SPOT SIZE, PROPAGATION CONSTANT AND EFFECTIVE INDEX OF SINGLE-MODE W-FIBERS

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ABSTRACT

The propragtion constant in W-fibers is calculated numerically through the calculation of the spot size using a Gaussian distribution for the electric field. This propagation constant helped in finding the effective refractive index of these fibers. The obtained results assure that this technique applies only for single-mode fibers.

keywords: Spot size, Gaussian distribution, propagation constant, effective refractive index, single-mode fibers.

I.INTRODUCTION

The spot size is an important parameter for characterizing single mode fiber properties which takes into account the wavelength dependent field penetration into the fiber cladding[1]. It is a better measure of the functional properties of the single-mode fiber than the core radius. It is sometimes called the fundamental radius and hence it is regarded as the single-mode analog of the fiber core radius in multimode fibers.

The spot size, r_0, of the fundamental mode is an important parameter as it directly yields an estimate of the propagation constant, β, and consequently the effective refractive index, n_{eff}, it can also be used to calculate the fraction of power transmitted through the fiber core [2]. It directly yields an estimate of the splice losses, microbend loss and waveguide dispersion of the fiber [3].

For step and graded index sigle-mode fibers, the field is well approximated by a Gaussian distribution [4], and the spot size is taken as half the distance between the opposite e^{-1} field amplitude points in relation to the corresponding values on the fiber axis. The fiber under consideration is characterized by a refractive index profile of the form:

$$n^2(r)=n_1^2\{1-2\alpha(r/a)^2+2m\alpha(r/a)^4\} \qquad r< a, \quad (1\text{-a})$$

$$=n_1^2\{1-2\alpha\}=n_2^2 \qquad r\geq a, \quad (1\text{-b})$$

where n_1 is the axial refractive index, a is the core radius, r is the radial position, α and m are two tailoring parameters. It is clear that, at the core-clad interface (r=a), there is a sudden change in the value of the refractive index. This results in a shift in the zero dispersion wavelength to lower values [5]. This type of fibers based on silica is fabricated by the modified chemical vapor deposition (MCVD) and exhibits low loss and high information capacity [6]. Section II presents the model used to determine the spot size. The quadratic variation (m=0) is studied, firstly, to check the validity of this technique and the obtained results are compared with the published ones giving a fair agreement as discussed in Sec.III. Secondly, the parameter m is taken into consideration to study the biquadratic variation which represents the W-type fiber. Finally, some conclusions on this work are given in Sec.IV.

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II. THEORY

A. Spot-Size and Propagation Constant

In the cylindrical homogemeous core waveguides under the weak guidance conditions, the solutions of the wave equation, giving the electric and the magnetic fields, are separable in the r, ϕ and z coordinates. The radial variation, $\psi(r)$, of electric field can be obtained by solving the wave equation[1] :

$$\{\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}+k^2n^2(r)-\beta^2-\frac{\mathscr{L}^2}{r^2}\}\ \psi(r)=0, \quad (2)$$

where k $(=2\pi/\lambda)$ is the wave number, λ is the free space wavelength, n(r) is the fiber core refractive index profile, β is the propagation constant and $\mathscr{L}$ is the mode number.

Our main objective is to find a good approximation for the electric field $\psi(r)$ and the propagation constant β of the fundamental mode ($\mathscr{L}=0$) on fibers with core refractive index profiles $n(r)$. Since the function $\psi(r)$ has a maximum at r=0 and decreases to zero as r increases for the graded index profiles [2], $\psi(r)$ can be approximated by a Gaussian distribution of the form[4]:

$$\psi(r)\simeq(\frac{2}{\pi r_0^2})^{1/2}\exp[-(r/r_0)^2], \quad (3)$$

where r_0 is the spot size and the factor precerding the exponential function is arbitrary and is chosen for normalization.

The propagation constant, β, can be obtained through the function $\psi(r)$ as [4]:

$$\beta^2=\frac{\int_{\infty}^{0}\{-(\frac{d\psi}{dr})^2+k^2n^2(r)\psi^2\}\ r dr}{\int_{\infty}^{0} r\ \psi^2 dr}, \quad (4)$$

Since the value of the spot size, r_0, corresponds to the maximum value of the propagation constant β [2], therefore:

$$\frac{\partial\beta^2}{\partial r_0}=0 \quad (5)$$

Equation (3) is substituted into Eq.(4) and the quantity β^2 is found in terms of r_0 and then used in Eq.(5), resulting in an equation of a root r_0, which can be calculated numerically. The obtained values of the spot size r_0 are then substituted back into Eq.(4) to get the propagation constant β numerically also.

