The Resonant Tunneling Diode characterization for high frequency communication systems

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ABSTRACT

In this paper, a detailed derivation process is proposed to characterize the Resonant Tunneling Diode (RTD) for high frequency regime. The proposed model is used to design and analyze a simple microwave oscillator based on the RTD using the commercial circuit simulation software, ADS from Agilent Technologies. The simulation is carried out using different equivalent circuit models; the proposed model, the original constant RC model, and the series/parallel double RC model, which is an alternative to the quantum-inductance RLC model. This is performed in terms of oscillation frequency and output power against resonant circuit elements, considering the CPU time. A comparison between the simulation results of the three models indicates that the proposed model is simple, accurate, and appropriate to investigate the behavior of the RTD at high frequency without any singularity and convergence problems. Also, its complexity (CPU time) is less than that of the series/parallel double RC model and higher than that of constant RC model. However, the constant RC model is inaccurate, especially in high frequency regime. In addition, the proposed technique can be easily incorporated into computer aided circuit design software, such as SPICE and ADS software, to simulate circuits containing RTD in high frequency regime. In brief, this work adds important contributions to the accurate characterization and modeling of RTDs and analyzes its based circuits in high frequency regime by addressing the problems of the current RTD equivalent circuit models.

1. Introduction

Oscillation is among the simplest of dynamic behaviors that has been used to study a wide variety of physical phenomena. In theoretical physics, the word oscillator refers to a physical object or quantity oscillating sinusoidally, or at least periodically, for a long time, ideally forever, without losing its initial energy [1–4].

An important type of oscillator widely used today is the electronic oscillator that converts direct current (DC) power into radio frequency (RF). It consists of an active device and a resonant circuit that determines the frequency of oscillation. The active device can be either a negative differential resistance (NDR) device to compensate for the losses of the resonator such as Gunn diode, RTD, and so on, or a transistor with appropriate feedback to cause instability [3]. The two approaches are identical, but the negative-differential resistance is the most famous used approach in microwave oscillators.

RTDs are different from other active devices in that they exhibit their NDR from DC up to the highest operating frequencies. Some researches [2,3] exploited RTDs using different material types resulting in an oscillation frequency in the range of THz but with a severe limitation in the output power. Nonetheless, the published spectra of free-running RTD oscillators are much broader than those of, for example, free-running Gunn oscillators [5].

Since the work of L. L. Chang et al. [6], an increasing interest has been given to study the characterization of RTD behavior for high frequency operation. This is considered through three different approaches, as follows.

i. Computationally complicated quantum simulator approach [7–12], where the small and large signal responses of RTD have been obtained numerically by solving Poisson and Schrodinger equations self consistently using a harmonic balance technique.

ii. RF measurements approach [13–16].

iii. Equivalent circuit approach [14,17–19].

The current researches [20–26] of RTD for high frequency applications show that a significant growth of frequency has been achieved and the RTD is a candidate for THz sources. In Refs. [20,21], the experiment...
and theory of sub-THz and THz oscillators are described with RTDs integrated on planar circuits and harmonic oscillation up to 1.02 THz were obtained at room temperature. In Refs. [22, 23], the oscillation frequency is further increased up to 1.42 THz using thin barriers and quantum wells and by optimizing the thickness of the collector spacer. In Ref. [24], the oscillation frequency is extended up to 1.55 THz by reducing the length of the antenna integrated with the RTD. However, the conduction loss around the antenna increases with decreasing antenna length [25]. In Ref. [26], by reduced conduction loss together with an optimum RTD structure, a fundamental oscillation of up to 1.92 THz is obtained, which is the highest fundamental oscillation frequency of room-temperature to date [26].

Despite the tremendous progress in fabrication process of the RTDs which improves their structures for obtaining higher frequency and output power and increasing its use for THz sources, there is still lack of modeling of the RTD, particularly at high frequency. This is considered the main motivation of this work to propose a model that has competitive advantages over the current models and to be characterized by higher accuracy. Also, it recommended to have the ability to be implemented into a computer aided circuit design software, such as SPICE and ADS software, to simulate circuits containing RTD in high frequency regime without any singularity or convergence problems.

In this paper, an RTD model has been proposed as an alternative for the well-known and well verified quantum-inductance RLC model, which provides a better understanding of the mechanisms that limit the transient behavior of the RTD. This model overcomes the disadvantages of the quantum-inductance RLC model, which suffers from a singularity and the ambiguity of the negative quantum inductance at NDR that gives rise to convergence problems and makes it useless when used in circuit simulators. Also, a simple microwave oscillator based on an RTD device is designed and simulated using the ADS simulator [27]. For the sake of simplicity, the RTD frequency dependent circuit elements are realized exploiting Symbolically Defined Devices (SDD) components [27].

The simulation is carried out using different equivalent circuit models, original constant RC model, series/parallel double RC model which is alternative to the quantum-inductance RLC model and the proposed model. This is performed to address the problems with the current RTD equivalent circuit models.

The rest of the paper is organized as follows. In Section 2, an overview of nonlinear equivalent circuit models is introduced. In Section 3, high frequency characterization parameters are defined to describe the dependency of these circuit elements on the frequency. Section 4 characterizes the RTD device under test. Analysis of the RTD based oscillator is introduced in Section 5. The RTD oscillator circuit simulation results are presented and discussed in Section 6. Section 7 is devoted for the main conclusions of this work. An appendix is added to describe the ADS Simulation.

