ON THE SOLITON TRANSMISSION IN NONLINEAR INHOMOGENEOUS GRADED-REFRACTIVE INDEX MEDIA

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ABSTRACT

In this paper, we investigate the soliton transmissi of optical pulses in nonlinear inhomogeneous graded-refractive index fibers (biquadratic profile). Avoiding the linear fashon approximation, the radial dependence of the field is treated. The obtained solutions are parametrically controlled in such a manner that both the light soliton and the dark one exist in both the anomalous dispersion region and the normal one. The controlling parameters affect the power required to achieve soliton propagation, the bit rate, and the group velocity.

I. INTRODUCTION

Recently, two decades ago, the propagation of solitary waves in nonlinear dispersive optical fibers has become of great importance. This pulse-type stationary solutions of nonlinear dispersive wave equations has attract the attention both mathematically and physically [1,2,3].

Solitons in optical communications are now considered to have a potential application to a high-bit-rate transmission system [4] with an additional important merit of automatic pulse reshaping during the course of transmission [5,6]. The radial dependence of the field had been linearly treated in the previous theories [7,8] where an equation for the scalar envelope $\varphi(Z,t)$ of the field that depends on axial position Z and the time t of the form

$$a \frac{d^2 \varphi}{d \eta^2} + b\varphi + C \varphi^3 = 0 \qquad (1)$$

where $\eta = Z - v_g t/v_g T$, v_g is the group velocity, T is the pulse duration, and a,b, and c are appropriate constants. This equation yields both light (a is positive)- and dark (a is negative)-solitons of the form sech $P\eta$ and tanh $p\eta$ respectively for anomalous and normal dispersion region of the fiber. Fibers with either quadratic or step refractive index in the radial direction had been considered [7,8].

El-Halafawy et.al. [9] analyzed the soliton propagation in the basis of the same linear fashon through graded fibers of biquadratic refractive index $n(r,w)$ of the form

$$n(r,w) = n(w)[1 - \alpha r^2 + m \alpha r^4] \qquad (2)$$

where w is the angular frequency, α and m are positive controlling parameters. The parameter α has a threshold value α_{th} related to fiber radius, R, thus for α less than α_{th} the soliton propagation is impossible. The controlling parameters α and m yield both light and dark solitons for both anomalous and normal dispersion regions of the fibers. The radial dependence was treated exactly of lossless dispersive step-index fiber [10,11]. Both light and dark solitons are found in the anomalous dispersion region of the fiber. In the present paper we analyze the soliton propagation in dispersive biquadratic-refractive-index fiber avoiding the linear fashon treatment of the radial dependence. Section II describes the basic model and deals with the mathematical formulation, analysis, and computational techniques. The results and discussion are exposed in section III. The main conclusions are pointed out in section IV.

II. BASIC MODEL AND ANALYSIS

It is now a well-established fact that the space-time evaluation of the complex amplitude $E(r;Z,t)$ of an optical-field envelope in a glass fiber obeys the nonlinear differential equation [7,12]

$$[\nabla_\perp^2 + \frac{\partial^2}{\partial z^2} + 2jq\frac{\partial}{\partial z} - q^2 + (n^2(r,w)/n^2(w)) \, .$$

$$\{(k_o^2 + 2jk_o k_o' x \frac{\partial}{\partial t} - (k_o'^2 + k_o k_o'') \frac{\partial^2}{\partial t^2}\}]A(r,z,t)$$

$$= \frac{-2n_2}{n_o} k_o^2 |A|^2 A \qquad (3)$$

where

$$E(r,z,t) = A(r,z,t)e^{-jwt} \qquad (4)$$

The electric field amplitude is given by A(r,z,t). The quantities k_o and k_o are given by:

$$k_o' = \left.\frac{\partial k}{\partial w}\right|_{w=w_o} \qquad (5)$$

$$k_o'' = \left.\frac{\partial^2 k}{\partial w^2}\right|_{w=w_o} \qquad (6)$$

where $k = wn(r,w)/c$ (7)

