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Pump depletion impact in fiber Raman amplifier

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In this paper, forward stimulated Raman scattering (SRS) is theoretically discussed and investigated in a distributed Raman fiber amplifier (DRFA). This is carried out with consideration of the pump depletion due to the SRS process. The effect of pump depletion on the performance of Raman amplifiers is clarified. It appears above threshold values of pump power of 330 mW, and fiber length of 135 km, respectively. Analytical expressions are derived for threshold depletion length and threshold depletion pump power in forward pumped Raman amplifier. In the backward pumped amplifier, an expression for the safe fiber length is derived such that the signal power can be amplified without losing its characteristics. Analytical expressions are also derived for the unity gain amplifier length in both forward and backward pumped amplifiers and for the maximum transmission length without a repeater for bidirectional pump amplifiers. The maximum unrepeatered transmission length in this case was found to be about 472 km at signal power of 0.1 µW and pump power of 0.5 W.

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1. Introduction

Raman fiber amplifiers (RFAs) are being increasingly used in optical telecommunication networks. Raman amplifier exploits the optical fiber itself as amplification medium. The gain mechanism in Raman amplification is governed stimulated Raman scattering (SRS), a process by which energy is transferred from a wavelength to a longer one through a nonlinear scattering process [1]. Gain flexibility, i.e., the possibility of achieving amplification at any wavelength in any fiber, is one of the key advantages of RFAs. Raman gain can be achieved in any conventional transmission fiber if suitable amount of pump lasers are available. The pump wavelength determines the gain spectrum of a Raman amplifier. Generally, RFA has low noise and broad gain bandwidth characteristics due to which it has been recognized as an enabling technology for optical fiber communication systems. Broadband and low noise figure Raman amplifiers can be achieved using multi-pumping schemes [2, 3]. Moreover, distributed Raman amplification provides a significant improvement in the noise performance and an increase in the signal power budget in transmission fibers.

The origin of SRS and outlined properties of Raman gain were discussed by Bromage [1]. Bit error rate performance of RFAs both as booster amplifiers and as preamplifiers in optical communication systems were studied by Aoki [4]. Thyagarajan et al. [5] proposed a novel fiber design with broad band flat effective Raman gain spectrum over a relative 3-dB bandwidth. The analytical process was based on the assumption that, pump depletion due to the stimulated process is neglected [1, 4, 5]. However, when stimulated Raman amplification is used as the direct optical amplification in optical fiber transmission systems, an optical signal at Stokes frequency exists from the beginning, and hence, it may become important to take into account the pump depletion due to the stimulated process.

In this paper, the signal amplified by SRS is theoretically investigated taking into account the pump depletion for silica. The dispersion effect on the gain and the effect of nonlinear phenomena other than stimulated Raman scattering are ignored as a first approximation. Section 2 presents the mathematical model. The obtained results are displayed and discussed in Sec. 3. The possibility of increasing unrepeatered transmission distance is investigated. Further, the model is verified through a comparison with previously reported results. Section 4 concludes this work.

2. Theory

In this section, the forward Raman amplification is analyzed when the Stokes signal with power much larger than photon energy exists from the beginning. The Raman scattering process can be stimulated in both forward and backward directions in optical fibers if the pump power exceeds a threshold value [6], i.e.

$$P_p(\omega) \geq \frac{\alpha_s \cdot A_{eff}}{g_R(\omega)}, \tag{1}$$

where $g_R(\omega)$, α_s, and A_{eff} are the gain coefficient, the fiber loss parameter in the signal wavelength and the effective area, respectively. For both forward and backward Raman scattering, we consider that the signal is pulse modulated and that the pump is a continuous wave (CW).

2.1 Forward Raman amplification

Forward stimulated Raman scattering process has been already theoretically analyzed by Bromage [1] under the assumption that pump depletion due to the stimulated process is neglected. However, when stimulated Raman amplification is used for the optical amplifier in optical transmission systems, the optical signal at the Stokes frequency exists from the beginning. In this case, since the signal will be amplified and the power may reach the pump power, we have to consider the pump depletion due to the stimulated process in order to analyze the process more precisely. We consider a single mode fiber and a pump with a power $P_p(0)$ injected at $z = 0$ and travelling in the $+z$ direction. We assume that $\alpha_s = \alpha_p = \alpha$ because the signal and pump wavelengths will be held around the 1.55 µm wavelength region in extremely low- loss fibers. The signal power and pump power for the forward travelling waves are according to the method used by Mochizuki [7].