2. Nonlinear equivalent circuit model for high frequency characterization of RTD

An overview of the equivalent circuits for RTD published so far indicates that most of them have been applied for characterizing the RTD in the small-signal regime [8, 9, 14, 17, 28, 29]. Also, their equivalent circuit elements extraction approaches are based on fitting the equivalent circuit model with measured S-parameter data over the frequency under restricted bias conditions in the three regions of RTD current – voltage characteristics or using numerical analysis.

The simplest equivalent circuit model introduced to represent a double barrier quantum well (DBQW) RTD was early proposed by Ref. [30]. This was used to describe Esaki tunnel diodes using a parallel combination of conductance and capacitance as illustrated in Fig. 1. The shortcomings of the simple model have been overcome in Ref. [17] to modify the simple equivalent circuit of Fig. 1 by introducing an inductance in series with the RTD conductance. This model is called parallel inductance equivalent circuit model. But, it suffers from a singularity that gives rise to convergence problems and makes it useless when used in circuit simulators.

The small-signal series/parallel double-RC equivalent circuit (SPDRC), shown in Fig. 2 [31] has been optimized as a full mutual alternative for the RTD quantum-inductance equivalent circuit model to overcome the mentioned shortcomings of the parallel inductance equivalent circuit model.

Actually, it has been shown experimentally that the real and imaginary parts of RTD admittances change with frequency, instead of having a constant value. So, this simple model is not applicable for RTD at high frequencies [13]. Therefore, we need to modify the values of the simple (G-C) equivalent circuit, shown in Fig. 1, to change with the frequency instead of having a constant value.

The large signal and nonlinear equivalent circuit model proposed by Ref. [32] is shown in Fig. 3. The aim of this model is to modify the values of the simple (G-C) equivalent circuit, shown in Fig. 1, to change with the frequency instead of having a constant value.

3. High frequency characterization parameters

In this section, a simplification of the analytical expressions of the proposed equivalent circuit elements is introduced through new parameters to describe the dependency of these elements on the frequency.

At first, we define two parameters that determine the relation between the simple equivalent circuit model elements G(V) and C(V) and the proposed equivalent circuit model elements Geq(V, f) and Ceq(V, f). This is carried out using the new parameters α(f) and β(f) that are assumed as

\[
\alpha(f) = \frac{G_{eq}(V, f)}{G(V)} \quad \beta(f) = \frac{C_{eq}(V, f)}{C(V)}
\]  

(1)

where \( G_{eq}(V, f) \) is given by

\[
G_{eq}(V, f) = \frac{R_0}{(R_0^2 + (\omega L_0)^2)} - \frac{G(V)}{1 + (\omega L_0)^2}
\]

(2)
where $C_{eq}(V,f)$ is given by Ref. [32].

where $C_0$ is defined as

$$C_0 = \frac{\text{Area}}{\varepsilon'_w + \frac{\varepsilon_p}{\tau_c} + \frac{\varepsilon_D}{\tau_e}}$$

$I_w$ is the width of the quantum well, $l_b$ is the width of the barrier, $l_D$ is the width of the depletion region, and $\varepsilon_w$, $\varepsilon_p$, and $\varepsilon_D$ are the dielectric constants of the quantum well, barrier, and depletion region, respectively [29]. $C_w$ depends on the differential conductance and is given by Refs. [14,29,33]. $C_v$ is the escape rate through the collector barrier.

So, the factor $\alpha(f)$ can be written as

$$\alpha(f) = \frac{C_{eq}(V,f)}{G(V)} = \frac{1}{1 + (\omega f)^2}$$

4. Characterization of RTD device under test

RTDs are typically realized in the III-V compound material systems and can also be realized using the Si/SiGe materials system. However, they are predicted to be theoretically inferior to III-V RTDs, because they have a lower peak current density and peak-to-valley current ratio. Some typical III-V materials used to realize RTDs are GaAs/AlGaAs, InAs/AlSb, and InGaAs/AlAs. However, InGaAs/AlAs/InP is usually used instead of InAs/AlSb because of its mature fabrication and growth technologies. More details and comparisons can be found in Refs. [7,34,35].

To establish the performance investigation of an actual oscillator circuit, one need, firstly, to characterize the RTD device used in such investigation.

The data given by Ref. [34] is the best available experimental results on the electrical performance of an InAs/AlSb-RTD. The DC - current-voltage characteristics and the device structure are the known data. The characterization operation starts with fitting the measured RTD DC current – voltage characteristic curve given in Ref. [34] through the entire bias range. Then, the equivalent circuit elements are calculated using a set of mathematical expressions as functions of the DC bias. Therefore, to reproduce the real device behavior accurately over the entire bias voltage range with easily fitting process, we proposed an analytical and differentiable model appropriate for the RTD device under test. Using LAB Fit Curve Fitting Software [36] to approximate the currents between measurement points, the I-V characteristic given in Ref. [34] was fitted. The resultant I-V curves with the calculated differential conductance are shown in Fig. 5.

The I-V curve shows a peak valley current ratio (PVCR) $(I_p/I_v)$ of about 3.4 at room temperature, and a peak current density of $2.8 \times 10^5 \text{ A cm}^{-2}$. The peak voltage, peak current, valley voltage and valley current are given by $V_p = 1.22 \text{ V}$, $I_p = 8.29 \text{ mA}$, $V_v = 1.496 \text{ V}$, $I_v = 2.43 \text{ mA}$, respectively. The area of the given device is $2.96 \mu\text{m}^2$.