In the present analysis, the index of refraction is assumed under the form

$$n(r,w) = n(w)(1 - \alpha\rho^2 + \alpha m \rho^4) \qquad (8)$$

where $\rho(= r/R)$ is the normalized radial position and R is the fiber radius. The scalar amplitude (envelope) A(r,z,t) is obtained as

$$[\nabla_{\perp}^2 + \frac{\partial^2}{\partial z^2} - \beta^2 \frac{n^2(r,w)}{n^2(w)} k_o^2 - (k_o'^2 - k_o k_o'') \frac{\partial^2}{\partial t^2}] A(r,z,t)$$

$$= - \frac{2 n_2}{n_o} k_o^2 |A|^2 A \qquad (9)$$

The use of

$$\eta = Z - v_g t / v_g T \qquad (10)$$

$$\varphi(r,z,t) = \left(\frac{2n_2 R^2 k_o^2}{n_o}\right)^{0.5} A \qquad (11)$$

in Eqn. (9) yields:

$$[\nabla_\rho^2 + a \frac{\partial^2}{\partial \eta^2} + b(1-\alpha\rho^2+m\alpha\rho^4)^2 - C]\varphi + \varphi^3 = 0 \qquad (12)$$

with

$$\nabla_\rho^2 \equiv \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho}$$

$$a = R^2 \{ v_g^{-2} - (k_o'^2 + k_o k_o'') \} T^{-2} \qquad (13)$$

$$b = R^2 k_o^2 , \qquad (14)$$

and

$$C = R^2 \beta^2 \qquad (15)$$

To obtain either light soliton solution φ_L or dark one φ_D even in and on the same spirit as [11,12] we represent $\varphi(\rho, \eta)$ in the form

$$\varphi_L(\rho,\eta) = \operatorname{sech}\eta \sum_{n=0} R_n(\rho) \operatorname{sech}^{2n} \eta \qquad (16)$$

$$\phi_D(\rho,\eta) = \tanh\eta \sum_{n=0}^{n=\infty} R_n(\rho)\,\mathrm{sech}^{2n}\eta \qquad (17)$$

These solutions whatever the value of η must yield

$\phi(1,\eta) = 0.0$, i.e., at $r = R$.

The substitution of Eqn. (16) in Eqn.(12) then equating to zero the coefficient of each power of $\mathrm{sech}^2\eta$ result the following respectively:

$$\sum_{n=0}^{\infty} S^{2n}\left[R_n'' + \frac{1}{\rho}R_n' + \gamma_n^2 R_n\right]$$

$$-2S^2\sum_{n=0}^{\infty}(n+1)(2n+1)S^{2n}R_n + S^2\{\Sigma R_n S^{2n}\}^3 = 0 \qquad (18)$$

where $S \equiv \mathrm{sech}\,\eta$ and $\gamma_n^2 = b(1-\alpha\rho^2 + m\alpha\rho^4)^2 - C + a(2n+1)^2$

$$R''_o + \frac{1}{\rho}R'_o + \gamma_o^2 R_o = 0 \qquad (19)$$

$$R_1'' + \frac{1}{\rho}R_1' + \gamma_1^2 R_1 = 2R_o(a - \frac{1}{2}R_o^2) \qquad (20)$$

$$R_2'' + \frac{1}{\rho}R_2' + \gamma_2^2 R_2 = 6R_1(2a - \frac{1}{2}R_o^2) \qquad (21)$$

with the boundary conditions

$$\frac{dR_n}{d\rho} = 0 \quad \text{at } \rho = 0$$

$$R_n = 0 \quad \text{at } \rho = 1 \qquad (22)$$

The same sort of relationships are obtained on the substitution of Eqn. (17) in Eqn. (12).

