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# RADIAL TEMPERATURE DISTRIBUTION IN BIQUADRATIC-INDEX OPTICAL FIBERS

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ABSTRACT

Radial temperature distribution in biquadratic-index optical fibers is studied and investigated. The radial temperature profile is obtained by solving the energy balance differential equation for a wide range of the controlling parameters: launching conditions and fiber parameters.

## 1. Introduction

When a laser beam is propagated through an optical fiber, a fraction of the laser energy is absorbed by the medium. The whole fiber was originally at a constant temperature $T_o$ , the ambient temperature. However, this absorbed power heats the fiber and alters the temperature along the laser path. These temperature deviations are thought to create new temperature profiles both axially and radially. The temperature distribution was studied in step-index multimode optical fibers when light is launched into the core[1]. For biquadratic-index fibers, only the axial temperature distribution was investigated[2]. Till now, to the best of our knowledge, no work is published about the effect of the laser propagation on the radial profile of temperature. The temperature radial distribution at any cross-section has been proved to be a function of two quantities:the ray trajectory position at the same cross-section, and the hot spot parameter. The aim of this work is to find out the radial temperature distribution when a laser is allowed to propagate through the fiber taking into consideration the effect of both the fiber's parameters and the launching conditions.

## 2. The Ray Trajectory

To consider the propagation of light within an optical fiber utilizing the ray theory model it is necessary to take account of the refractive index of the medium. The biquadratic refactive index spatial profile n of a fiber is given by[3]:

$$n^2(\rho)=n_o^2(1-2\alpha\rho^2+2m\alpha\rho^4), \qquad (1)$$

where $n_o$ is the axial refractive index, $\alpha$ and $m$ are two constant parameters, and $\rho$ is the normalized radial position; $\rho$ $(=r/R)$, with $r$ the radial position and $R$ the fiber radius.

The ray trajectory position is a function of the launching conditions, and the normalized axial propagation distance; $\eta(=z/R)$ ,with $z$ the axial position from the launching point. It is defined by the equation[4]:

$$\frac{\partial^2\rho}{\partial\eta^2} = \frac{1}{2n_o^2N_o^2}\frac{\partial n^2}{\partial\rho}, \tag{2}$$

where $N_o$ is the direction cosine of the incident ray.
For more simplification, we use:

$$\rho_m = \rho\sqrt{2m}\,, \tag{3.a}$$

and

$$\eta_m = \eta\sqrt{2\alpha}/N_o \tag{3.b}$$

in Eq.(2) and get:

$$\frac{\partial^2\rho_m}{\partial\eta_m^2} + \rho_m - \rho_m^3 = 0. \tag{4}$$

The solution of Eq.(4) is assumed under the series form:

$$\rho_m = \sum_{l=0}^{\infty} C_l\eta_m^l\,, \tag{5}$$

With the launching conditions:

$$\rho_m\big|_{\eta=o.} = \rho_o = C_o \ , \qquad (6.a)$$

and

$$\frac{\partial \rho_m}{\partial \eta_m}\Big|_{\eta=o.} = \rho_o' = C_1 \ . \qquad (6.b)$$

When Eqs. (5) and (6) are inserted into Eq.(4), the differential equation can be solved numerically to calculate the modified trajectory radial position which when divided by $\sqrt{2m}$, we obtain the exact value of the ray position as a function of $\eta_m$. The function $\rho_m(\eta_m)$ varies periodically during propagation and the point at which $\rho_m$ is minimum corresponds to a maximum temperature rise in the fiber.

## 3. The Hot Spot Parameter

When a continuous wave Gaussian laser beam is injected into an optical fiber, the laser intensity $I(\rho,z)$ varies within the fiber according to the integral equation[5]:

$$\frac{I(\rho,z)}{I(\rho,0)} = \exp\left[-\sigma z - \int_o^z \left(\frac{\partial}{\partial\rho} + \frac{1}{I}\frac{\partial I}{\partial\rho}\right) \cdot \int_o^{z'} \frac{1}{n_o}\frac{\partial n}{\partial \rho}\, dz'' dz'\right] \ , \qquad (7)$$