The analytical mathematical expressions that are extracted from LAB Fit Curve Fitting Software [36] are given by

$$I_{PVCR}(V) = F_1(V)[A_1 V^{\beta_1} + C_1 e^{\beta_1 V}]$$

$$I_{NDV-PVCR}(V) = F_2(V)[A_1 V^{\beta_1} + C_1 e^{\beta_1 V} + E_1]$$

$$I(V) = \frac{\text{Area}}{2.96} \left| G_{PVCR}(V) + G_{NDV-PVCR}(V) \right|$$

where $\tau_c$ is the escape rate through the collector barrier.

The two parameters $\alpha(f)$ and $\beta(f)$, that characterize the behavior of RTD at high frequency, versus frequency are displayed in Fig. 4 $\alpha(f)$ is almost equal to one at very low frequencies and $\beta(f)$ is almost equal to one at very high frequencies.

![Fig. 4. The ratio between the conductances of the two models $\alpha(f)$ versus frequency $f$ (dashed line) and the ratio between the capacitances of the two models $\beta(f)$ versus frequency $f$ (solid line). $C_w = 2.88\mu\text{F}$ and $\tau_c = 30\,\text{fs}$.](image)

![Fig. 5. The fitted I-V curve of the Brown model [34], together with the calculated differential conductance $G$. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)](image)
I_{FB1} represents the current through the first positive differential region, while I_{NDP,B2} represents the current through the negative differential region and the second positive differential region in the I-V curve.

We proposed the mathematical functions F_1(V), F_2(V), F_3(V) and F_4(V) that are used to smooth out the I-V characteristics and NDC curves observed. The functions F_1(V) and F_2(V), are given by:

\[
F_1(V) = \left(\frac{1}{\gamma}\right) \tan^{-1}(\gamma (V - V_p)) \left(+\frac{\pi}{2}\right)
\]

where
\[
F_1(V) = \begin{cases}
\approx 1 & \text{at } V < V_p \\
\approx \frac{1}{2} & \text{at } V = V_p \\
\approx 0 & \text{at } V > V_p
\end{cases}
\]

(11)

(12)

Similarly, the functions F_3(V) and F_4(V) can be derived and given by:

\[
F_3(V) = \left(\frac{1}{\gamma}\right) \tan^{-1}(\gamma (V - V_p)) \left(+\frac{\pi}{2}\right)
\]

where
\[
F_3(V) = \begin{cases}
\approx 1 & \text{at } V > V_p \\
\approx \frac{1}{2} & \text{at } V = V_p \\
\approx 0 & \text{at } V < V_p
\end{cases}
\]

(13)

(14)

The fitting parameters are: A_1 = 0.6232 \times 10^{-2}, B_1 = 2.632, C_1 = -0.6019 \times 10^{-3}, D_1 = 6.727, A_2 = 140, B_2 = -7.86, C_2 = 0.247 \times 10^{-2}, D_2 = 0.898, E_2 = -0.8327 \times 10^{-2}.

The differential conductance, G(V), which is defined as dI/dV is deduced and given by

\[
G_{\text{POS}}(V) = F_1(V)\left[\frac{d}{dV}(\text{Re} C_{\gamma} R_s e^{i\omega V})\right]
\]

(15a)

\[
G_{\text{NDP,B2}}(V) = F_2(V)\left[\frac{d}{dV}(\text{Re} C_{\gamma} R_s e^{i\omega V})\right]
\]

(15b)

\[
G(V) = \frac{\text{Area under graph}}{\pi \gamma}\left[\text{G}_{\text{POS}}(V) + \text{G}_{\text{NDP,B2}}(V)\right]
\]

(15c)

G_{\text{POS}} represents the differential conductance through the first positive differential region, while G_{\text{NDP,B2}} represents the differential conductance through the negative differential region and the second positive differential region of the I-V curve. The functions F_3(V) and F_4(V) are given by

\[
F_3(V) = H_1 \times F_1(V)
\]

where
\[
H_1 = \frac{2}{\gamma} \tan^{-1}(\gamma (V - V_p))
\]

(16)

(17)

and
\[
F_4(V) = \begin{cases}
\approx 1 & \text{at } V < V_p \\
\approx 0 & \text{at } V \geq V_p
\end{cases}
\]

\[
F_4(V) = H_2 \times F_2(V)
\]

where
\[
H_2 = \frac{2}{\gamma} \tan^{-1}(\gamma (V - V_p))
\]

(19)

(20)

and
\[
F_4(V) = \begin{cases}
\approx 0 & \text{at } V \leq V_p \\
\approx 1 & \text{at } V > V_p
\end{cases}
\]

(21)

5. Analysis of RTD based oscillator

In this section, the theoretical intrinsic maximum oscillation frequency of the RTD device is adequately predicted by a lumped-element of different equivalent-circuit models. Then, the influence of circuit elements on the RTD oscillator parameters is analytically investigated. These parameters include the oscillation frequency, the maximum oscillation frequency, and the condition for oscillation to occur. This investigation is carried out for different equivalent circuit models. Also, the condition for maximum power is derived for different realistic equivalent circuits including the proposed one with different categories of RTD oscillators.