The linear imaginary part of Eq. (3) has been employed to derive an expression for the constant envelope velocity v_g under a radially-averaged form,

$$v_g = \frac{\beta}{k_o k_o^{`}\left(1+\frac{m^2 \alpha^2}{5} - \frac{\alpha^2 m}{2} + \frac{\alpha^2}{3} + \frac{2\alpha m}{3} - \alpha\right)} \qquad (23)$$

where β is the phase constant. The use of the perturbation theory to the third order for single mode fiber yields [13]

$$\beta^2 = k_o^2 - \frac{2k_o}{R}\sqrt{2\alpha} + \frac{2m}{R^2} + 4.5\,\frac{m^2}{\sqrt{2\alpha}R^3 k_o^2} + \frac{79\;\; m^3}{8\,\alpha R^2 k_o^2} \qquad (24)$$

For R_n we assume a solution of the form

$$R_n = \sum_{i=0}^{\infty} a_{ni}\,\rho^i \qquad (25)$$

The use of Eqn. (25) in Eqns. (19),(20), (21),... yields.

$$a_{n1} = a_{n3} = a_{n5} = \ldots = 0, \qquad (26)$$

and for even terms case of $n = 0$.

$$\sum_{i=0} 4(i+1)^2 a_{02(i+1)} \rho^{2i} + \{b(1-\rho^2 + m_\alpha \rho^4)^2 - c + a\} \times \sum_{i=0} a_{02i} \rho^{2i} = 0 \qquad (27)$$

with $i = 1, 2, 3, 4 \ldots$.

On equating the coefficients of the same power of ρ^2 the even terms of R_n are obtained as:

a_{0m} functions of $a, b, c, \alpha, m, a_{oo}$

a_{1m} functions of $a, b, c, \alpha, m, a_{oo}, a_{10}$

a_{2m} functions of $a, b, c, \alpha, m, a_{oo}, a_{10}, a_{20}$

.......etc.

where $m = 2, 4, 6, 8 \ldots$.

The use of the boundary conditions given by Eqn. (22) with the restriction that $dR/d\rho = 0.0$ yields the coefficients a_{nm}.

The peak power, P_o, of the pulse is obtained as [10]

$$P_o = \frac{1}{2} n_o \left(\frac{\varepsilon_o}{\mu_o}\right)^{\frac{1}{2}} S A_{max}^2 \qquad (28)$$

where n_o is the refractive index, S is the cross sectional area of the fiber, ε_o and μ_o are the permittivity and permeability of free space, and A_{max} is the maximum electric field which is obtained as follows:

$$\varphi(\rho,\eta)=R_o(\rho)\operatorname{sech}\eta+R_1(\rho)\operatorname{sech}\eta^3+R_2(\rho)\operatorname{sech}^5\eta \quad (29)$$

As $\rho=\eta=0.0$, φ possesses a maximum value φ_{max} where

$$\varphi_{max}=\sum_{n=0} a_{no} \quad (30)$$

The use of Eqn. (30) in Eqn. (11) yields

$$A_{max}=\sum_{n=0} a_{no}\Big/\sqrt{\frac{2n_2R^2k_o^2}{n_o}} \quad (31)$$

III. RESULTS AND DISCUSSION

The most important expansion coefficient of the functions $R_n(\rho)$ is the coefficient a_{oo}, on which all other coefficients, of different modes, have a direct dependence. This coefficient is derived as:

$$a_{oo}=\sqrt{2\ a} \quad (32)$$

and a is given by Eqn. (13). The quantity "a" must be positive to give a real solution. Thus, a criterion for the existence of solution is derived as: Soliton solution is obtained if the quantity a_{oo}^2 is a positive one. The use of Eqn. (13) into Eqn. (32) yields

$$a_{oo}^2=2R^2\left(v_g^{-2}-(k_o'^2+k_ok_o'')\right)/T^2 \quad (33)$$

where v_g, k_o, k_o', and k_o'' are functions of R, α, m, and λ. Thus for fixed values of both R and λ, the parameters α and m control the sign of the quantity a_{oo}^2 whatever the type of dispersion (normal or anomalous) or the type of soliton (light or dark). The authors carried out this approach carefully [9] and also in this paper the same approach is applied yielding the same sort of results as in Ref. [9], and not presented here.

