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CHROMATIC DISPERSION IN GRADED-INDEX SINGLE-MODE OPTICAL FIBERS

MOUSTAFA H. ALY
Department of Engineering Mathematics and Physics
Faculty of Engineering
University of Alexandria, Alexandria, EGYPT
Member of the Optical Society of America (OSA)

Abstract

The chromatic dispersion in single-mode optical fibers is modelled and investigated. The studied fibers are characterized by a core refractive index of the graded type, including quadratic and biquadratic variations of the refractive index with the radial position. The index profile has two controling parameters which are tailored to give minimum dispersion. The obtained results are compared with the published ones, showing a fair agreement.

I. Introduction

Dispersion of the transmitted optical signal causes distortion for both digital and analog transmission along optical fibers. It causes broadening of the transmitted pulses. The intermodal dispersion results from the propagation delay differences between modes within a multimode optical fiber. Under purely single mode operation, there is no intermodal dispersion and therefore, the pulse broadening is solely due to the intramodal mechanisms. Intramodal or chromatic dispersion may occur in all types of fibers and results from the finite spectral linewidth of the optical source. There may be propagation delay differences between the different spectral components of the transmitted signal. The delay differences may be caused by : the dispersive properties of the fiber material (material dispersion ; D_m), and the guidance effects within the fiber structure (wave-guide dispersion; D_w).

In the present work, both types are investigated separately for single mode optical fibers. The material dispersion is studied through the material dielectric function $\epsilon(\omega)$, while the waveguide dispersion is studied, in a new approach, through the dependence of both the propagation constant β and the normalized frequency V on the wave-length λ. The two types are then added to each other giving the chromatic dispersion.

The core of the fibers under investigation has a graded index profile of the form:

$$n^2(\rho) = n_c^2(1 - 2\alpha\rho^2 + 2m\alpha\rho^4) \quad , \qquad (1)$$

where ρ ($=r/a$) is the normalized radial distance from the fiber axis, a is the core radius, n_c is the refractive index at the fiber axis, α and m are two tailoring parameters. The parameters α and m have nonzero values for the biquadratic variation of the refractive index, but for a quadratic variation, m is equal to zero while α is equal to 2Δ; where Δ is the relative refractive index difference between the core and the cladding. Figure 1. illustrates the biquadratic variation of the core refractive index for the considered fibers. This figure shows a W-form of the index profile with one maximum n_c at the core axis and another maximum n_1 at the core edge, and with one minimum at a certain position r_m. The values of the tailoring parameters α and m are adjusted according to the values of n_1/n_c, n_2/n_c, and r_m/a.

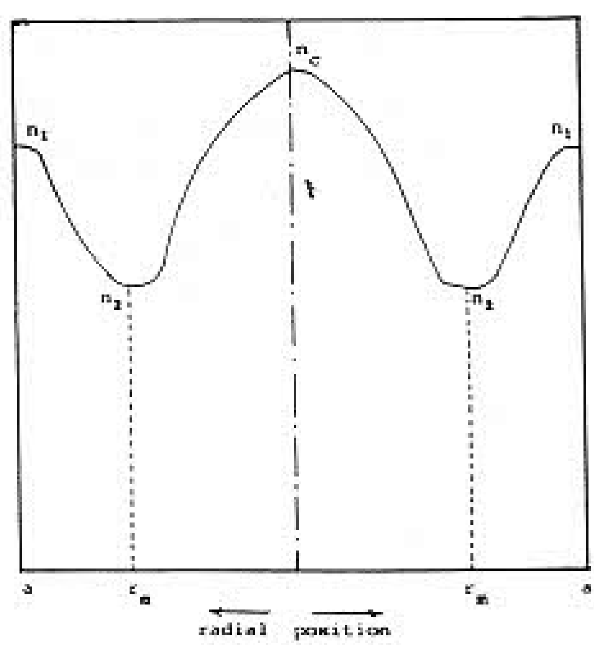

Fig.1. Core refractive index profile.