5.1. Intrinsic maximum oscillation frequency

It is informative to calculate the theoretical maximum oscillation frequency which is one of the important figures of merit for DBRTD oscillators. The maximum oscillation frequency, f_{max} of an intrinsic DBRTD is adequately predicted by a lumped-element of the equivalent-circuit model. The condition that the real part of the impedance of this circuit vanishes determines the maximum frequency that is the upper limit for oscillation as a function of the circuit elements [37]. The intrinsic maximum oscillation frequency for the three small signal RTD models mentioned before can be determined as follows.

5.1.1. Simple RC model

For the small signal equivalent circuit which consists of a parallel combination of RTD conductance and capacitance, the input impedance can be written as:

\[
Z_{in} = R_s + \left(\frac{1}{G + j \omega C}\right)
\]

(22)

The real part of the input impedance is

\[
\text{Re}(Z_{in}) = R_s = \left(\frac{\omega^2 C_{\gamma} R_s + G + G R_s}{G + \omega C}\right)
\]

(23)

At R_s = 0, the maximum oscillation frequency, as a function of the differential conductance, G, is obtained as:

\[
f_{\text{max}} = \frac{\omega}{2 \pi} \left(\frac{\omega^2 C_{\gamma} R_s + G + G R_s}{\omega^2 C_{\gamma} R_s + G^2}\right)
\]

(24)

where G is negative during NDR.

The maximum oscillation frequency, f_{max}, is calculated using MATLAB and is displayed with G in Fig. 6, where C_\gamma = 2.88 \, \Omega, R_s = 3 \, \Omega and G is calculated using Eq. (15c). The figure shows that this model exhibits an intrinsic maximum oscillation frequency of about 7.6 THz at the boundary of the NDR.

5.1.2. Quantum-inductance (RLC) model

For RLC small signal equivalent circuit, the input impedance is deduced and given by

\[
\text{Re}(Z_{in}) = \text{Re}\left\{R_s + \left(R_0 + j \omega L_0\right)\left(\frac{1}{j \omega C_0}\right)\right\}
\]

(25)

Equating Eq. (25) to zero and using MATLAB gives the graph of maximum oscillation frequency versus differential conductance G as shown in Fig. 6. It is clearly seen that this model exhibits an intrinsic maximum oscillation frequency of about 2.1 THz.

5.1.3. The proposed model

The real part of the input impedance is derived as:

\[
\text{Re}(Z_{in}) = R_s + \frac{G_{\gamma}}{G_{\gamma} + \left(\omega C_{\gamma}\right)}
\]

(26)
Letting the real part of the input impedance equals to zero, the derived oscillation frequency is the same as that given in Eq. (24) by replacing \( C_D \) and \( G \) with \( C_{eq} \) and \( G_{eq} \), respectively.

When the maximum oscillation frequency is plotted against the differential conductance, it was found to be exactly like that of quantum-inductance RLC model as shown in Fig. 6. This is expected because the two models are alternative to each other [32].

Comparing the results of Fig. 6, it is clear that the original constant RC circuit model tends to overestimate the maximum oscillation frequency.

5.2. Parallel RTD based oscillator circuits

A simplified equivalent circuit of an oscillator consists of an RTD connected to a bias circuit and a load \( RL \) via a resonating LC circuit is used. An example of this type of circuits is schematically represented in Fig. 7. This circuit is called a parallel RTD oscillator circuit, where RTD is connected in parallel with the load.

In this case of RTD oscillators, the RTD provides the required capacitance and negative differential conductance for the resonance to occur. The lumped element \( LB \) realizes the parasitic inductance of biasing lines and the blocking capacitor \( C_{BL} \) acts like a short circuit at high frequency to couple only the high frequency output signal to the load.

The resistance \( R_s \) with conductance \( G \) and capacitance \( C_D \) are the lumped elements of the simple RC small signal equivalent circuit model of the RTD. The lumped element \( LR \) is designed to select the frequency of oscillation and \( RL \) is the load resistance. Finally, the VDC is the DC bias applied to the oscillator circuit to supply energy and determine the operating point of the RTD at the negative differential region.

The real part of resultant impedance of the circuit shown in Fig. 7 can easily be deduced as:

\[
\text{Real}(Z_{in}) = \text{Real} \left( j\omega L_B + \left( R_L + \frac{1}{G + j\omega C_D} \right) \left( R_L + j\omega L_B + \frac{1}{j\omega C_{BL}} \right) \right)
\]

An input DC voltage of 1.4 V is used that makes the voltage across the tunnel diode lies in the middle of the negative differential region and as a result, the conductance \( G \) is negative. The other elements are set as \( LB = 1 \mu H, R_s = 3 \Omega, C_{BL} = 500 \mu F \) and \( CD = 2.8 \mu F \) [34].

The input impedance has a minimum and negative value when plotted versus the oscillation frequency at different values for \( LR \) and \( RL \). As a result, the RTD oscillator circuit acts as a series resonant circuit and the minimum value of \( Z_{in} \) occurs at the oscillation frequency.