B. Effective Refractive Index

The effective refractive index, n_{eff}, can be considered as an average over the refractive index of the medium [7]. For single-mode fibers, it is defined by the ratio of the propagation constant of the fundamental mode, β, to that of the wavenumber, k [8]:

$$n_{eff}=\beta/k \quad (6)$$

III. RESULTS and DISCUSSION

A Spot Size and Propagation Constant

Following the procedure described in Sec.II, Eq.(5) is solved numerically using the increment search method [9], yielding the spot size, r_o, at different values of the affecting parameters for both quadratic and biquadratic refractive index profiles. The values of the tailoring parameters α and m, Eq.(1-a), are chosen as follows:

The parameter $\alpha(<<1)$ is the relative index difference between the core and the clad refractive indices, while the parameter m takes the values between m=0 for the quadratic refractive index and $m\leq1.0$ for the biquadratic one, such that $n(a)\leq n_1$.

To discuss the results, one must recall the normalized frequency, V, and its cutoff value, V_c, which are important parameters in dealing with single-mode fibers. On the basis of the equal volume method described by Snyder[2], V_c for the fibers expressed by Eq.(1-a) is obtained as:

$$V_c = 2.405\sqrt{1/(0.5 + m/3)} \quad , \tag{7}$$

which coincides with the case of the quadratic index when m=0 [10].

The described method used to determine the spot size was first examined in case of quadratic variation in the refractive index, Figure (1), showing a complete agreement with the work of Marcuse [11]. Figure (1) also displays a sample of the results when the biquadratic refractive index variation ($m \neq 0$) considered.

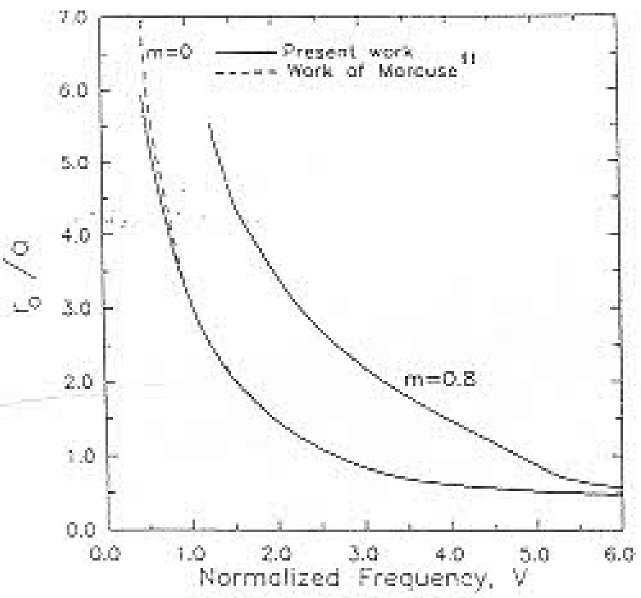

Figure 1. Variation of the spot size with the normalized frequency for quadratic and biquadratic refractive index profiles.

To apperciate the accuracy of using the Gaussian distribution in the present work, the obtained propagation constant, β, was compared to that obtained by Jacobsen [12], using the perturbation theory, which has the form:

$$\beta^2 = n_1^2k^2 - 2n_1k(2\alpha)^{1/2}/a + 2m/a^2 + 9m^2/2a^3n_1k(2\alpha)^{1/2} + 79m^3/8a^4n_1^2k^2\alpha \,. \tag{8}$$

This comparison is shown in Figure (2), where it is clear that the values of β in both methods are very close especially for small values of m. This is because the use of the Gaussian approximation yields an increased error for values of $m>0$.