II. Method of Analysis

A - Material Dispersion

For optical fibers in which the refractive index difference between the core and the cladding is small, the material dispersion D_m is given by [1,2] :

$$D_m = -\frac{\lambda L}{c}\frac{d^2n}{d\lambda^2} \quad , \qquad (2)$$

where λ is the wavelength, L is the fiber length, c is the free space speed of light and n is the cladding refractive index.

It has been shown that for frequencies > ω_L, the lattice optical frequency, the dielectric function for glasses can be represented as [3,4] :

$$\epsilon(\omega) = n^2 = 1 + \frac{E_d E_0}{E_0^2 - \hbar^2\omega^2} - \frac{E_l^2}{\hbar^2\omega^2} \quad , \qquad (3)$$

where $\hbar$ is Planck's constant divided by 2π, ω is the angular frequency, E_0 is the average electronic energy gap, E_d is the electronic oscillator strength and E_l is the lattice oscillator strength.

Equation (3) is twice differentiated w.r.t. λ, and the result is substituted in eq.(2) giving the material dispersion D_m as a function of the wavelength λ. From definition, it is clear that the material dispersion is independent of the fiber parameters (a, α and m), but it only depends on the fiber material through the material constants E_0, E_l, and E_d.

B - Waveguide Dispersion

Again, for optical fibers in which the refractive index difference between the core and the cladding is small, D.Marcuse [4] has separated the waveguide dispersion as :

$$D_w = \frac{L\ n}{c\ \lambda}\ V\ \frac{d^2(bV)}{dV^2} \quad , \qquad (4)$$

where V is the normalized frequency and b is the normalized propagation constant.

Equation (4) has been solved by C.T.Chang [5] using the Bessel and modified Hankel function relationships and their derivatives. Using another way, L.G.Cohen et al. [6] have solved eq.(4) through a Taylor expansion for the propagation constant. K.V.Subramaniyam et al. [7] used the effective refractive index method in solving eq.(4) to get the waveguide dispersion and, consequently, the chromatic dispersion. In the present work, we introduce a different approach to solve this equation starting from the definitions of both V and b.

The normalized frequency, V, is defined as [8] :

$$V = ak\ (n_c^2 - n^2)^{1/2} \quad , \qquad (5)$$

where k is the wave number ($=2\pi/\lambda$; with λ the wavelength). The normalized propagation constant, b, is defined as [9] :

$$b = [(\beta/nk)^2 - 1] \ / \ [(n_c/n)^2 - 1] \quad , \qquad (6)$$

where β is the propagation constant which is obtained using the perturbation theory, to the third order, in the form [10] :

$$\beta^2 = k^2 - \frac{2k(2\alpha)^{1/2}}{a} + \frac{2\alpha}{a^2} + \frac{9\ m^2}{2a^3k(2\alpha)^{1/2}} + \frac{79\ m^3}{8\ a^4k^2\alpha} \qquad (7)$$

Now, it is clear that both V and b are functions of λ. So, one can get the quantity $d^2(bV)/dV^2$, eq. (4), from the properties of the parametric differentiation as follows :

$$\frac{d^2(bV)}{dV^2} = V\ \frac{d^2b}{dV^2} + 2\ \frac{db}{dV} \quad , \qquad (8)$$

where

$$\frac{db}{dV} = \frac{db}{d\lambda} \ / \ \frac{dV}{d\lambda} \quad , \qquad (9)$$

and

$$\frac{d^2b}{dV^2} = \left(\frac{d}{d\lambda}\ \frac{db}{dV} \right) / \ \frac{dV}{d\lambda} \quad . \qquad (10)$$

From eqs.(5) and (6) one can get :

$$\frac{db}{dV} = - \frac{C_2}{C_1} \left(\lambda^3\beta^2 + \lambda^4\beta\ \frac{d\beta}{d\lambda} \right) \quad , \qquad (11)$$

where

$$C_1 = 2\pi a\ (n_c^2 - n^2)^{1/2} \quad , \qquad (12)$$

and

$$C_2 = 2\ (4\pi^2 n_c^2 - 4\pi^2 n^2)^{-1} \quad , \qquad (13)$$

The use of eq.(11) in eq.(10) yields :

$$\frac{d^2b}{dV^2} = - \frac{C_2}{C_1^2} \left\{ 6\beta\lambda^5 \frac{d\beta}{d\lambda} + 3\beta^2\lambda^4 + \lambda^6\beta\ \frac{d^2\beta}{d\lambda^2} + \lambda^6 \left(\frac{d\beta}{d\lambda}\right)^2 \right\} \quad , \qquad (14)$$

where β, the propagation constant, and its derivatives w.r.t. λ are obtained from eq.(7).