$$P_s(z) = \frac{P_s(0)\cdot\exp\left[\frac{g_R\cdot P_p(0)}{\propto}\cdot(1-e^{-\propto z})-\alpha\cdot z\right]}{1+\frac{\nu_p}{\nu_s}\cdot\frac{P_s(0)}{P_p(0)}\cdot\exp\left[\frac{g_R\cdot P_p(0)}{\alpha}\cdot(1-e^{-\alpha z})\right]} \tag{2}$$

and

$$P_p(z) = \frac{P_p(0)\cdot e^{-\propto z}}{1+\frac{\nu_p}{\nu_s}\cdot\frac{P_s(0)}{P_p(0)}\cdot\exp\left[\frac{g_R\cdot P_p(0)}{\propto}\cdot(1-e^{-\propto z})\right]} \tag{3}$$

The Raman interaction transfers the pump power to the signal power. P_p and P_s represent the pump and signal powers of waves at frequencies υ_p and υ_s and both are functions of the propagation distance z.
In case of

$$\frac{\nu_p}{\nu_s}\cdot\frac{P_s(0)}{P_p(0)}\cdot\exp\left[\frac{g_R\cdot P_p(0)}{\propto}\cdot(1-e^{-\propto z})\right] \ll 1 \tag{4}$$

the signal power becomes [7]

$$P_s(z) = P_s(0).e^{\left\{\frac{g_R\cdot P_p(0)}{\propto}\cdot(1-e^{-\alpha z})-\alpha z\right\}} \tag{5}$$

Equation (5) is similar to that obtained by several researchers [1, 8-10] under the assumption that pump depletion due to stimulated process is neglected.

2.1.1 Threshold depletion length

The effect of pump depletion appears above a certain value of fiber length denoted as L_{dep}. When the amplifier length is below L_{dep}, the effect of pump depletion can be ignored. The pump depletion impact is represented as the dominator of Eq. (2). This impact can be considered in case of

$$\frac{\nu_p}{\nu_s}\cdot\frac{P_s(0)}{P_p(0)}\cdot\exp\left[\frac{g_R\cdot P_p(0)}{\propto}\cdot\left(1-e^{-\alpha\cdot L_{dep}}\right)\right] \geq \frac{1}{10} \tag{6}$$

So, L_{dep} can be obtained as

$$L_{dep} \approx \frac{-1}{\propto}\cdot\ln\left[1-\frac{\propto}{g_R\cdot P_p(0)}\ln\left(\frac{\lambda_p.P_p(0)}{10.\lambda_s.P_s(0)}\right)\right] \tag{7}$$

2.1.2 Unity gain length

The maximum unrepeatered transmission distance $L_{u(fw)}$ can be evaluated when the power is selected such that the Raman gain is at unity and is just sufficient to compensate for fiber losses. $L_{u(fw)}$ is obtained from Eq. (2) when signal power, $P_s(L_{u(fw)}) = P_s(0)$, as

$$L_{u(fw)} = \frac{-1}{\propto}\cdot\ln\left[e^{-\frac{g_R\cdot P_p(0)}{\propto}}+\frac{\nu_p}{\nu_s}\cdot\frac{P_s(0)}{P_p(0)}\right], \quad \alpha L \gg 1 \tag{8}$$

2.1.3 Threshold depletion pump power

The effect of pump depletion also appears after a certain pump power value to be called threshold depletion pump power and denoted as $P_{p(dep)}$. The value of $P_{p(dep)}$ is a function of the fiber length, fiber loss coefficient and input signal power as follows. Starting from the dominator of Eq. (2) and according to the defined threshold depletion impact, we have

$$\frac{\nu_p}{\nu_s}\cdot\frac{P_s(0)}{P_{p(dep)}}\cdot\exp\left[\frac{g_R\cdot P_{p(dep)}}{\propto}\cdot(1-e^{-\alpha\cdot z})\right] \geq \frac{1}{10}$$

So, $P_{p(dep)}$. can be obtained as

$$P_{p(dep)} \approx \frac{10.P_s(0).\lambda_s}{\lambda_p}\cdot\exp\left[\frac{g_R\cdot P_{p(dep)}}{\alpha}\cdot(1-e^{-\alpha\cdot z})\right] \tag{9}$$

2.1.4 Gain for forward Raman amplification

Here, the amplifier gain, G, due to stimulated Raman amplification is defined by $G = P_s(L)/P_s(0)$ [1]. When pump depletion is considered, the amplification gain is obtained as follows [7]

$$G = \frac{\exp\left[\frac{g_R\cdot P_p(0)}{\propto}\cdot(1-e^{-\propto L})-\alpha\cdot L\right]}{1+\frac{\nu_p}{\nu_s}\cdot\frac{P_s(0)}{P_p(0)}\cdot\exp\left[\frac{g_R\cdot P_p(0)}{\alpha}\cdot(1-e^{-\alpha z})\right]} \tag{10}$$

In case of negligible pump depletion, amplification gain becomes [7]

$$G = \exp\left[\frac{g_R\cdot P_p(0)}{\alpha}\cdot(1-e^{-\alpha L})-\alpha L\right] \tag{11}$$

Equation (11) is the same as that derived by Bindal et al. [10] under the assumption that pump depletion due to stimulated process is neglected.

