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DARK SOLITONS IN SINGLE MODE OPTICAL FIBERS

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Abstract

The dark soliton transmission of optical pulses in nonlinear inhomogeneous optical fibers is modelled and parametrically analyzed. The tailoring parameters of the refractive index have great influence on the minimum power and the high group velocity required for a stable soliton transmission.

1. Introduction

Normally, a light pulse spreads out as it travels along an optical fiber. This happens because parts of the light pulse travel at different speeds in the fiber: an effect known as dispersion. As the intensity of the pulse increases, another effect comes into play, which squeezes the pulse. If the intensity of the pulse is increased to a certain level, the two effects balance each other and the pulse becomes a soliton.

Light solitons were first produced in the 1930s. Since then, theoretical physicists have wondered whether the same effect could be produced in dark pulses. By a dark pulse, they mean a short interruption in a steady beam of light. In 1987, Dieter Kroekel and his colleagues in IBM's Watson Research in New York became the first to produce dark pulses [1].

In the present paper, a theoretical work is carried out on the propagation of dark solitons in single mode fibers. The fiber under consideration is of a biquadratic refractive index. The tailoring parameters are selected through the calculation of the group velocity of the soliton pulses and the accompenied power.

Section II deals with the mathematical model. In Section III results are illustrated and the discussion outlies how the tailoring parameters of the refractive index affect the physical properties of the propagating dark soliton pulses.Conclusions are given in Section IV.

2. Basic Model and Analysis

For a three-dimensional wave propagation in a nonlinear medium, the wave equation for the electric field $E(r,z,t)$ is given by [2]:

$$\nabla^2 E - (1/c^2)(\partial^2 D_L/\partial t^2) = (2n_2 n_0/c^2).\partial^2/\partial t^2(|E|^2 E) , \qquad (1)$$

where $n_2 |E|^2$ is the nonlinear part of the refractive index,

$$n(r,\omega,E) = n(r,\omega) + n_2 |E|^2 . \qquad (2)$$

The inhomogeneity of the medium is taken to be represented by

$$n^2(r,\omega) = n_0^2(\omega)[\ 1 - 2\alpha \rho^2 + 2m\alpha \rho^4], \qquad (3)$$

with ρ $(=r/R)$ the normalized radius and R the fiber's core radius. Nonlinearity and inhomogeneity are contained within the wave equation to offset the undesired effects of longitudinal and transverse types of dispersion, respectively.

In Eq.(1), the dispersion of the medium is included in the linear part of the dielectric as

$$D_L(r,z,t) = (\ 1 - 2\alpha \rho^2 + 2m\alpha \rho^4) \int n^2(t-t') E(r,z,t')dt' , \qquad (4)$$

with $n_0 \equiv n(\omega)$, where ω is the frequency of the electric field

$$E(r,z,t) = \hat{e} A(r,z,t) \exp[i(\beta z - \omega_0 t)], \qquad (5)$$

where β is the propagation constant and $\hat{e}$ is a unit vector.

The amplitude of the electric field is defined in the frequency domain by

$$A(r,z,t) = A_\omega(\rho,z)\exp[-i\omega t] \quad . \qquad (6)$$

Transferring to this domain, Eq. (4) becomes

$$D_L(r,z,\omega) = (1 - 2\alpha\rho^2 + 2m\alpha\rho^4)\int n^2(\omega+\omega_0)E(r,z,\omega)\,d\omega \quad . \qquad (7)$$

Substituting from Eq.(5) in Eq.(7), one can get

$$D_L(r,z,\omega) = \hat{e}(1 - 2\alpha\rho^2 + 2m\alpha\rho^4)\exp[i(\beta z-\omega_0 t)] \cdot \int n^2(\omega+\omega_0)A_\omega(\rho,z)\exp[-i\omega t]\,d\omega \quad . \qquad (8)$$

