real value in case of bound modes and is a complex quantity in case of leaky modes [1]. The imaginary part represents the attenuation of the ray during propagation. The mode is bound when $n_1k \geq \beta > n_2k$, where n_1 and n_2 are respectively, the axial and cladding refractive indices and k ($=2\pi/\lambda$) is the wave number and λ is the free space wavelength. The bound modes propagate in the core only. The region of propagation is confined between the inner and the outer caustics, where the caustic is the point in the fiber core at which the ray is reflected. At this point the radial propagation vector, k_r, is zero [5]. If the clad is depressed, a third caustic could exist in it, hence a part of the power propagates in the clad and is lost and the mode, in this case, will be a leaky one. The cutoff frequency is defined by the frequency at which $\beta=\beta_{cutoff} = n_2k$.

The fiber under consideration is characterized by a refractive index of the biquadratic form:

$$n^2(r) = n_1^2\{1 - 2m(r/a)^2 + 2m\alpha(r/a)^4\} \qquad 0 \le r \le a \quad , \qquad (1\text{-a})$$

$$= n_1^2\{1 - 2m + 2m\alpha\} \qquad r \ge a \quad , \qquad (1\text{-b})$$

where a is the core radius, m and α are two tailoring parameters. This type of fiber enhances the confinement factor and decreases the binding loss due to the deppressed clad. The value of the parameter m is very small ($<< 1$) and the value of the parameter α can be chosen to give minimum total impulse response width [6], but still we've $\alpha < 1.0$ for $n(r) < n_2$.

Section II deals with the basic model and analysis. The obtained results are presented and discussed in Sec.III and the effect of the fiber parameters on the mode attenuation is studied. Section IV demonstrates the conclusions on the present work.

MODEL AND ANALYSIS

Determination of the Mode Attenuation

The ray optics approach is used to calculate the attenuation in the leaky modes. In this method, the path of the photon in the fiber core is considered. The photon upon its path along the fiber is reflected at the inner and outer caustics. If $\beta > \beta_{cutoff}$, there will be only two caustics in the core, otherwise, a third one exists in the clad. The positions of the caustics, r_1, r_2 and r_3, are the roots of the equation [5]:

$$k_r^2 = n^2(r)\, k^2 - \beta^2 - (\ell^2/r^2) = 0 \qquad , \qquad (2)$$

where ℓ is the mode number.

In case of leaky mode, the photons reaching the second caustic have a probability to tunnel through a forbidden region { $k_r^2 < 0$ } and begin to propagate in the clad from the third caustic, r_3. A portion of the photon power incident at r_2 tunnels to r_3 and the tunneling coefficient, T, is given by [5]:

$$T = \exp\left(-2\int_{r_2}^{r_3} \sqrt{|\,k_r^2(r)|} \quad r\, dr\right) \qquad , \qquad (3)$$

Near cutoff, the rays will still be confined to the core far from the clad, so the refractive index, in this region, can be considered to have quadratic variation (i.e. negligible effect of the parameter α). Therefore, using the ray optics approach [5], the longitudinal distance, L, between two successive contacts of the photon with the second caustic is calculated as:

$$L = \frac{\pi a \beta}{n_1 k \sqrt{2m}} \qquad (4)$$

When photons reach the second caustic, a portion of the optical power is reflected and the other is transmitted to the radiation caustic (the third one). Since the transmitted power to the clad represents the waveguide loss, therefore, the attenuation coefficient can be obtained by:

$$\frac{T}{L} = \frac{n_1 k \sqrt{2m}}{\pi a \beta} \exp\left(-2 \int_{r_2}^{r_3} \sqrt{|k_r^2(r)|} \; r \, dr\right) \qquad (5)$$

It is clear that, to determine the attenuation coefficient, Eq.(5), we have first to get the propagation constant, β.

Determination of the Propagation Constant, β

A closed form for the propagation constant, β, is derived using the WKB method [7] yielding:

$$\beta^2 = n_1^2 k^2 \left\{ 1 + \frac{8m\left(1 - \sqrt{1 + 6\alpha\left\{\frac{1}{2V} + \frac{\alpha \ell^2}{V^2} + \frac{(M+0.5)}{V}\right\}}\right)}{3\alpha} \right\}, \qquad (6)$$

where V is the normalized frequency.

Equation (6) indicates that, any mode is defined with two numbers; ℓ and M, where ℓ determines the number of maxima of the oscillatory field in Φ direction and M+1 determines the number of maxima of the oscillatory field in the radial direction, r. More details are given in appendix 1.

Another method is used to find the propagation constant, β, by solving the wave equation, for the electric field in both core and clad, using the infinite series form. A complete derivation is given in appendix 2.