Where $\sigma$ is the absorption coefficient of the fiber material. We assume that the initially Gaussian beam obeys the relation:

$$I(\rho,z) = \frac{I_o}{F^2} \exp\left[-\sigma z - \frac{\rho^2}{F^2}\right] \ , \qquad (8)$$

where $F$ and $I_o$ are, respectively, the hot spot parameter and the laser source intensity. Equation (8) is subjected to the initial conditions:

$$F = 1. \qquad \text{at} \qquad z = 0. \;, \qquad (9.a)$$

and

$$\rho = \rho_o \qquad \text{at} \qquad z = 0. \;. \qquad (9.b)$$

The use of Eqs. (8) and (9) into Eq. (7) gives, after some algebraic manipultion :

$$\frac{dF}{dz} = \frac{\frac{zF^3}{2n}\frac{\partial^2 n}{\partial \rho^2} - \frac{zF\rho}{n}\frac{\partial n}{\partial \rho} - F\rho\frac{\partial \rho}{\partial z}}{(F^2 - \rho^2)} \;, \qquad (10)$$

The first and second derivatives of n are obtained from Eq. (1). The value of the hot spot parameter is calculated through a numerical solution for the differential equation (10) using the Runge-Kutta method with the boundary conditions given by Eq. (9).

## 4. The Radial Temperature Distribution

The radial profile of the temperature in the fiber is found by solving the energy balance differential equation (6):

$$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial T}{\partial \rho}\right) = -\frac{\alpha I_o R^2}{K F^2} \cdot \left(\frac{\rho_o}{\rho}\right)^2 \cdot \exp\left[-2\sigma n_o N_o z - \frac{\rho^2}{F^2}\right], \qquad (11)$$

where K is the thermal conductivity of the fiber material, and T is the absolute temperature at the radial position $\rho$.

For simplicity we use:

$$M = -\frac{\sigma I_o R^2 \rho_o^2}{K F^2} \exp\left[-2\sigma n_o N_o z\right] , \tag{12}$$

hence Eq. (11) may be rewratten as :

$$\frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial T}{\partial \rho}\right) = \frac{M}{\rho^2} \exp\left[-\frac{\rho^2}{F^2}\right] . \tag{13}$$

Direct integration of Eq. (13) gives:

$$\rho \frac{\partial T}{\partial \rho} = M \int_{\rho_w}^{\rho} \exp\left[-\frac{\rho^2}{F^2}\right] \frac{d\rho}{\rho} , \tag{14}$$

keeping in mind that $T_w$, the absolute temperature at $\rho_w$, has a maximum value, so $(\partial T/\partial \rho)_{\rho_w}$ equals zero. For a certain set of parameters ($\alpha, m, R, I_o, \sigma, \rho_o, \rho_o$)and the same cross section and radial position, the R.H.S. of Eq.(14) may be assigned to a constant A. Again through direct integration Eq.(14) yields:

$$T(\rho) = T_w - A \ln\left[\frac{\rho_w}{\rho}\right] , \tag{15}$$

where $T(\rho)$ and $T_w$ are taken at the same cross section.

## 5. Results And Discussion

Calculations are started by assigning the indepentent set of parameters ( $\alpha$ , m , R , $I_o$ , $\sigma$, $\rho_o$ and $\rho_o^*$) and then proceeding till reaching a complete radial profile of the temperature at different values of $\rho$ . Along the fiber, the temperature rises gradually with the decrease of $\rho_w$. Figure 1 illustrates both behaviors. At each cross section there is a different temperature rise which is concentrated at $\rho_w$ and is slightly declined at other radial positions.

A wide range of values of the independent variables has been investigated in order to determine their effect on the radial temperature profile. It is obvious that whatever the change in the values of the independent parameters is, the same temperature distribution results although a change in the maximum temperature was observed. A typical radial temperature profile is shown in Fig. 2 . The effect of $\alpha$,R,$I_o$, $\rho_o$ , $\rho_o^*$ and $\sigma$ on the radial distribution temperature are displayed on Figs. 3 to 8 .

Two remarks are to be noticed in the results. Firstly, the decrease of the radial temperature is so small since the thickness of the fiber is in the order of tens of micrometers . Another explanation can be deduced from Eq.(15). The constant A that determine the temperature reduction is a small quantity and so is the logarithmic term $\ln(\rho_w/\rho)$. The net result is that T ( $\rho$ ) is approximately equal to its axial value whatever the value of $\rho$ is. Secondly, the change in the maximum temperature with the change of any parameter from the independent set of parameters means that the elements of this set affect mainly the axial temperature distribution but not the radial profile.