Therefore

$$(-1/c^2)\,\partial^2 D_L/\partial t^2 = \hat{e}[1 - 2\alpha\rho^2 + 2m\alpha\rho^4]\exp[i(\beta z-\omega_0 t)] \cdot \int k^2(\omega)A_\omega(\rho,z)\exp[-i\omega t]\,d\omega \quad , \qquad (9)$$

where

$$k^2(\omega) = \omega^2 n^2(\omega)/c^2 \quad , \qquad (10)$$

Expanding $k(\omega)$ around $k(\omega_o) = k_o$, we get

$$k^2(\omega) = k_o^2 + 2k_o k_o' \omega + (k_o'^2 + k_o k_o'')\omega^2 \quad , \tag{11}$$

with k_o, k_o', and k_o'' are the dispersion relations.
As an operator, ω is equivalent to $i\partial/\partial t$, then Eq.(9) can be rewritten as

$$(-1/c^2)\partial^2 D_L/\partial t = \hat{e}\,[\,1 - 2\alpha\,\rho^2 + 2m\alpha\,\rho^4]\exp[i(\beta z - \omega_o t)]$$
$$\cdot \int [\,k_o^2 + 2ik_o k_o'\partial/\partial t - (k_o'^2 + k_o k_o'')\partial^2/\partial t^2]\,A_\omega(\rho,z)\exp[-i\omega t]\,d\omega \;. \tag{12}$$

Substituting back into the wave equation, we get

$$[\,\nabla_\perp^2 + \partial^2/\partial z^2 + 2i\beta\partial/\partial z - \beta^2 + (\,1 - 2\alpha\,\rho^2 + 2m\alpha\,\rho^4)(k_o^2 + 2ik_o k_o'\partial/\partial t$$
$$-(k_o'^2 + k_o k_o'')\partial^2/\partial t^2)]\,A(r,z,t) = -2(n_2/n_o)k_o^2 \cdot |A|^2 A(r,z,t) \;. \tag{13}$$

To solve Eq.(13), we use the method of separation of variables as follows

$$A(r,z,t) = \psi(r)\cdot\phi(z,t) \;, \tag{14}$$

Equation (13) turns to be

$$\phi\nabla_\perp^2\psi + \psi[\,\partial^2/\partial z^2 + 2i\beta\partial/\partial z - \beta^2 + (\,1 - 2\,\alpha\rho^2 + 2m\,\alpha\rho^4)$$
$$\cdot(k_o^2 + 2ik_o k_o'\;\partial/\partial t - \{k_o'^2 + k_o k_o''\}\;\partial^2/\partial t^2)\,]\,\phi$$
$$= -2(n_2/n_o)\,k_o^2\,|\psi\phi|^2\,\psi\,\phi \quad , \tag{15}$$

where

$$\nabla_\perp^2 = (1/R^2)(\,\partial^2/\partial\rho^2 + 1/\rho\;\partial/\partial\rho\,) \;. \tag{16}$$

The propagation constant, β , is calculated through the perturbation theory to the third order as [3] :

$$\beta^2 = k_o^2 - 2k_o\sqrt{2\alpha}/R + 2m/R^2 + 9m^2/2R^3k_o\sqrt{2\alpha} + 79m^3/8R^4k_o^2\alpha \quad . \quad (17)$$

The first step is to determine the radial part of the electric field. We start with the separation of the r-dependent terms of Eq.(15):

$$[\nabla_\perp^2 - \beta^2 + k_o^2 (1 - 2\alpha\rho^2 + 2m\alpha\rho^4)] \psi(\rho) = 0 \quad . \quad (18)$$

Assuming $\psi(\rho)$ to be a simple series of the form

$$\psi(\rho) = \exp[-\rho^2/2].(1 + \sum_{\ell=1} C_{2\ell} \rho^{2\ell}) . \quad (19)$$