2.2 Backward Raman aqmplification

In case of backward stimulated Raman amplification, it should be mentioned that, the system is designed such that it does not induce pump depletion due to the backward stimulated process because the signal level will fluctuate if pump depletion occurs. Hence, one can neglect the pump depletion. The assumption will be satisfied when we deal with a weak signal and choose the appropriate pump power.

Let the incident pump wave travels in the -z direction from z = L to z = 0 with the signal wave propagating in the +z direction. The pump and signal powers for a backward travelling wave are given by [7, 11]

$$P_s(z) = P_s(0)\cdot \exp\left[\frac{g_R\cdot P_p(L)}{\alpha}\cdot e^{-\alpha L}(e^{-\alpha z}-1) - \alpha\cdot z\right] \quad (12)$$

and

$$P_p(z) = P_p(L)\cdot e^{-\alpha(z-L)} \quad (13)$$

2.2.1 Unity gain length

One can derive an expression for the maximum repeater spacing distance, $L_{u(bw)}$, such that the Raman gain is unity. $L_{u(bw)}$ is obtained from Eq. (12) in case of α L >> 1 and signal power, $P_s(L_{u(bw)}) = P_s(0)$. $L_{u(bw)}$ can be obtained as

$$L_{u(bw)} = \frac{g_R\cdot P_p(L_{u(bw)})}{\alpha^2} \quad (14)$$

2.2.2 Safe fiber length

One can obtain a mathematical expression for fiber length such that

$$P_s(z_{min}) = \frac{P_s(0)}{10} \quad (15)$$

where [7]

$$z_{min} = \frac{1}{\alpha}\cdot \ln\left[\frac{\alpha}{P_p(L)\cdot g_R}\right] + L \quad (16)$$

as reported by several researchers [6, 7, 12, 13].

The safe fiber length in a backward pump amplifier is defined as the maximum length of the fiber such that the signal power can be amplified without losing its characteristics, {i.e. $P_s(L_s) \geq (P_s(0)/10)$}. Starting from Eq. (12) and substituting z_{min} from Eq. (16), L_s can be obtained as

$$L_s \approx \frac{-1}{\propto}\cdot \ln\left[\frac{0.03825.\alpha}{P_p(L)\cdot g_R}\right] \quad (17)$$

2.3 Maximum unrepeatered transmission distance in bidirectional pump Raman amplification

Bidirectional pump Raman amplification uses forward and backward Raman amplification in the optical transmission systems. Signal and pump powers can be derived from Eqs. (2), (3), (12), and (13). The maximum long unrepeatered spacing is derived by different mathematical assumptions.

The result in Ref. [7] was obtained by considering forward Raman pump only in the range from z = 0 to z = z_{min} and backward pump only from z = z_{min} to z = L. However, the former assumption is not good since in the range z = z_{min} there is almost no forward pump but the backward pump starts to affect the signal. The second assumption by Chi et al. [13] considered forward pump only from z = 0 to z = L/2 and backward pump only from z = L/2 to z = L. To obtain a more accurate result, we consider forward pump only from z = $L_{u(fw)}$ to z = L. When using signal power $P_s(z)$ in Eq. (12) at z = z_{min} , the signal power becomes

$$P_s(z_{min}) = P_s(0)\exp\left[\frac{g_R\cdot P_p(L)}{\alpha}\cdot e^{-\alpha L}(e^{-\propto z_{min}}-1) - \alpha z_{min}\right. \quad (18)$$

Let $L \to L - L_{u(fw)}$ and $z_{min} \to z_{min} - L_{u(fw)}$

Then, $P_s(z_{min})$ becomes

$$P_s(z_{min}) = P_s(0)\cdot \exp\{\frac{g_R P_p(L)}{\alpha}\cdot e^{-\alpha(L-L_{u(fw)})}$$

$$.\left(e^{-\alpha(z_{min}-L_{u(fw)})}-1\right) - \alpha\cdot(z_{min}-L_{u(fw)}) \quad (19)$$

The maximum transmission length, L_{max}, without a repeater can be obtained through Eqs. (8) and (9) by substituting z = z_{min} and $\alpha.(L - L_{u(fw)}) >> 1$ as

$$L_{max} = \frac{1}{\alpha}.\left[1 - \ln\left[\frac{\alpha}{g_R\cdot P_p(L)}\right] - \ln\left[\frac{P_s(z_{min})}{P_s(0)}\cdot\left(e^{-\frac{g_R\cdot P_p(0)}{\alpha}} + \right.\right.\right.$$
$$\nu p\cdot Ps(0)\nu s\cdot Pp(0) \quad (20)$$

3. Results and discussion

3.1 Forward pump Raman amplification

3.1.1 Threshold depletion length

When a weak signal is transmitted through to an optical fiber with a strong pump, it will be amplified due to the Raman interaction between the pump and the signal, i.e. SRS. The data used here are taken from Refs. [7] and [14].