RESULTS AND DISCUSSION

The propagation constant, β, at cutoff, obtained by the WKB method and by the series method is used to calculate the cutoff value of the normalized frequency, V_{cutoff}, in the LP_{01} mode and the results are plotted against the parameter α in Fig.1 showing a fair agreement, especially when α becomes closer to 1. The reason is that, as $\alpha \rightarrow 1$ we have to operate at large values of V to become just above cutoff and the caustics r_1 and r_2 become closer to the fiber axis, i.e., the ray is confined near the axis and consequently the effect of the term containing α, Eq.(1-a), has a negligible effect on propagation. The effect of α is to modify the value of the normalized frequency, V, which will increase by a factor $1/(1 - \alpha)$. The comparison is made when $\ell=0$, but the derived WKB expression is generalized for $\ell \neq 0$.

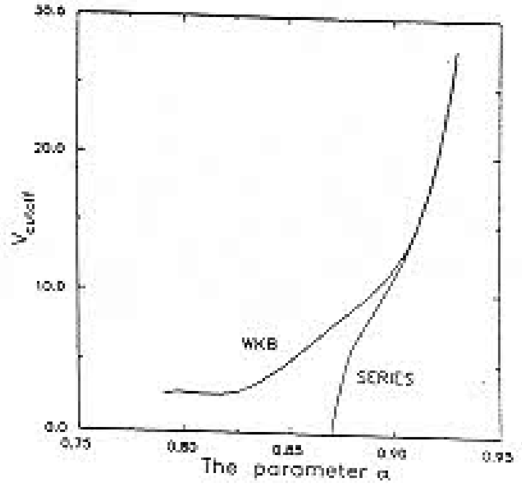

Fig.1. Variation of the cutoff value of the normalized frequency with the parameter α, in the fundamental mode.

The obtained values for the propagation constant, β, Eq.(6), and the caustics obtained by solving Eq.(2) using the increment search method [8], are used in Eq.(5) to get the attenuation coefficient, where only the three dominanat modesd; LP_{01}, LP_{11} and LP_{02} are considered. The results are displayed in Figs.2-4 at different values of the affecting parameters. For each mode we used a specified range of λ after which, the attenuation increases monotonicaly.

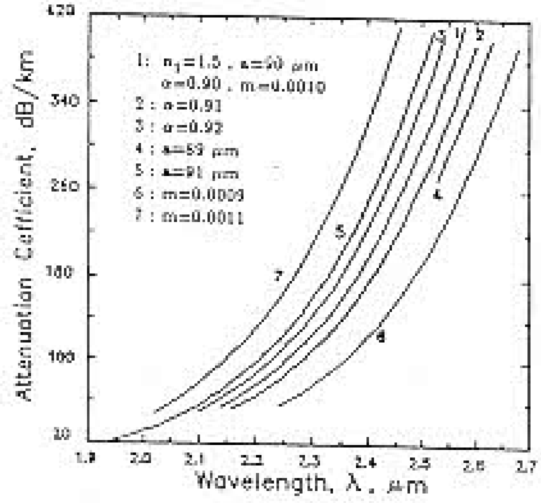

Fig.2. Variation of the attenuation coefficient with the wavelength at different values of the affecting parameters, in the LP_{01}.

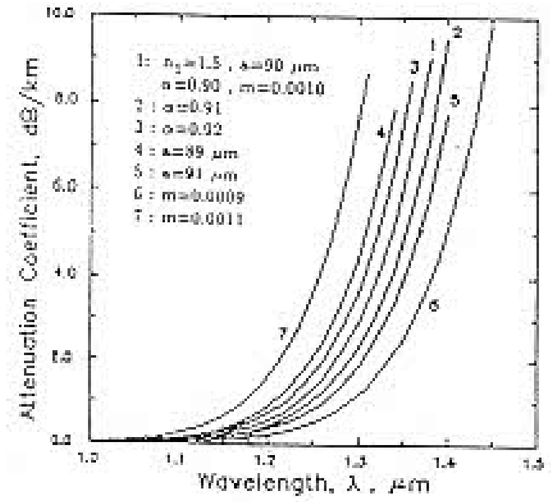

Fig.3. Variation of the attenuation coefficient with the wavelength at different values of the affecting parameters, in the LP_{11}.

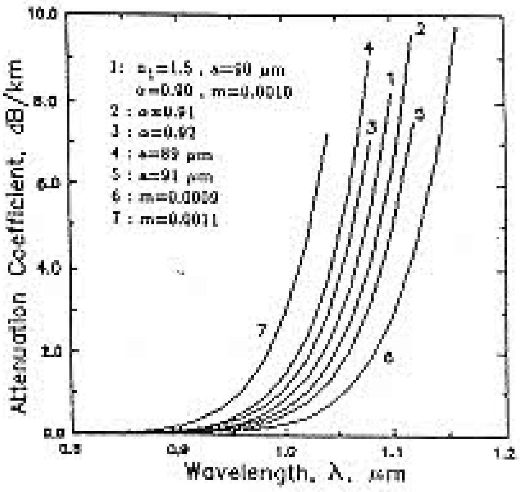

Fig.4. Variation of the attenuation coefficient with the wavelength at different values of the affecting parameters, in the LP_{02}.

The obtained results show that the attenuation increases abruptly from zero to the calculated value at β_{cutoff}. This is because as β is below cutoff the third caustic exists in the clad and when β is above cutoff the third caustic exists at infinity and therefore, the losses tend to zero.