The variations of the peak power P_o and the group velocity v_g are investegated at λ =10.6 m and different values of the set of parameters α, m, R. The expansion coefficients of the first three modes are given in table I.

a_{oo}= 1.428 $\times 10^{+1}$	a_{10}= -1.319 $\times 10^{-4}$	a_{20}=1.127 $\times 10^{-7}$
a_{02}=1.376	a_{12}=1.288 $\times 10^{-5}$	a_{22}=-1.375$\times 10^{-7}$
a_{04}=1.486	a_{14}=2.056 $\times 10^{-4}$	a_{24}=1.191 $\times 10^{-8}$
a_{06}=4.656 $\times 10^{-1}$	a_{16}=-1.174 $\times 10^{-4}$	a_{26}=1.575 $\times 10^{-8}$
a_{08}=8.164 $\times 10^{-2}$	a_{18}= 4.148 $\times 10^{-5}$	a_{28}=-3.032$\times 10^{-9}$
a_{010}=2.716$\times 10^{-2}$	a_{10}=-1.599$\times 10^{-5}$	a_{210}=-6.32$\times 10^{-10}$
a_{012}=5.312$\times 10^{-3}$	a_{112}=5.420$\times 10^{-6}$	a_{212}=7.720$\times 10^{-10}$
$\langle a_{om} \rangle$=1.398	$\langle a_{1m} \rangle$ =-7.992$\times 10^{-5}$	$\langle a_{2m} \rangle$=5.128 $\times 10^{-8}$

TABLE I. The expansion coefficients of the first three modes where λ = 10.6 μm, R = 5 μm, T = 1 n sec., α = 0.1 and m = 0.55

From this table, it is clear that all the expansions of the different modes are convergent, but it is clearly portrayed that the zero mode (n=0) is the most dominant one. Also, the average value of the coefficients, $< a_{nm} >$, points to the same conclusion. The variations of the quantity a_{oo}^{2} against the parameter α at the assumed set of parameters are displayed in Fig. 1, where it is clear that there is a threshold value for the parameter to obtain a stable soliton solution. Both the variations of the group velocity v_g and the peak power P_o possess respectively minimum value and a maximum value with the variations of the fiber raduis R and the asumed set of parameters around $R \approx 7.5\ \mu m$. as displayed in Figs. 2 and 3.

From the previous results it remarked that there is a critical values of α , α_{cr}, at which the values of the group velocity and the peak power are independent of the parameter m.

The critical values of the parameter α_{cr} are displayed in Fig. 4 where it is clear that α_{cr} is related to the fiber radius.

IV. CONCLUSION

For the best design of single mode fiber for digital transmission (high v_g and low P_o) at $\lambda = 10.6\ \mu m$, the following must be taken into consideration:

i. The radius R is smaller than 7.5 μm.

ii. The parameter must be of minimum value.

iii. The parameter m must be of maximum value.

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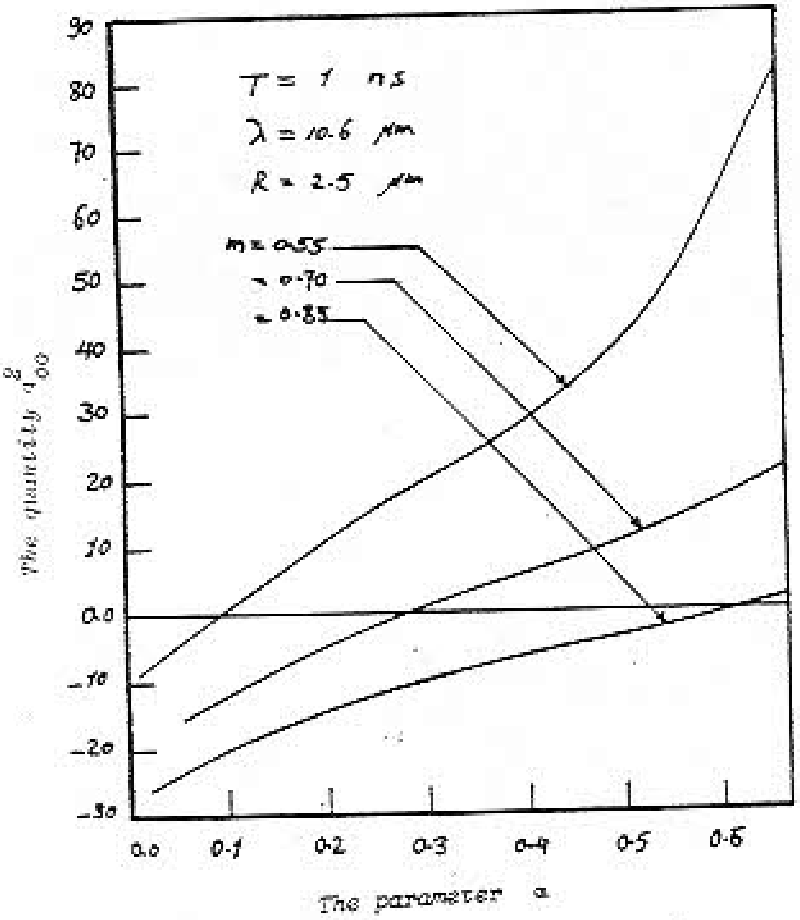