The effect of pump depletion on signal power along the silica fiber is considered with both cases when pump depletion is considered and when it is neglected at fiber loss α = 0.20 dB/km and $P_p(0)$ = 0.3 W and at different values of input signal power as shown in Fig. 1.

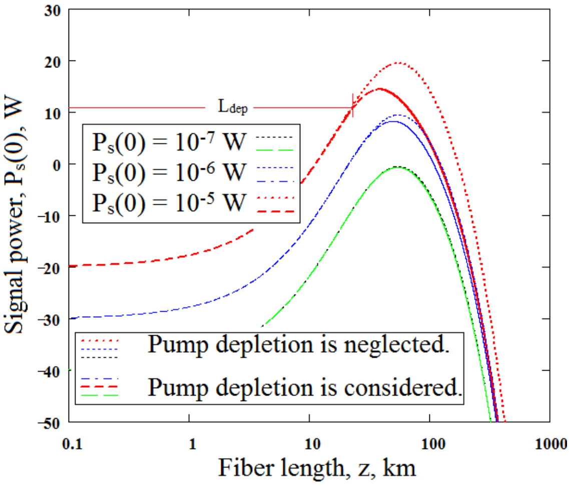

Fig. 1. Signal power as a function of fiber length at different values of input signal power.

As can be seen from Fig. 1, the signal power is affected more by pump depletion at high values of $P_s(0)$. The signal power decreases due to the pump depletion effect above L_{dep} values of 135, 36.72, and 21.79 km for $P_s(0)$ of 10^{-7}, 10^{-6}, and 10^{-5} W, respectively. Pump depletion is caused by the interaction between the pump and the amplified signal. The weak signal can be amplified to the level of the pump depletion when the fiber length is sufficient.

The threshold depletion length decreases with the input signal power, thus decreasing fiber loss as shown in Fig. 2. After L_{dep} reaches saturation values of 27.257, 21.79, and 18.659 km, the fiber loss α is 0.25, 0.20 and 0.15 dB/km, respectively. If the power of the input signal transmitted through the fiber is high and the fiber loss is low, the signal power will reach the level of pump depletion within a short distance.

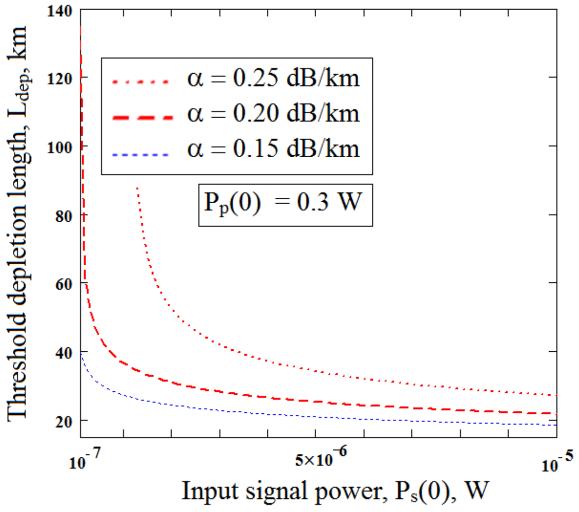

Fig. 2 Threshold depletion length against input signal power at different values of fiber loss α.

3.1.2 Unity gain length

The unity gain length increases linearly with pump power below the saturated pump power value P_{sat}, as shown in Figs. 3 and 4. Figure 3 shows the effect of pump power on the unity gain length in both cases of: with and without pump depletion. At a low pump power $P_p(0) < 330$ mW, pump depletion does not play any significant role in the signal power amplification process. However, at high pump power, the signal gets amplified dramatically when the pump interacts significantly with the signal. These results coincide with that obtained by Raghuwanshi et al. [11].

The unity gain length, $L_{u(fw)}$, as a function of the pump power for several values of fiber loss is displayed Fig. 4. After the pump power reaches saturation values P_{sat} of 0.35, 0.45, and 0.6 W, the unity gain length becomes almost constant due to the pump depletion effect at $L_{u(sat)}$ values of 468, 335, and 234 km and fiber loss α values of 0.15, 0.20, and 0.25 dB/km, respectively.