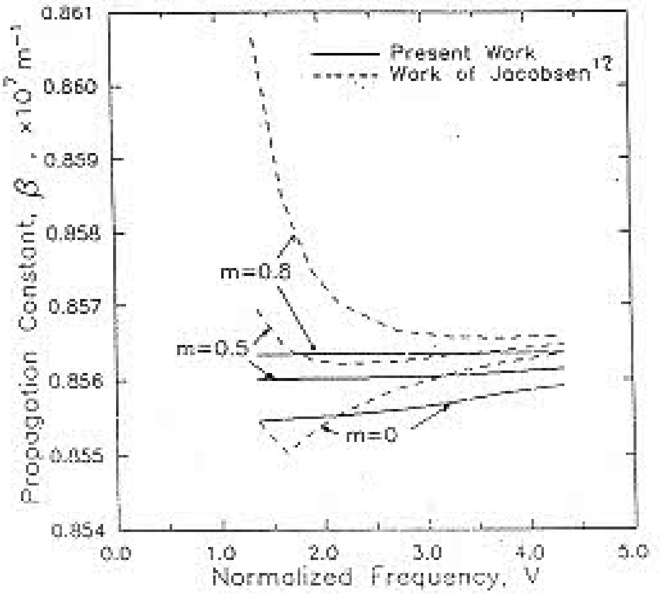

Figure 2. Comparison of the propagation constant of the present work with that obtained by the perturbation theory.

The effect of the different parameters on the spot size was then investigated and the obtained results are represented in Figures (3)- (6).

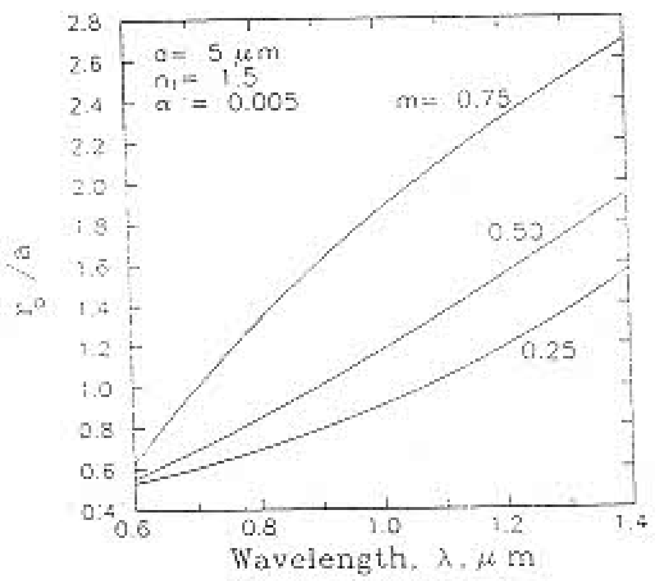

Figure 3. Variation of the spot size with the wavelength at different core radii.

From Figure (3), it is clear that as the core radius, a, increases, the ratio r_0/a decreases. This is because the increase in the radius, a, will turn the fiber to operate as a multimode one when the V number $\{ =2\pi a n_1 \sqrt{2\alpha} / \lambda \}$ exceeds the cutoff value, of the single-mode operation, and the Gaussian approximation will not apply as mentioned before.

This is the case also when the wavelength, λ, decrease because decreasing λ will increase the V number over its cutoff value yielding a multimode operation for the fiber.

Concerning the effect of the tailong parameter m, Figure (4), it is clear that the ratio r_0/a increases with m because the increase in m will decrease the cutoff value and the fiber will be a multimode one and again the Gaussian approximation will not hold.

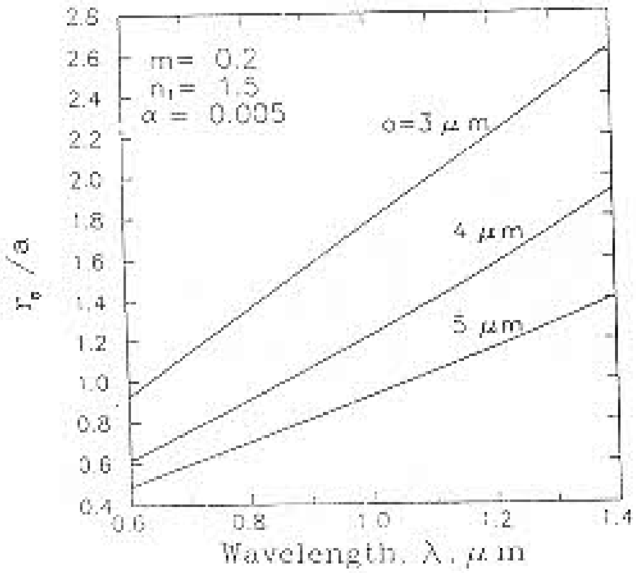

Figure 4. Variation of the spot size with the wavelength at different values of the tailoring parameter m.