As can be seen from figures, the increase in the core radius causes a decrease in attenuation. This is because the increase in the core radius makes the electromagnetic barrier, which is the region of forbidden propagation, wider and so, the attenuation decreases. The increase in α lowers the cutoff wavelength for all modes causing an increase in the loss for the same range of freguency. Increasing the value of the parameter m, the modes become more confined near the axis of fiber and so the attenuation decrease.

CONCLUSION

This work presents the calculation of the electromagnetic tunneling loss of leaky modes in graded index, biquadratic, multimode fibers characterized by two tailoring parameters. The obtained results show that the propagation of fundamental mode is accompanied by a large attenuation in the other modes (LP_{11} & LP_{02}), so, it is suitable for mode filtering operation.

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APPENDIX 1

Determination of the Propagation Constant Using WKB Method

When dealing with weakly guiding waveguides, electromagnetic propagation is described by local plane waves. The ray or local plane wave trajectory lies between the inner and outer caustics of radii r_1 and r_2 .The modal field oscillates only between these two points. The WKB eigenvalue equation, at which β is a solution, is given by [7]:

$$\int_{r_1}^{r_2} \left((n^2(r)\, k^2 - \beta^2) r^2 - \mathcal{L}^2 \right)^{\frac{1}{2}} \frac{dr}{r} = (2M + 1)\, \frac{\pi}{2} \qquad , \qquad (7)$$

where $\mathcal{L}$ and M are the mode numbers.

If the two caustics are too close to the fiber axis, which is the case when the parameter α is very close to 1, the terms containing $(r/a)^4$ will be very small. Hence, neglecting all terms of order higer than r^4 and solving Eq.(2) one can get r_1 and r_2 which are the limits of integration.

The refractive index n(r) of Eq.(1) is substituted in Eq.(7) and the integral gives:

$$1 = \left\{ \frac{V}{2m} \left(1 - \frac{\beta^2}{k^2 n_1^2} \right) - 2\mathcal{L} \right\} \frac{\pi}{4} + \frac{1}{2}\, \frac{m \alpha k^2 n_1^2 a}{\sqrt{2mk^2 n_1^2}} \left(\mathcal{B}^2 \frac{\pi}{2} + \mathcal{A}^2 \pi \right) , \qquad (8)$$

where V is the normalized frquency given by $n_1 ka \sqrt{2m}$,

$$\mathcal{A} = \frac{(k^2 n_1^2 - \beta^2)}{4mk^2 n_1^2} \quad \text{and} \quad \mathcal{B} = \frac{\sqrt{(k^2 n_1^2 - \beta^2)^2 - 4\,(2mk^2 n_1^2)\,\frac{\ell^2}{a^2}}}{4mk^2 n_1^2} \quad .$$

Substitution with I, $\mathcal{A}$ and $\mathcal{B}$ in Eq.(7) gives:

$$\frac{3\alpha VH^2}{128m^2} + \frac{VH}{8m} - \left(\frac{\ell}{2} + \frac{\alpha \ell^2}{V} + M + \frac{1}{2}\right) = 0 \quad , \tag{9}$$

with $\quad H = 1 - \beta^2/n_1^2 k^2 \quad .$

Solving Eq.(9) for H and rearranging, one can get β^2 as indicated in Eq.(6), where the negative sign of the root was chosen because β^2 must be greater than $n_1^2 k^2$.

APPENDIX 2

Determination of the Propagation Constant Using the Series Method

The exact derivation of the propagation constant, β, is found through the solution of the wave equation, for the electric field, in both core and clad. The solution in the core is represented by an infinite series of the form:

$$E(r) = \sum_{n=0}^{\infty} a_n\,(r/a)^{l+n} \quad . \tag{10}$$

where a_n are constants. Assuming that $a_0 = 1$, one can get:

$$a_2 = \frac{w^2}{\{(\ell + 2)^2 - \ell^2\}} \quad , \tag{11}$$

$$a_4 = \frac{2ma^2k^2n_1^2 + w^2\,a_2}{\{(\ell + 4)^2 - \ell^2\}} \quad , \tag{12}$$

and in general:

$$a_{n+6} = \frac{2ma^2k^2n_1^2(a_{n+2} - \alpha a_n) + w^2\,a_{n+4}}{(n + \ell + 6)^2 - \ell^2} \quad , \tag{13}$$

where $w = a(\beta^2 - k^2 n_2^2)^{1/2}$.

The solution of the wave equation in the clad, where the refractive index is constant must be a monotonicaly decreasing function and has the form [1]:

$$E(r) = K_{\ell}(wx) \qquad (14)$$

where K_{ℓ} is the modified Bessel function of order ℓ and $w = a(\beta^2 - k^2 n_2^2)^{1/2}$.

Applying the boundary conditions at the interface between core and clad, one can get:

$$\sum_{n=0}^{\infty} (n + 2\ell) a_n = 0 \qquad (15)$$

The propagation constant, β, which is included in the constants a_n, is obtained numerically through this equation.