The recurrence formula between the coefficients C_ℓ' can be obtained as

$$C_\ell = (1/\ell^2)[(2\ell - 2 + X1) C_{\ell-2} + X2.C_{\ell-4} + X3.C_{\ell-6}] \quad , \quad (20)$$

with $C_{-2}=C_{-4}=0$, $C_o=1$, ℓ is an even number,given that

$$X1 = R^2 (\beta^2 - k_o^2) \quad , \quad (21)$$

$$X2 = 2\alpha R^2 k_o^2 - 1 \quad , \quad (22)$$

and

$$X3 = - 2m\alpha R^2 k_o^2 \quad . \quad (23)$$

The next step is to find out the second part of the electric field. To eliminate the r-dependence from Eq.(15), we have to multiply it by $\psi(\rho)\,d\rho$ and integrate over the region within which the pulse is confined, i.e., from $\rho = 0$ **to** $\rho = \rho_o$. Doing this one can get:

$$[\ \partial^2/\partial z^2 + 2i\beta\ \partial/\partial z + Q\ (\ 2ik_o\dot{k}_o\ \partial/\partial t - \{\dot{k}_o^2 + k_o\ddot{k}_o\ ]\ \partial^2/\partial t^2)]\ \phi(z,t)$$
$$= -2(n_2/n_o)\ k_o^2\ |\phi|^2\ \delta\ \phi(z,t) \quad , \qquad (24)$$

where

$$Q = \{\int_0^{\rho} [\ 1 - 2\alpha\rho^2 + 2m\alpha\rho^4]\ \psi(\rho)d\rho\ \}/\{\ \int_0^{\rho} \psi(\rho)d\rho\ \} \quad , \qquad (25)$$

and

$$\delta = \{\ \int_0^{\rho} \psi^3\ (\rho)d\rho\ \}\ /\ \{\ \int_0^{\rho} \psi(\rho)d\rho\ \}\ . \qquad (26)$$

Equation (24) is now a function of both z and t. If η is a function defined by

$$\eta = (1/\tau)[t - z/V_g] \quad , \qquad (27)$$

where τ is the pulse duration and V_g is the group velocity, $\varphi\,(z,t)$ becomes $\varphi\,(\eta)$. Since $\varphi(\eta)$ must be real for solitons to exist, the imaginary part of Eq.(24) is

$$\beta\, \partial/\partial z + 2Qk_o \grave{k}_o\, \partial/\partial t = 0 \ , \qquad (28)$$

from which V_g can be obtained as

$$V_g = \beta/2Qk_o \grave{k}_o \qquad . \qquad (29)$$

Equation (24) admits a dark soliton solution :

$$\phi(\eta) = \phi_o \tanh \eta \quad , \qquad (30)$$

A substitution of Eq.(30) in Eq.(24) results in

$$\phi_o^2 = \{\ Q\ [\grave{k}_o^2 + k_o \acute{k}_o\ ] - [\ 1/V_g^2\ ]\ \}/\ \{ (n_2/n_o)\tau^2 k_o^2\ \delta \} \qquad . (31)$$

By Eq.(31), the definition of the electric field, given by Eq.(14), is obtained.

The peak power, P_o, in watts, of a soliton pulse is given by [4] :

$$P_o = 0.5\ \varepsilon_o\ n_o^2\ V_g\ S\ \phi_o^2 \qquad , \qquad (32)$$

where ε_o is the permittivity of free space, S is the cross sectional area of the fiber core, and φ_o, defined by Eq.(31), is the peak electric field at the core's center.

3. Results and discussion

Before we detect the allowable values of the tailoring parameters(α and m) that give suitable values of the group velocity and the peak power, we have to be sure that the quantity φ_o^2 given by Eq.(31), has a positive value. This limits the ranges over which α and m can spread.

Equation (31), representing φ_o^2 necessitates that there must exist threshold values for the tailoring parameter α for possible transmission($\varphi_o^2 > 0$ and consequently $P_o > 0$). These values α_{th} , are detected for a fiber of radius 4μ m and the variation of α_{th} with m is illustated in Fig.1.

For values of α higher than α_{th} , the group velocity V_g ,Eq.(29), and the peak power P_o, Eq.(32), are directly calcuated. The variations of V_g and P_o with both α & m are shown in Figs.2-7.