In negative differential resistance oscillator circuits, the conditions of oscillation are [39].

\[
\text{Real}(Z_{in}) < 0, \quad \text{Imag}(Z_{in}) = 0
\]

If we equate the real part of the input impedance in Eq. (28) to zero, one can obtain the maximum oscillation frequency. For \( LR = 14 \mu H, RL = 31 \Omega, CD = 2.8 \mu F, CBL = 500 \mu F, LB = 1 \mu H, Rs = 3 \Omega \), and using Eq. (28), the maximum oscillation frequency versus the conductance through the negative differential region for the given oscillator circuit is calculated using MATLAB and is displayed in Fig. 8. A maximum oscillation frequency of about 7.6 THz is noticed which is the same as the intrinsic one of the RC model.

From the previous analysis, it is noted that the RTD based oscillator circuit shown in Fig. 7 is very complex when analyzed. This is why this circuit has to be simplified. The simplification is shown in Fig. 9, where the bias line inductance \( LB \) is assumed large and so acts like an open circuit at high frequencies of oscillation and the capacitance \( C_{BL} \) acts like short circuit at high frequencies. For the large signal equivalent circuit, Fig. 9(a), the voltage controlled current source \( I(V) \) is modeled by Eq. (10c).

5.2.1. Linear analysis of the parallel RTD circuit using simple RC model

One of the most useful methods of gaining an insight into the fundamental mechanism of a NDR oscillator is to analyze the basic circuit of Fig. 9 in terms of a linear differential equation for the instantaneous output voltage.

The large signal RF equivalent circuit can be described by the following second order differential equation that describes the RTD oscillation.
\[
\frac{d^2 V(t)}{dt^2} + 2\alpha \frac{dV(t)}{dt} + \omega^2 V(t) = 0
\]  
(29)

where \(V(t)\) is the time-dependent voltage across the RTD and

\[\alpha = \frac{R_s C_D + L_g G + R_L C_D}{2 L_g C_D}\]

(30)

\[f_{osc} = \frac{1}{2\pi} \sqrt{\frac{1 + G(R_s + R_L)}{L_g C_D}}\]

(31)

where \(\alpha\) describes the damping factor of the oscillator and \(f_{osc}\) is the oscillation frequency.

For positive values of \(\alpha\), oscillations will decay with time [40]. Then, for high frequency oscillations, the condition for oscillation is

\[\alpha / C_v^2 < 0\]

leading to

\[R_L < \frac{R_s}{C_v^2} - R_i\]

(32)

But, to obtain a sustained oscillation, with constant amplitude, the damping factor \(\alpha\) must be equal zero.

At \(\alpha = 0\), the restriction on the circuit elements becomes

\[R_L = \frac{L_g |G|}{C_D} - R_i\]

(33)

and the oscillation frequency is

\[f_{osc} = \frac{1}{2\pi} \sqrt{\frac{1}{L_g C_D}} G^2 C^2 D\]

(34)

A detailed analysis to understand the RF oscillation using the small-signal analysis which depends on the circuit shown in Fig. 9(b) is described below. In this case, the current source as shown in Fig. 9(b) is replaced with a conductance \(G\) and then the resultant impedance becomes

\[Z_v = Z_{RTD} + Z_L\]

\[= \frac{1}{G + j\omega C_D} + R_s + R_L + j\omega L_g\]

(35)

Then, the real part of the input impedance is deduced as

\[\text{Real}(Z_v) = \frac{G + (R_s + R_L)(G^2 + (\omega C_D)^2)}{G^2 + (\omega C_D)^2}\]

(36)

At \(\text{Real}(Z_v) = 0\), the maximum frequency is

\[f_{\text{max}} = \frac{|G|}{2\pi C_D} \sqrt{\frac{1}{|G(R_s + R_L)|} - 1}\]

(37)

This frequency corresponds to \(f_{\text{max}}\) the cutoff frequency of the diode, which can be reduced to Eq. (34). Above this frequency, the device can no longer supply power to the circuit. From Eq. (37), the condition of oscillation is

\[R_L < |R_s| - R_i\]

(38)

Another characteristic frequency is when the imaginary part of \(Z_v\) becomes zero where

\[\text{Imag}(Z_v) = \frac{\omega L_g G^2 - \omega C_D + \omega^2 L_g C_D}{G^2 + (\omega C_D)^2}\]

(39)

At \(\text{Imag}(Z_v) = 0\), the oscillation frequency is

\[f_{osc} = \frac{1}{2\pi} \sqrt{\frac{1}{L_g C_D} - \frac{G^2 C^2 D}{C_D}}\]

(40)

From Eq. (40), the condition of oscillation is

\[\left|\frac{R_L}{C_D}\right| > \sqrt{\frac{L_g}{C_D}}\]

(41)

while the other condition \(\text{Real}(Z_v) < 0\) leads to the condition:

\[R_L < \frac{|G|}{G^2 + (\omega C_D)^2} - R_i\]

(42)

One of the most important figures of merit of the RTD is to obtain high output power. The output power delivered to the load for the circuit shown in Fig. 9(b) is given by Ref. [17] as

\[P_L = \frac{1}{2} \left( \frac{|V|}{|R_s - Z_{RTD}|} \right)^2 \text{Real}(Z_{RTD})\]

(43)

Let the resistance \(R_{RTD}\) and reactance \(X_{RTD}\) be the real and imaginary parts of \(Z_{RTD}\) and \(R_L\) and \(X_L\) be the real and imaginary parts of \(Z_L\).
To obtain the maximum output power delivered to the load, the load impedance $Z_L$ must be equal to complex conjugate of the RTD impedance $Z_{RTD}$. $R_L$ and $X_L$ at which the maximum power occurs can be determined as follows.