Fig. 1. The variations of the quantity a_{00}^2 with the parameter α

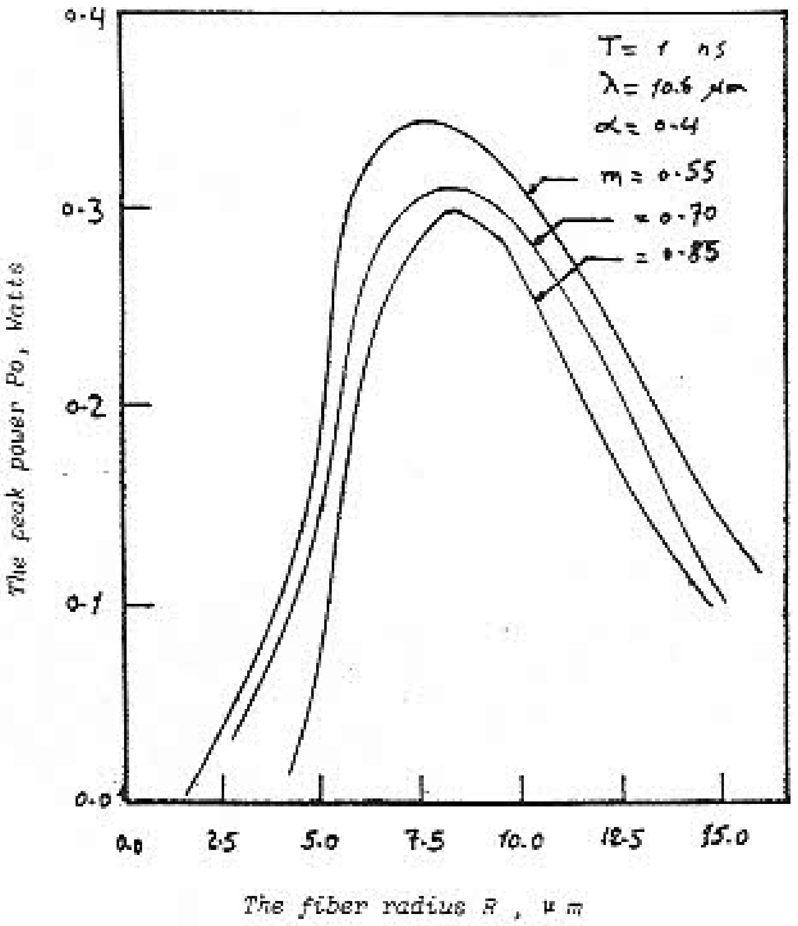
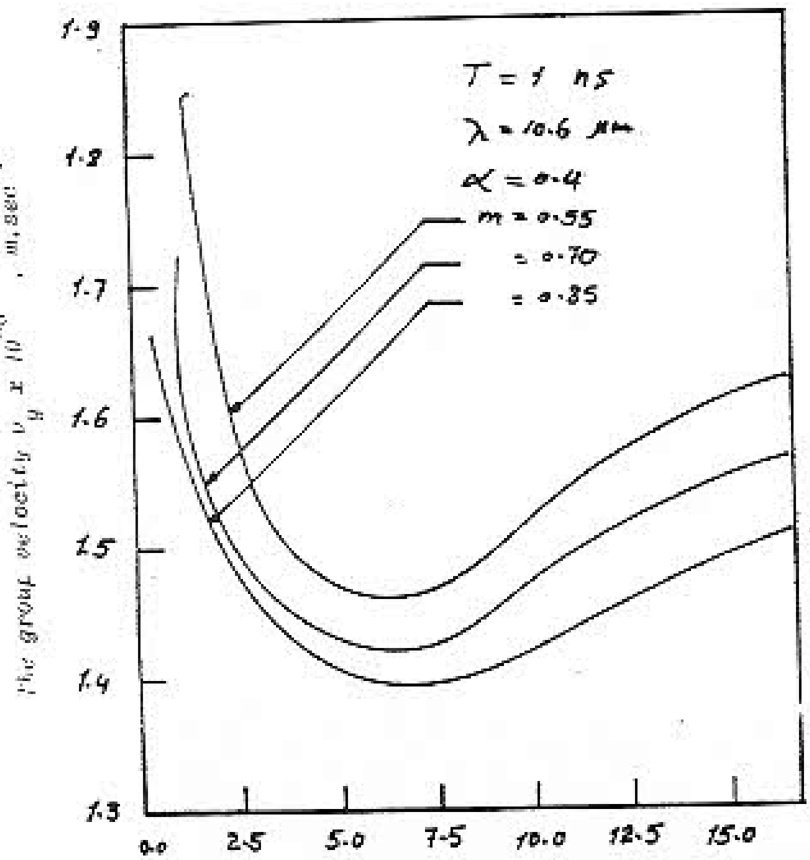