## 6. Conclusions

Numerical calculations for a biquadratic index optical fiber are

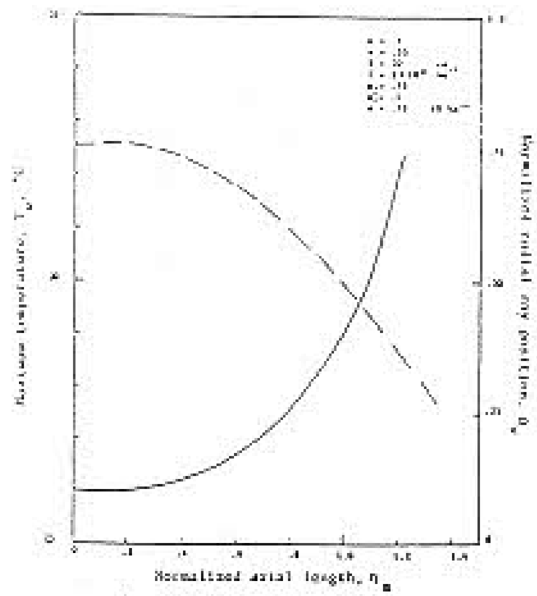



Fig.1 Variation of $T_w$ and $\rho_w$ with $\eta_m$

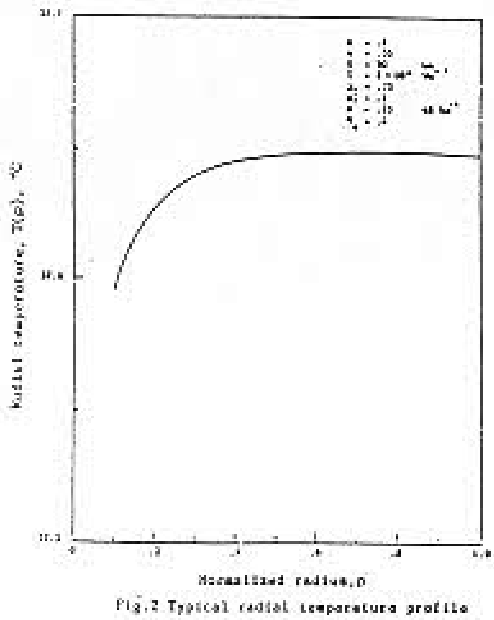



Fig.2 Typical radial temperature profile

Fig.4 Radial temperature profile : Effect of [illegible]

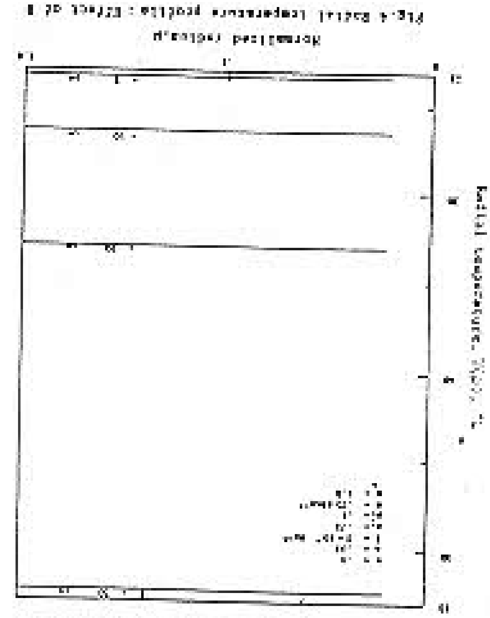
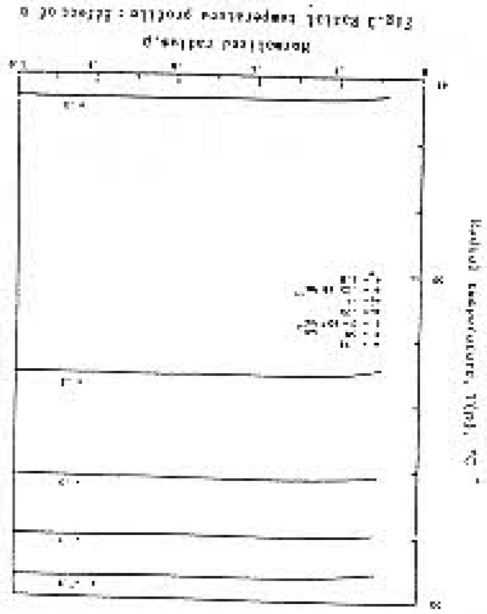


Fig.3 Radial temperature profile : Effect of [illegible]