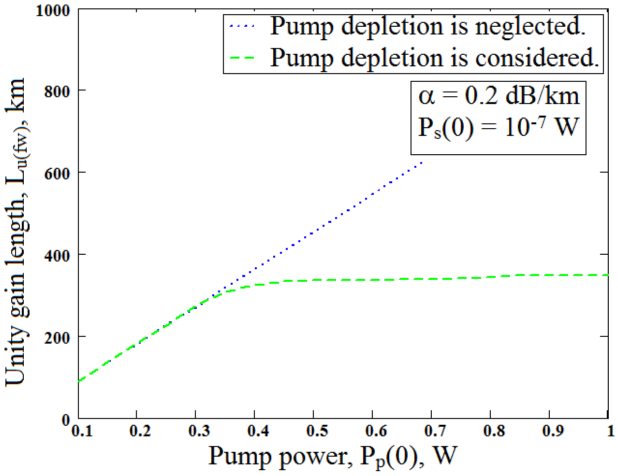

Fig. 3. Effect of pump depletion on unity gain length.

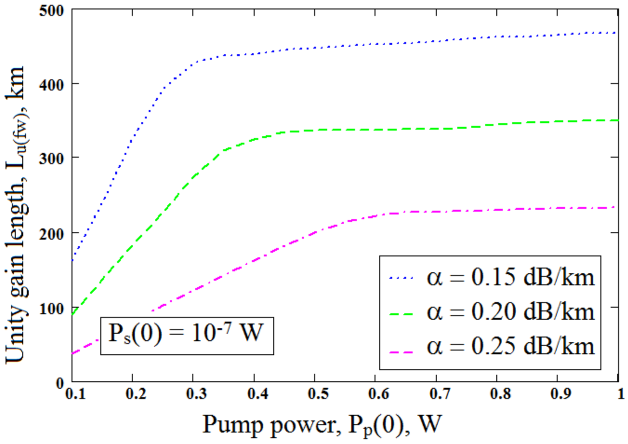

Fig. 4. Unity gain length against input pump power at different values of fiber loss α..

The same analysis mentioned above is repeated but pump depletion is included. It is observed that, the unity gain length increases when the fiber loss decreases. This is expected since the amplified signal power will return back to its input value within a short distance if the fiber loss coefficient is high (e.g., α = 0.3 dB/km). Furthermore, if the value of input signal power is low (e.g., 10^{-7} W), the amplified signal power will decrease to its original input value over a long distance.

3.1.3 Threshold depletion pump power

The signal power (in dBm) linearly increases with the input pump power below $P_{p(dep)}$ at α = 0.2 dB/km and z = 50 km, as illustrated in Fig. 5. The threshold depletion pump power decreases with the input signal power. Further, for an input signal power of 10^{-7}, 10^{-6}, and 10^{-5} W, the values of $P_{p(dep)}$ are, respectively, 0.330, 0.330, and 0.195 W.

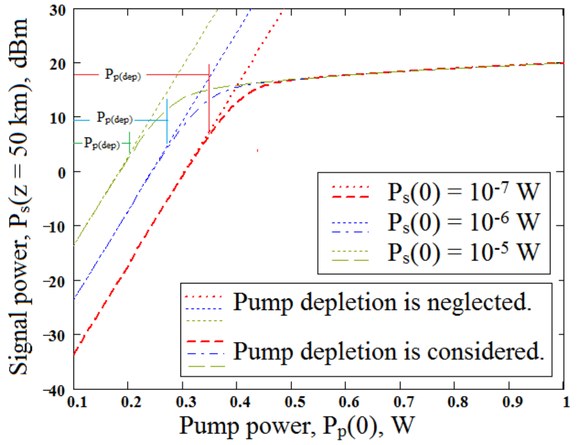

Fig. 5. Signal power against input pump power at different values of input signal power.

The gain increases with fiber length until the maximum signal gain is reached. This is called the gain saturation point of the signal [11], as shown in Fig. 6.