The waveguide dispersion D_w, eq.(4), can easily be obtained with the aid of eqs.(5), (11) and (14) as a function of the wavelength λ.

The addition of the material dispersion D_m, eq.(2), and the waveguide dispersion D_w, eq.(4), gives the chromatic dispersion D_t as a function of the wavelength λ.

III. Results and Discussion

In the first step, the material dispersion has been calculated using eqs.(2) and (3). The fiber material is chosen to be silica with the constants E_d, E_0 and E_l respectively equal to 14.62, 13.3 and 0.127 eV [11]. The variation of the material dispersion with the wavelength λ is displayed in Fig.2, giving a very good agreement with the results of Wemple [12] with the same zero material dispersion point λ = 1.22 μm.

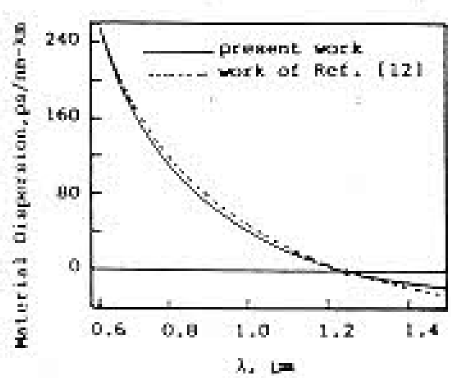

Fig.2. Material Dispersion of Silica Fibers.

In the second step, eqs.(4) through (14) have been used to calculate the waveguide dispersion at different sets of fiber parameters. We have started the calculations with the case of the quadratic index fiber, where the values of the controling parameters are :

$\alpha = 2\Delta$ and $m = 0$

The values obtained for the waveguide dispersion are added to that obtained for the material dispersion giving the chromatic dispersion. The chromatic dispersion is displayed in Fig.3 against the wavelength λ, where a fair agreement is noticed when compared with the work of D.Marcuse et al. [13]. This supports the new approach and so one can extend it for the case of the biquadratic refractive index, eq.(1).

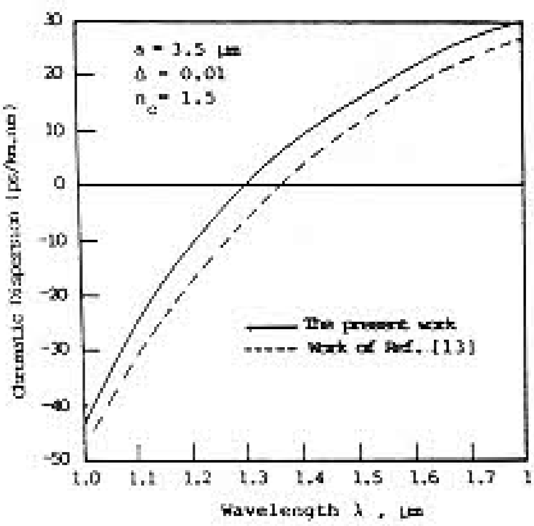

Fig.3. Chromatic Dispersion for quadratic index single-mode optical fibers.

The values of the tailoring parameters , α and m, of the core refractive index are adjusted in the ranges :

$$0.01 \le \alpha \le 0.34 \qquad , \qquad (15)$$

and

$$0.55 \le m \le 0.85 \qquad (16)$$

The corresponding values of n_1, n_2 and r_m, Fig.1, are :

$$0.95\ n_c < n_1 < 0.98\ n_c \qquad , \qquad (17)$$

$$0.90\ n_c < n_2 < 0.96\ n_c \qquad , \qquad (18)$$

and

$$r_m = 0.7\ a \qquad (19)$$

The waveguide dispersion D_w is calculated as a function of the wavelength λ, at different sets of fiber parameters, including α, m, and the core radius a. The obtained values are added to the calculated material dispersion to get the total chromatic dispersion. It must be kept in mind that the chromatic dispersion depends on the fiber parameters through the dependence of the waveguide dispersion D_w on them. This is because the material dispersion D_m depends only on the fiber material and not on the fiber parameters. A sample of the obtained results are shown in Fig.4, which displays the variation of the chromatic dispersion with the wavelength λ at different values of the core radius a.