1. The condition $X_L = -X_{RTD}$ leads to

$$L_R = \frac{C_D}{G^2 + (\omega C_D)^2}, \quad f_{osc} = \frac{1}{2\pi} \sqrt{\frac{C_D - L_R G^2}{L_R C_D}}$$ (44)

Then, for positive oscillation frequency, the following condition must be satisfied

$$|R_D| > \sqrt{\frac{L_R}{C_D}}$$ (45)

2. The condition $R_L + R_R = |R_{RTD}|$ leads to

$$R_L = \left| G - R_G \frac{G^2 - \omega^2 R_G C_D^2}{G^2 + (\omega C_D)^2} \right|$$ (46)

and the oscillation frequency is the same as that indicated in Eq. (37).

By differentiating the power equation with respect to $R_L$ and equating to zero, the expression for $R_L$ at which the maximum power occurs is obtained as

$$R_L^2 = R_G^2 + (\omega L)^2 + \frac{1 + 2 R_G G^2 - \omega^2 L_R C_D}{G^2 + (\omega C_D)^2}$$ (47)

This relation is equivalent to that derived before in Eq. (33). From this relation, the load resistance versus the oscillation frequency, $f_{osc}$, given by Eq. (40) and the inductance $L_R$ at which maximum power occurs are shown in Fig. 10.

Fig. 10 shows that at $RL \approx 56 \Omega$, the maximum output power occurs at $LR = 14$ pH and an oscillation frequency of 433 GHz is achieved. Also, the oscillation frequency decreases with increasing $RL$.

5.2.2. Parallel RTD based oscillator circuit using RLC model

Using the RLC model instead of the RC model in the simplified circuit shown in Fig. 9, the input impedance is deduced and the condition that $\text{Real} (Z_{in}) = 0$ leads to the maximum oscillation frequency as

$$f^2 = \left( \frac{1}{2\pi} \right)^2 \frac{1}{2 L_R^2} \left( \frac{1}{C_D L_Q} - \frac{R_{D}^2}{2 L_Q^2} \right)$$ (48)

while the condition that $\text{Imag} (Z_{in}) = 0$ leads to

$$f^2 = \left( \frac{1}{2\pi} \right)^2 \frac{1}{2 L_R^2} \left( \frac{1}{C_D L_Q} + \frac{R_{D}^2}{2 L_Q^2} \right)$$ (49)

By equating the two oscillation frequencies which are given in Eq. (48) and Eq. (49), the load resistance is calculated and then the oscillation frequency at which the input impedance becomes a minimum is determined. The relation between the load resistance versus the inductance $LR$ and the oscillation frequency is shown in Fig. 11.

5.2.3. Parallel RTD based oscillator circuit using the proposed model

In this section, we apply the proposed model on the parallel RTD oscillator circuits. Utilizing the proposed model in the simplified circuit shown in Fig. 9, to find the oscillation frequency at which $\text{Imag} (Z_{in}) = 0$, yields the same oscillation frequency given in Eq. (34), by replacing $C_D$ and $G$ by $C_{eq}$ and $G_{eq}$ respectively.

Table 1 summarizes the oscillation frequency for the parallel RTD based oscillator circuit using the three different RTD models. These results confirm that the RLC model and the proposed model are equivalent to each other. Also, the oscillation frequency for the RC model with capacitance given by Eq. (8) is calculated to distinguish the effects of the quantum inductance on the oscillation frequency.

From Table 1, it is observed that the oscillation frequency for the RC model is higher than that for the RLC or the proposed model for values less than 668 GHz, which occurs at $LR = 10$ pH. Below this frequency, the oscillation frequency for the RC model becomes lower than that for the
6. RTD oscillator circuits: simulation results and discussion

The aim of this section is to simulate, study, and investigate RTD based oscillator circuits behavior at high frequency using the proposed RTD model. To achieve this, a simulation comparative study is carried out between the proposed model, the simple RC model, and the SPDRC model which is a full mutual alternative for the quantum-inductance RLC model. To achieve this, a simulation comparative study is carried out for the parallel topology of RTD oscillator circuits with studying the effects of the circuit elements and comparing the obtained results with those reported before.

6.1. Simulation setup and parameters

The RTD characterized with the I-V curve shown in Fig. 5 is used in this comparative study. I(V) characteristics is described by Eq. (10 c) and the differential conductance, \( G(v) \), is described by Eq. (15c). The junction capacitance, \( C_p \), is taken to be 2.8 fF, the escape rate through the collector barrier, \( \tau_c \), is 30 fs, the electron lifetime, \( \tau \), is assumed to be 90 fs, and the contact resistance, \( R_o \), is 3 \( \Omega \) [34].

The oscillator circuit, shown in Fig. 7, is built in the commercial advanced design system software (ADS) [27] and the transient time domain simulations are carried out.

To model the voltage dependent current source using ADS, one port symbolically-defined device (SDD) is used as shown in Fig. A.1. Also, the frequency dependent capacitor is implemented in ADS using two ports SDD device, as shown in Fig. A.3. Note that, VDC must lie between 1.23 V and 1.49 V to insure the NDR.