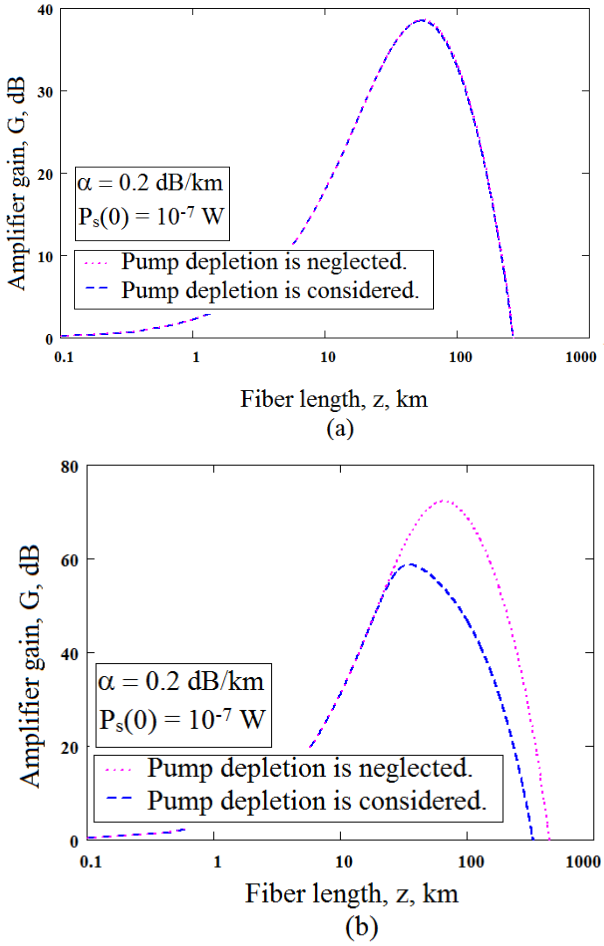

Fig. 6. Amplifier gain as a function of fiber length at (a) $P_p(0) = 0.3$ W and (b) $P_p(0) = 0.5$ W.

The effect of pump depletion appears after a certain value of $P_{p(dep)} = 0.330$ W at fiber loss 0.2 dB/km and $P_s(0) = 10^{-7}$ W. This does not mean that pump depletion is harmful in the amplification process, but it is essential to be considered as pump depletion can lead to performance degradation. Hence, pump depletion is defined as the amount of signal power loss [10].

The same abovementioned analysis is repeated but the signal wavelength is varied for cases when pump depletion is considered and when it is neglected. The input pump power Pp(0) is 0.3W in Fig. 7 (a) and 0.5 W in Fig. 7 (b). From Fig.7, $G_{max} = 50$ and 21 dB when pump depletion is considered and when it is neglected, respectively, at Pp(0) = 0.5 W. The value of $G_{max} = 14$ dB for both cases with Pp(0) = 0.3 W at signal wavelength $\lambda_s = 1.55$ μm is in a fair agreement with the results reported by Beshr et al. [14]. Finally, one can observe that the Raman amplifier gain spectrum is relatively flat but less in magnitude when the pump power is above a certain value of $P_{p(dep)}$. Figures 6 and 7 confirm that the gain is affected by the depletion pump power only if $P_p(0) > P_{p(dep)}$.

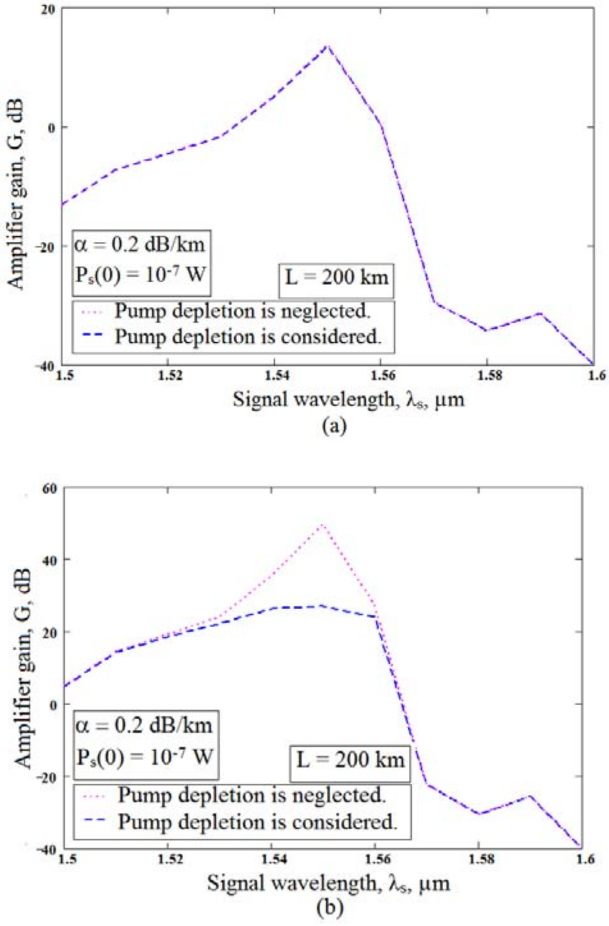

Fig. 7. Amplification gain as a function of signal wavelength at (a) $P_p(0) = 0.3$ W and (b) $P_p(0) = 0.5$ W.

3.2 Backward pump Raman amplification

3.2.1 Safe fiber length

Fig. 8 shows that when the safe fiber length increases, the backward pump power increases and the fiber loss coefficient decreases. This is because as the backward pump power increases, the signal requires more distance for attenuation to (1/10) of its original power. Also, as the fiber loss coefficient increases, the signal power reaches 0.1 $P_s(0)$ within a short fiber length. This can also be deduced from Eq. (17).