The previous calculations are repeated for fibers with radii 2.5 μ m and 5 μ m . The obtained values of V_g always exceed the permissible value (c/n) from which one can deduce that these values of R are totally unacceptable.

Finally, we must mention that in our calculations the series which determines the radial part of the electric field , Eq.(19), is truncated after 60 terms where a negligible effect of higher terms was noticed. Integrations found in Eqs.(25) and (26) are performed numerically with ρ_o = 0.6 is the upper lilmit of integration. This means the pulse confinement is within the entire core since a minimum value of the refractive index at ρ_o = 0.6 prevents the pulse existence outside this region.

4. Conclusion

The above mentioned results and discussion leads to the following

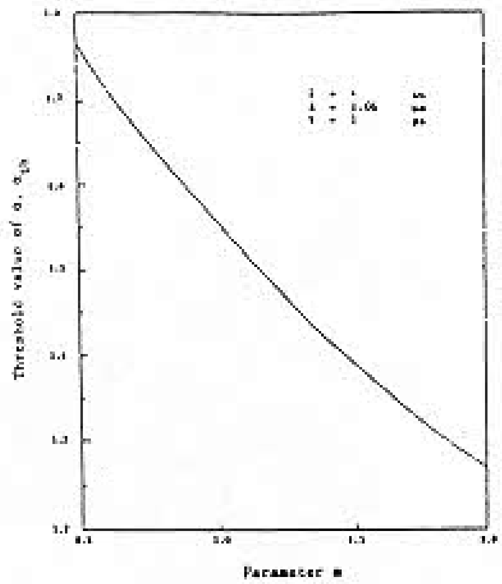

Fig.1 Variation of α_{th} with m

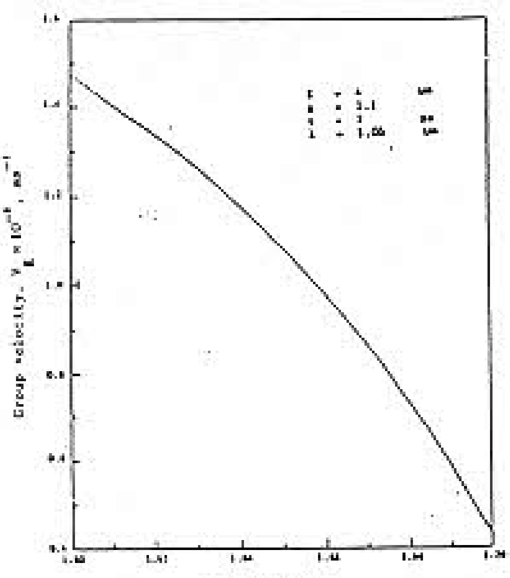