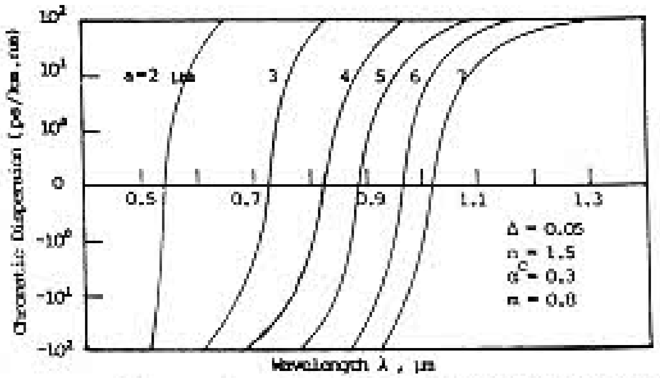

Fig.4. Chromatic dispersion for biquadratic index single-mode optical fibers (effect of a and Δ).

From Fig.4, one can predict the zero dispersion wavelength λ_o. The variation of the zero dispersion wavelength λ_0 with the fiber radius is drawn in Fig.5, from which

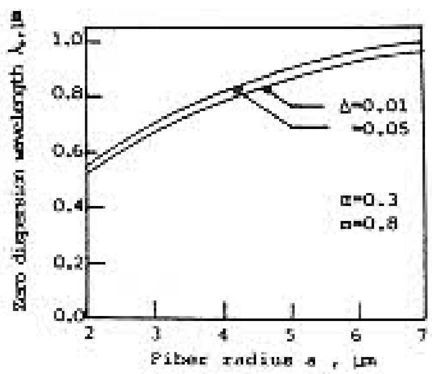

Fig.5. Variation of the zero dispersion wavelength λ_0 with the core radius a.

The effect of the two tailoring parameters of the refractive index, α and m, on the value of λ_0 is studied at constant values of a and Δ. It was found that the value of λ_0 decreases with m and increases with α, as shown in Fig.6.

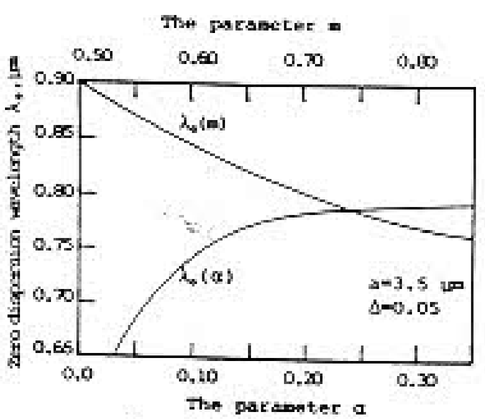

Fig.6. Variation of the zero dispersion wavelength λ_0 with the tailoring parameters α and m.

IV. Conclusion

In the present work, a new approach is introduced to calculate the waveguide dispersion, and consequently the chromatic dispersion, for graded index optical fibers. Quadratic and biquadratic variations of the core refractive index are considered. The obtained results are compared to the published ones [13] showing a good agreement. The introduced model is valid for values of the normalized frequency V in the range 1.2-3.6, which are consistent with that of the single mode graded index fibers [9]. For a given frequency λ, the chromatic dispersion decreases with the increase of the core radius a as shown in Fig.4.

The zero dispersion wavelength λ_0 is also studied. Appreciable dependence of λ_0 on the fiber parameters was noted, Figs.5 and 6. It was found that the value of λ_0 decreases with the increase of the parameter m and increases with the increase of each of the core radius a, the relative refractive index difference Δ, and the parameter α, where α and m are the tailoring parameters of the core refractive index.

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