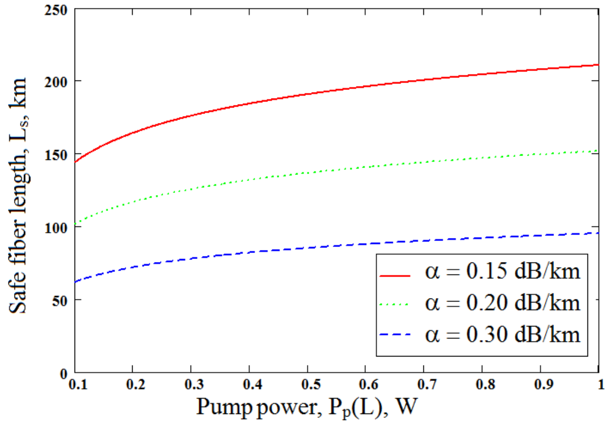

Fig. 8. Variation of safe fiber length with input pump power at different values of fiber loss.

3.3 Maximum unrepeatered transmission distance in bidirectional pump Raman amplification

Consider now the bidirectional pumping scheme, in which the fiber is pumped in both forward and backward directions. We assume in Fig. 9 that $P_p(0) = P_p(L) = P_p(\text{input})$ = Pump power,. The maximum fiber length without repeater is discussed as a function of the forward and backward pump power at several fiber loss values. It should be noted that the fiber length could not increase indefinitely, because of the saturation effect due to pump depletion.

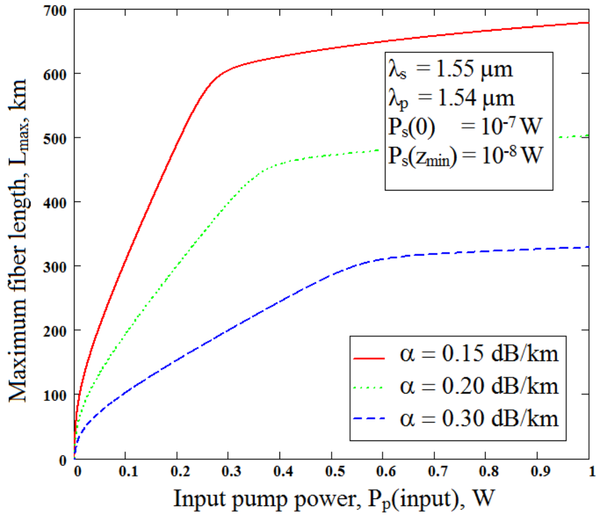

Fig. 9. Maximum fiber length without a repeater as a function of input pump power at $P_p(0) = P_p(L)$ for different values of fiber loss.

From Fig. 9, it is clear that the pump power increases linearly with the maximum fiber length without repeater until it reaches 0.29, 0.38, and 0.58 W at fiber loss values of 0.15, 0.20 and 0.30 dB/km, respectively. After these saturation values are reached, the maximum length without a repeater becomes almost constant due to pump depletion in the stimulated process. One can also observe that as the fiber loss coefficient decreases, the maximum fiber length without repeater increases.

Table 1 clarifies the effect of the forward and backward pump powers on the maximum unrepeatered transmission distance L_{max} at a certain fiber loss α of 0.2 dB/km. It should be noted that the forward pump power $P_p(0)$ has more effect on the maximum length without a repeater compared to the backward pump power $P_p(L)$. This is because backward amplification occurs only in a small range of fiber lengths between $z = z_{min}$ and $z = L$.

Table 1. Maximum length without a repeater L_{max}(km) at fiber loss α = 0.2 km/dB {λ_s = 1.55, λ_p = 1.54 μm, $P_s(0) = 10^{-7}$ W and $P_s(z_{min}) = 10^{-8}$ W}.

$P_p(0)$ **$P_p(L)$**	**0.1W**	**0.2W**	**0.3W**	**0.4W**	**0.5W**
0.1W	194.3	285.72	375.07	429.06	437.67
0.2W	209.4	300.77	390.13	444.17	452.72
0.3W	218.2	309.58	398.93	452.92	461.52
0.4W	224.4	315.83	405.18	459.17	467.77
0.5W	229.3	320.67	410.02	464.01	472.61

Fig. 10 displays the maximum fiber length with the signal wavelength at different values of pump power in the bidirectional case. It is clear that, as the pump power becomes relatively high (≥ 0.6 W), the maximum fiber length is almost constant in the wavelength range 1.50–1.55 μm, which is in agreement with Fig. 9. This means that at high pump power (≥ 0.6 W), the wavelength has no considerable effect on the amplifier maximum length.