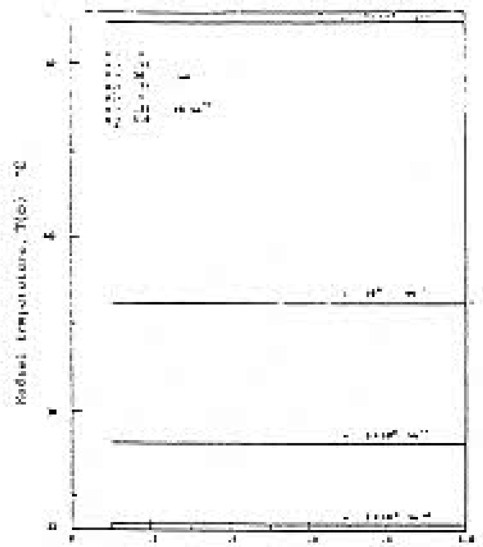



Fig.5 Radial temperature profile: Effect of I

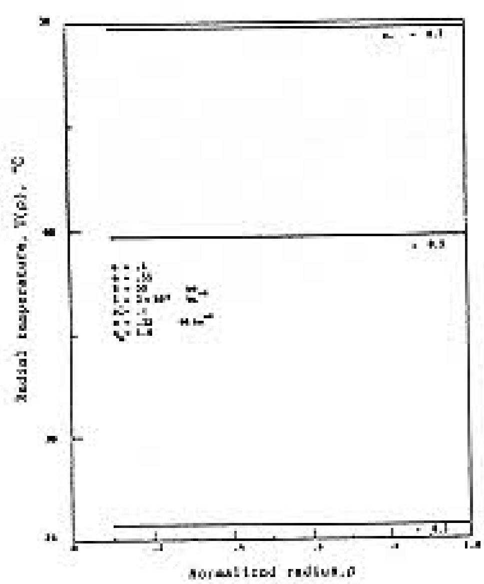



Fig.6 Radial temperature profile : Effect of $P_L$

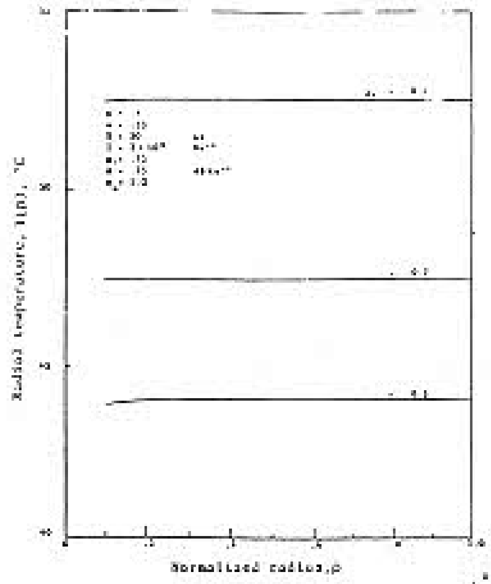


Fig.7 Radial temperature profile : Effect of $\rho_i$

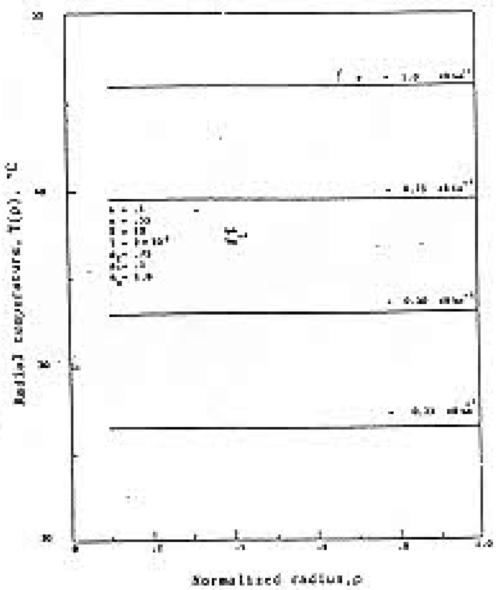


Fig.8 Radial temperature profile : Effect of $\sigma$

performed to obtain the temperature radial distribution at various cross sections of an optical fiber with certain values of the controlling parameters, and then at a specific cross section with the variation of these controlling parameters. We have shown that :

i. The axial temperature rise at any cross section is inversely proportional to the radial position.

ii. The radial temperature profile obeys a general behavior irrespective of the variations made in the controlling parameters.

iii. At a certain cross section, the temperature rise is directly proportional to all the controlling parameters except m and the launching angle. The tailoring parameter m has no effect on the temperature rise while an increase of the launching angle tends to reduce the temperature at the same cross section.

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