Fig.2 Variation of V_g with α

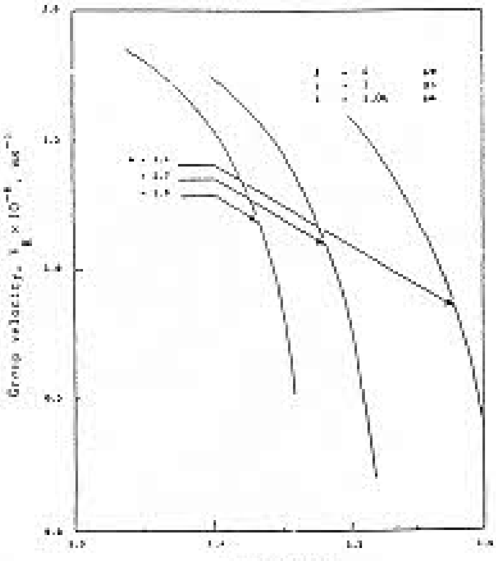

Fig.3 Variation of V_g with α

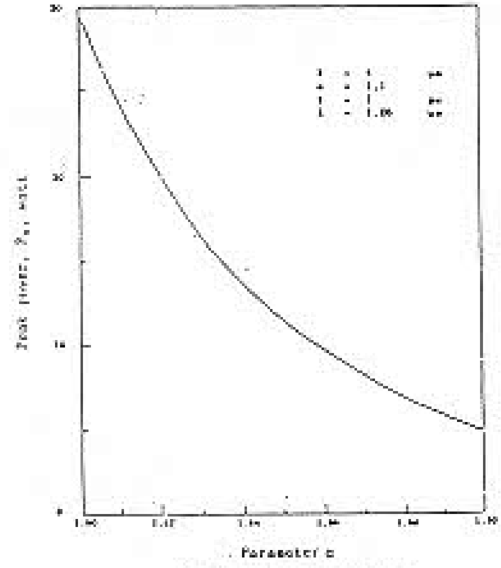

Fig.4 Variation of P_o with α

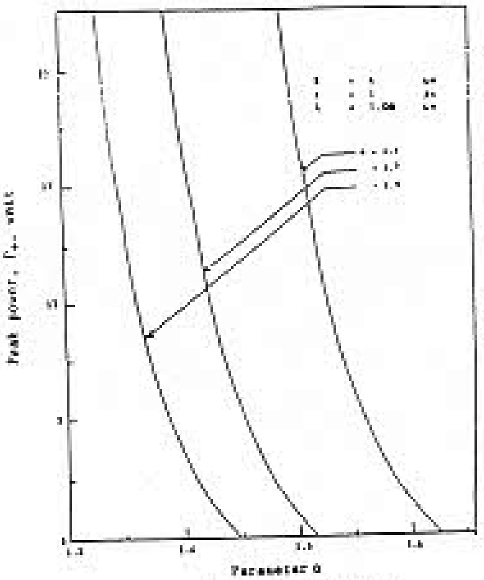

Fig.5 Variation of P_+ with θ

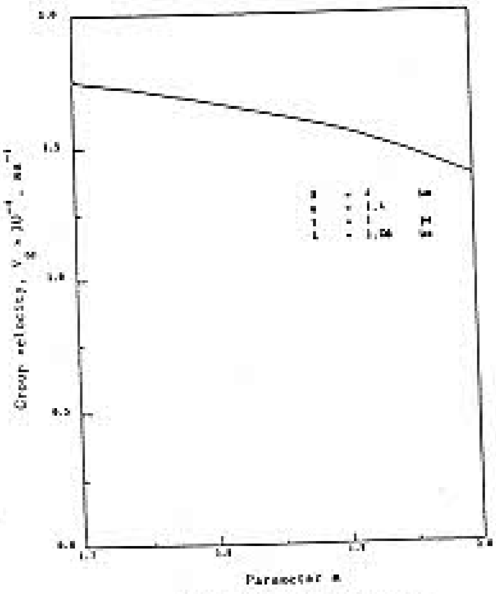

Fig.6 Variation of V_g with m

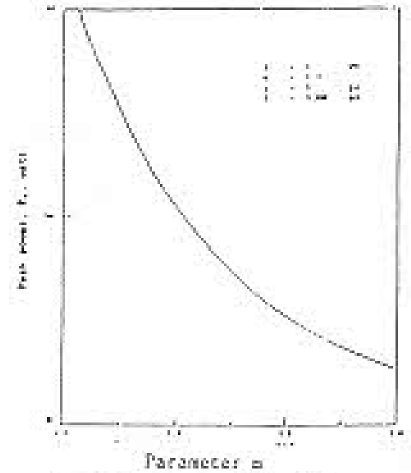

Fig.7 Variation of P_m with m

conclusions :

i - In any application, the parameter α must be tailored with a value higher than the threshold value α_{th} for a stable transmission.

ii - For an Nd:YAG laser source, λ = 1.06 μm , the fiber radius must be in the order of 4 μm to allow a dark soliton transmission.

iii- For a high quality communication process (high group velocity and low power), the controlling parameters α and m must be tailored with values:

and $1.3 < \alpha < 1.7$

$1.6 < m < 2.0$

References

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