The lumped element LB is used to isolate the high frequency oscillation from the DC bias and acts like an open circuit at the high frequency of oscillation. To choose its value, the output power and oscillation frequency against LB is investigated for LR in the range 3–20 \( \mu \)H for three different values of the load resistance 5, 10 and 20 \( \Omega \). The simulation was carried out at VDC = 1.40 V; the center of the NDR region. The dependency of both output power and oscillation frequency on both RL, LR, and the input DC bias, VDC, are varied as affecting parameters to determine both output power and oscillation frequency. The dependency of both output power and oscillation frequency on both RL, LR, and VDC is investigated for LR in the range 3–20 \( \mu \)H for three different values of the load resistance 5, 10 and 20 \( \Omega \). The corresponding simulation results are displayed in Figs. 13 and 14.

From Fig. 13, it is observed that the output power has a maximum value at a certain value of LR for each value of RL in both RC model and the proposed model. This agrees with the analytical results. Therefore, for maximum power to be delivered to the load, as the load resistance RL increases, the inductance LR must be increased, i.e., the output signal frequency decreases, as indicated in Fig. 10. It is noted, from Fig. 14, that of LB. Therefore, a value of 1 \( \mu \)H is used in all simulations to avoid the effect of LB. Also, the capacitance CBL is chosen to be 500 \( \mu \)F to act like short circuit at high frequency.

6.2. Simulation results and discussion

To simulate the parallel RTD based oscillator circuit shown in Fig. 7, the RTD is modeled using the simple RC equivalent circuit, shown in Fig. 1. Once again, the proposed equivalent circuit model shown in Fig. 3 is used. The values of the lumped inductance element, LR, the load resistance, RL, and the input DC bias, VDC, are varied as affecting parameters to determine both output power and oscillation frequency. The dependency of both output power and oscillation frequency on both RL, LR, and VDC is investigated for LR in the range 3–20 \( \mu \)H for three different values of the load resistance 5, 10 and 20 \( \Omega \). The simulation was carried out at VDC = 1.40 V; the center of the NDR region. The corresponding simulation results are displayed in Figs. 13 and 14.

As indicated in Fig. 12, at low values of LB, the output power increases and the oscillation frequency decreases with LB, while at higher values of LB, the output power and oscillation frequency are independent of LB. Therefore, a value of 1 \( \mu \)H is used in all simulations to avoid the effect of LB. Also, the capacitance CBL is chosen to be 500 \( \mu \)F to act like short circuit at high frequency.

![Fig. 12. Output power and oscillation frequency against LB at VDC = 1.4 V.](image)

![Fig. 13. Output power against LR at different values of RL with VDC = 1.4 V.](image)
the output oscillation frequency increases with decreasing both LR and RL. This agrees with the analytical results shown in Fig. 10. Also, this observation shows a fair agreement with those concluded by other researchers in Refs. [18,38].

However, the output power across the load RL is less than that obtained from simple RC model. This is because the proposed model contains the parameter $\alpha(f)$ that causes a reduction in the oscillation power at high frequency, as shown in Fig. 15.

Fig. 15 illustrates that the output power increases with $\alpha(f)$ over the range from 0.55 to 1 while at values of $\alpha(f) \leq 0.5$, no oscillation occurs.

The effect of the DC bias voltage within the range of the NDR is displayed in Fig. 16 for the RC model and in Fig. 17 for the proposed model. The simulation is carried out at different bias voltages: 1.3 V, 1.35 V and 1.4 V. It is noted that, at VDC = 1.4 V, the output power delivered to the load achieves maximum values compared with other DC values. For the RC model, a maximum power of about 337.5 $\mu$W occurs at RL = 31 $\Omega$. This is in a fair agreement with that reported in Ref. [18] which achieved a maximum output power of 343 $\mu$W, for the same RTD device under test. However, for the proposed model, a maximum power of ~246 $\mu$W occurs at RL = 27 $\Omega$.

On the other side, the dependence of the output power and oscillation frequency on RTD capacitance, $C_D$, is shown in Figs. 18 and 19 for RC model.

Fig. 16. Output power in $\mu$W against RL at different values of bias VDC for RC model with RL = 14 pH.

Fig. 17. Output power in $\mu$W against RL at different values of bias VDC for the proposed model with RL = 14 pH.

Fig. 18. Output power against LR at different values of $C_D$ at RL = 5 $\Omega$ and VDC = 1.4.
It is observed that the oscillation frequency decreases with increasing the capacitance while the output power increases with increasing the capacitance of RTD. The results shown in Fig. 19 that describe the relation between the RTD capacitance and the oscillation frequency confirm with that reported in Table 1.

The simulation results and the analytical ones shown in Fig. 10 and Table 1 are summarized in Table 2. The analytical oscillation frequency was calculated through Eq. (40) that was proved under the condition of maximum power for the simplified circuit, Fig. 9, while the simulated oscillation frequency is calculated from the simulation results for the whole circuit.

From Table 2, it is clear that there is a fair agreement between the analytical results and simulation ones, especially at high frequencies, for the oscillation frequency. It is observed that the percentages of errors are small and are acceptable at high frequencies. This confirms that the simplified circuit shown in Fig. 9 is fairly equivalent to the whole simulated circuit especially at high frequencies in the case of RC model.

Fig. 20 illustrates the output power and oscillation frequency dependence on the RTD area for RC model. It is clear that the output power increases while the oscillation frequency decreases with increasing the RTD area. The simulation results show that the RTD area is restricted to be less than 3.6 μm². Due to this restriction, the output power of the RTD oscillator is restricted to be less than a few tens of microwatts.