Fig. 2. The variations of the power P_o with the fiber radius R



The fiber radius R, μ m

Fig. 3. The variations of the group velocity v_g with the fiber radius R

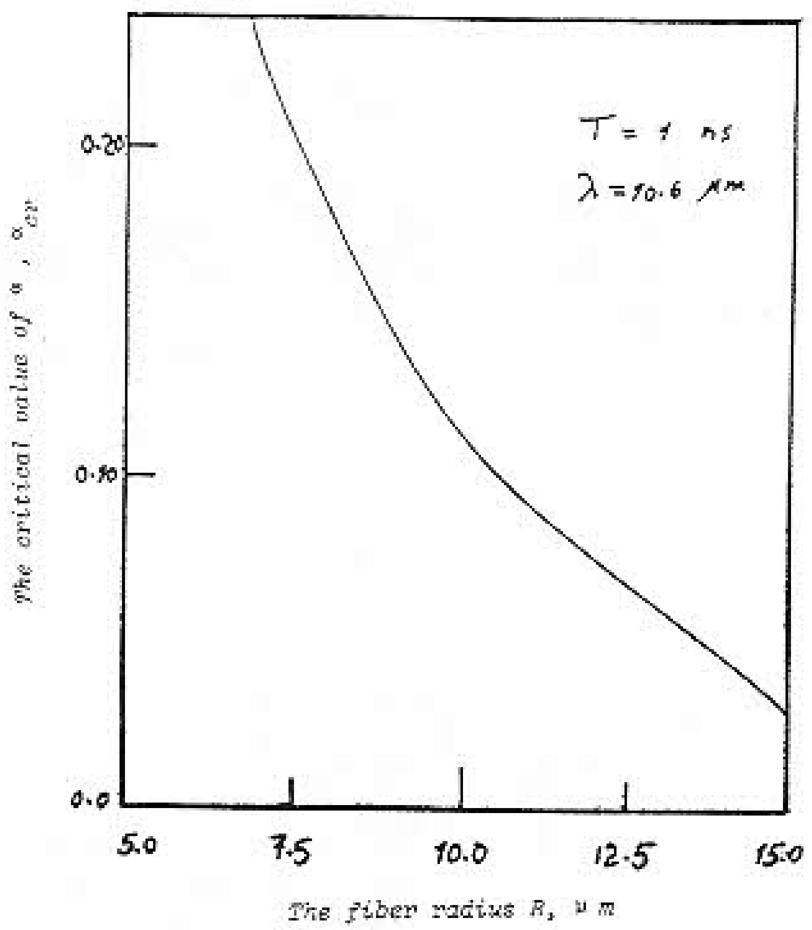


Fig.4 The variations of the parameter α_{cr} with the fiber radius, R