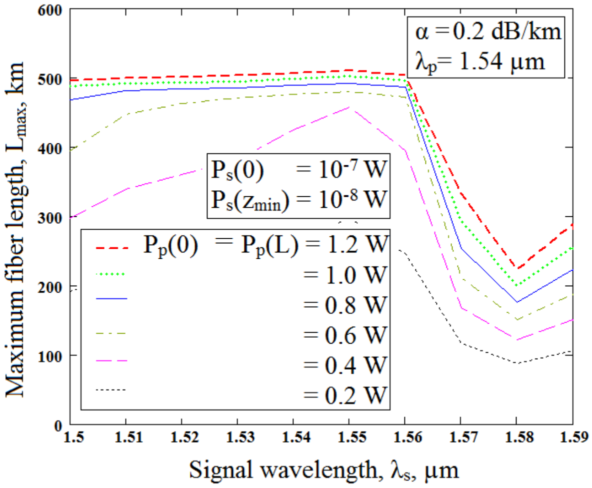

Fig. 10. Maximum fiber length without a repeater as a function of signal wavelength at different values of input pump power.

Fig. 11 shows the changes in the signal power, the forward pump power, and the backward pump power along a 400-km-long distributed Raman amplifier. This fiber length is obtained from Eq. (20) when $P_p(0) = P_p(L) = 0.3$ W, $\alpha_s = \alpha_p = \alpha = 0.2$ dB/km, $P_s(0) = 10^{-7}$ W, and $P_s(z_{min}) = 10^{-8}$ W. A realistic approach to an ideal distributed Raman amplifier is obtained by using a bidirectional Raman pumped amplifier [15].

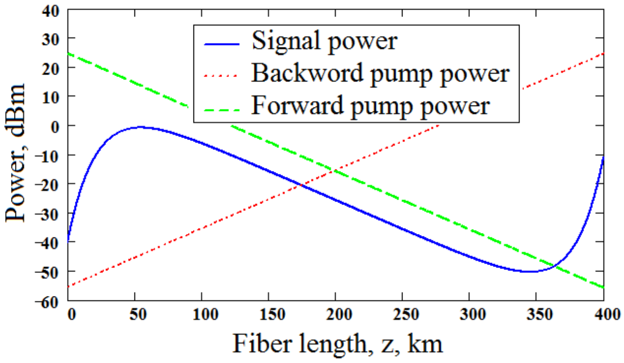

Fig. 11 Signal and bidirectional pump power variation as a function of fiber length at $\alpha = 0.2$ *dB/km,* $\lambda_s = 1.55$ *µm,* $\lambda_p = 1.54$ *µm,* $L = 400$ *km,* $P_p(0) = P_p(L) = 0.3$ *W,* $P_s(z_{min}) = 10^{-8}$ *W, and* $P_s(0) = 10^{-7}$ *W.*

The example shown in Fig. 11 yields an amplification gain of 29.271 dB which is not very high. This is expected since the intention is to increase fiber length without a repeater under the condition that signal power could not be reduced below a certain value $P_s(z_{min})$.

4. Conclusion

In this work, we derived analytical expressions for the threshold depletion length, L_{dep}, unity gain length, $L_{u(fw)}$, and threshold depletion pump power, $P_{p(dep)}$, for a signal amplified by forward Raman scattering while considering pump depletion. It is found that, the signal power, P_S, decreases after a certain value of fiber length L_{dep} of 135 km at $P_s(0) = 10^{-7}$ W, $P_p(0) = 0.3$ W, and $\alpha = 0.2$ dB/km. The amplified signal is affected by pump depletion if $P_p(0) > P_{p(dep)}$ for a signal power $P_s(0) = 10^{-7}$ W; $P_{p(dep)}$ is found to be 330 mW for a fiber length of 50 km. The value of $L_{u(fw)}$ is easily found numerically and is about 335 km at $P_s(0) = 10^{-7}$ W, $P_p(0) = 0.35$ W and $\alpha = 0.2$ dB/km. Furthermore, the forward pump amplifier gain is reduced by 24 dB due to the pump depletion at $\lambda_s = 1.55$ µm, $\lambda_p = 1.54$ µm, $P_p(0) = 0.5$ W, and $\alpha = 0.2$ dB/km.

We also derived analytical expressions for $L_{u(bwn)}$, safe length, L_S, in a backward pumped amplifier and the maximum transmission length, L_{max}, without a repeater for bidirectional pumped amplifier. The maximum unrepeatered transmission length, L_{max}, in the bidirectional pumped system was easily found numerically and is about 472 km at $P_s(0) = 10^{-7}$ W, $P_s(z_{min}) = 10^{-8}$ W, and $P_p(0) = P_p(L) = 0.5$ W.

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