The effect of the axial refractive index n_1 is studied separately in Figure (5) showing that as n_1 increases, the value of r_0/a decreases. This can be explained as follows: as n_1 increases, the ray tends to propagate closer to the fiber axis resulting in a decrease of r_0 and consequently in the ratio r_0/a. This occurs also when the tailoring parameter α increase, Figure (6)

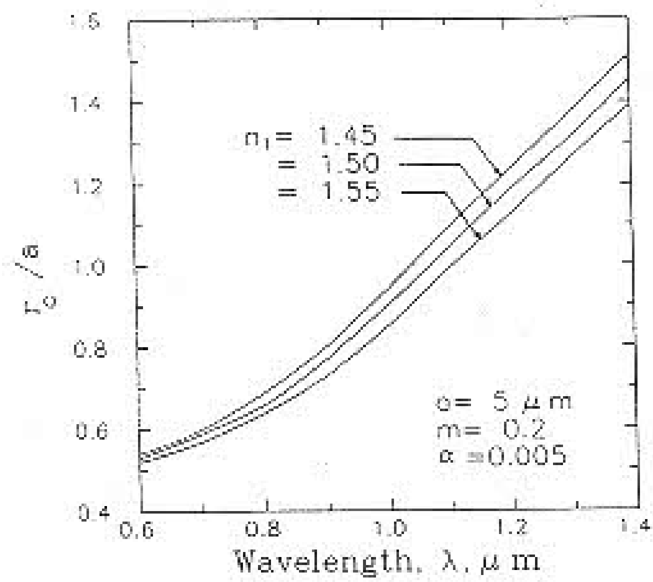

Figure 5. Variation of the spot size with the wavelength at different values of the axial refractive index.

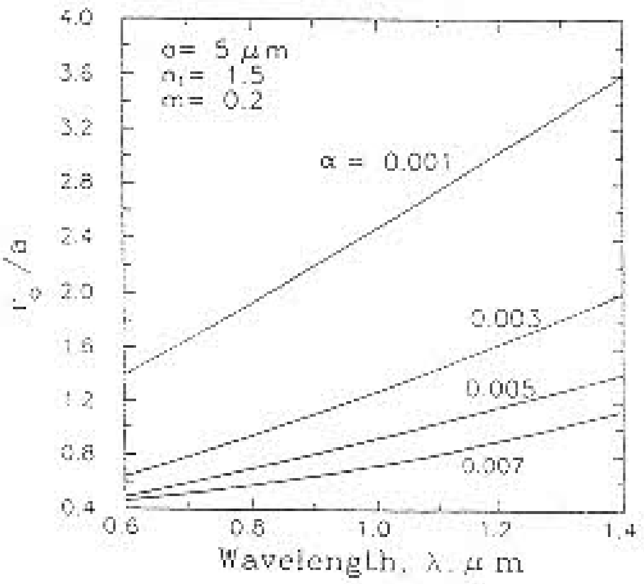

Figure 6. Variation of the spot size with the wavelength at different values of the tailoring parameter α.

B. Effective Refractive Index

Equation (6) is used to calculate the effective refractive index, n_{eff}, of graded index fibers. The variation of n_{eff} with the V number is represented in

Figure (7) for both quadratic (m = 0) and biquadratic ($m \neq 0$) variations of the refractive index. It is clear that, the value of the effective index increases with the increase in the tailoring parameter m, which can easily be explained by the positive sign in Eq.(1-a). In all cases, it can be noted that the value of n_{eff} varies over the interval $n_2 < n_{eff} < n_1$, which satisfies the variation of the propagation constant over the range $n_2k < \beta < n_1k$ for single mode fibers [1], where n_2 is the cladding refractive index.

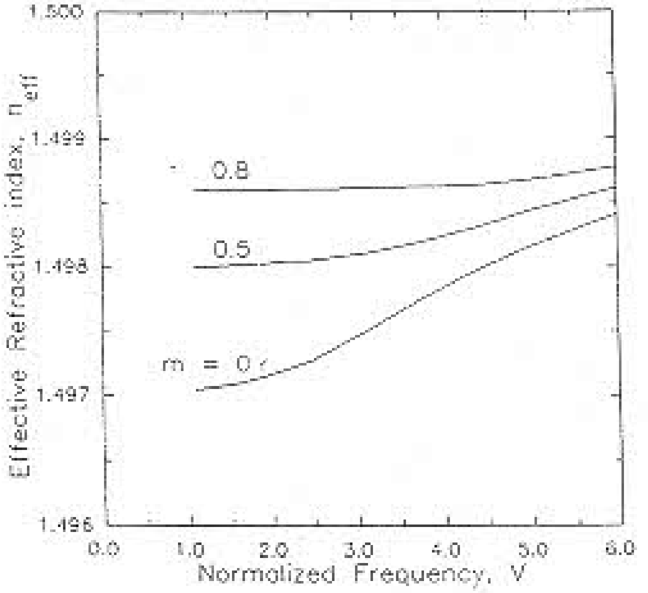

Figure 7. Variation of the effective refractive index with the normalized frequency for quadratic and biquaqdratic refractive index profiles.

IV. CONCLUSION

The Gaussian approximation was used in solving the wave equation in single-mode W-fibers to find the spot size and the propagation constant of these fibers. The accuracy of this approximation was studied by comparing the obtained propagation constant with the corresponding one obtained by the perturbation theory, giving an almost complete agreement. The effect of different parameters was studied showing that the spot size increases with the tailoring parameter m and the operating wavelength, λ, and decreases with the fiber radius, a, the axial refractive index, n_1, and the tailoring parameter α. The obtained results give an indication that this approximation applies only for single-mode fibers.

An important result of the described method is obtaining the variation of the effective index with the normalized frequency, V, for both quadratic and biquadratic (W) variations of the core refractive index.

REFERENCES

[1] John M. Senior, Optical Fiber Communications, Principles and Practice, *Prentice Hall, New York, 2nd ed.*, 1992.

[2] Allan W. Snyder, Optical Waveguide Theory, *Chapman and Hall Ltd., New York*, 1983.

[3] B.P. Pal, U.K. Das and N. Sharma, Effect of Variation in Cutoff Wavelength on the Performance of Single-Mode Fiber Transmission in the 1300 nm and 1500 nm Wavelength Windows", *J. Opt. Commun., vol. 10, pp. 108-114*, 1989.

[4] Allan W. Snyder, "Understanding Monomode Optical Fibers", *Proc. IEEE, vol. 69, No. 1, pp. 6-13*, 1981.

[5] M.J. Adams, *An Introduction to Optical Waveguides*, John-Wiley & Sons Ltd., New York, 1981.

[6] W.G .French, J.B. Mac Chesney, P.B.O, Conner and G.W. Tasker, "Optical Waveguides with Very Low Losses", Bell Syst. *Tech. J., vol. 53, pp. 951-954*, 1974.

[7] E.G. Neumann, "Single-Mode Fibers: Fundamentals", *Springer-Verlage*, 1988.

[8] A.N. Kaul, "Analysis of Single-Mode Graded Core Optical Fibers by the Effective Index Method", *J. Opt.Commun., vol. 11, pp. 26-28*, 1990.

[9] M.L. James, G.M. Smith and J.C. Woldford, "Applied Numerical Methods for Digital Computation", *Harper & Row Publishers, New York*, 1977.

[10] R. Okamoto and T. Okoshi, "Analysis of Wave Propagation in Optical Fibers Having Core with α-Power Refractive Index Distribution and Uniform Cladding", *IEEE Trans. Microwave Theory Tech.*, MTT-24, pp. 416-421, 1976.

[11] D. Marcuse, "Gaussian Approximation of the Fundamental Modes of Graded-Index Fibers", *J. Opt. Soc.* Am., vol. 68, p. 103, 1978.

[12] G. Jacobsen and J.J.R. Hansen, "Propagation constants and group delays of guided modes in graded-index fibers: a comparison of three methods", *Appl. Opt.* vol.18, No. 16, pp. 2837-2842, 1979.