Studying the RTD area dependency of the output power of the proposed model reached to the same conclusion stated before with the RC model with this parallel topology.

The analytical results listed in Table 1 for the proposed model and the simulation results are summarized in Table 3. A fair agreement is noticed between the analytical and simulation results for the oscillation frequency.

The comparison between simulation and analytical results in Tables 2 and 3 shows the oscillation frequency error is higher at low frequency. This is because the analytical results are derived based on the simplified small-signal RF equivalent circuit shown in Fig. 9 (b). In this simplified circuit, the inductance \( L_B \) is assumed large and acts like an open circuit at high frequency oscillation. Also, the capacitance \( C_B \) acts like a short circuit at high frequency. In simulation results, the inductance \( L_B \) and the capacitance \( C_B \) have been taken into account, as shown in Fig. 7. These neglected elements \( L_B \) and \( C_B \) can be play important roles in the simulation results at low frequency and, consequently, increases the error. Also, from the comparison, there is disparity between the two RL values, especially, at low values of RL. That is because the value of RL is calculated at the maximum power delivered to the load which depends

<table>
<thead>
<tr>
<th>LR (pH)</th>
<th>Simulation results</th>
<th>Analytical results</th>
<th>Percentage of error between simulated and analytical ( f_{osc} )</th>
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</thead>
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<tr>
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<td>( f_{osc} ) (THz)</td>
<td>RL (Ω)</td>
<td>( f_{osc} ) (THz)</td>
</tr>
<tr>
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<td>2.94</td>
<td>1.25</td>
</tr>
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<td>4</td>
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<td>18.27</td>
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<tr>
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<th>Analytical results</th>
<th>Percentage of error between simulated and analytical ( f_{osc} )</th>
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<td>RL (Ω)</td>
<td>( f_{osc} ) (THz)</td>
</tr>
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<td>0.409</td>
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</table>

Fig. 19. Oscillation frequency against LR at different values of CD at RL = 5 Ω and VDC = 1.4.

Fig. 20. The output power and oscillation frequency against RTD area at VDC = 1.4 V, RL = 31 Ω, LR = 14 pH, and CD = 2.8 fF.
on the oscillation frequency. Therefore, any error in the oscillation frequency is reflected on the value of RL, especially, at low values of RL. However, the error of RL is not of great influence on the values of the oscillation frequency as shown in the two tables, specially, at high frequency.

To complete the simulation comparison between the RTD equivalent circuit models, the parallel RTD based oscillator circuit, shown in Fig. 7, is simulated using the SPDRC model which is shown in Fig. 2. SPDRC model is a full mutual alternative for the quantum-inductance (RLC) model under certain conditions. Also, it is interesting to note that the proposed model is also a full mutual alternative for the SPDRC under the same conditions [19]. The conditions are

\[
R_{\text{W}} = -R_{\text{D}}, \quad \tau = L_{Q} G = R_{\text{W}} C_{W} \tag{50}
\]

where the capacitor \(C_{W}\) represents the effect of the stored charge in the well on the high frequency, while \(R_{W}\) is the resistance that determines the rate of charging or discharging in the well. Because the \(R_{W}C_{W}\) time constant is equal to \(L_{Q} G\) of the RLC model, so, it is also an indication of the quasibound-state life time in the quantum well. In this equivalent circuit, the lumped reactive elements are positive, thus removing the ambiguity of a negative inductance for the RLC model.

A comparison between the output power and the oscillation frequency for the three models are shown in Fig. 21 and Fig. 22. In these simulations, the three models are applied on the parallel RTD circuit.

From comparison in both figures, one can write:

- RC circuit model tends to overestimate the maximum oscillation frequency and the power delivered to the load.
- The proposed model curve displays correctly the same power roll off behavior as the experimental data reported in Ref. [17].
- The SPDRC equivalent circuit model curve displays almost the same power roll off behavior as the experimental data reported in Ref. [17]. However, there is a small disparity between it and the proposed model curve, especially at high frequency. This is because it is difficult to satisfy the impedance equality conditions, Eq. (50), during the whole simulation.

The average Central Processing Unit (CPU) time is computed for the three models. The average CPU time is taken for each model as an average of 15 runs for \(LR = 4\) pH–18 pH using Intel Core 2 Duo (3 GB RAM). The average CPU time for each model is shown in Table 4.

From Table 4, the proposed model takes ~79 s which indicates that the complexity of the proposed model (CPU time) is less than that of the SPDRC model (90.6 s) and higher than that of constant RC model (56 s). This result is because the constant RC model is inaccurate, especially in high frequency regime, while the proposed model more accurate.

7. Conclusion

This paper focuses on the I-V characterization, high frequency characterization, and small signal modeling of RTDs. Efficient contributions are achieved to the accurate characterization and modeling of RTDs and analyzing the based circuits in the high frequency regime without any singularity and convergence problems that arise with other current equivalent circuit models.

The obtained results are compared with the current RTD equivalent circuit models leading to the following conclusions:

- The proposed model displays correctly the same power roll off behavior as the experimental data reported in literature.
- The SPDRC equivalent circuit model displays almost the same power roll off behavior as the experimental data reported in literature. However, there is a small discrepancy with the proposed model curve, especially at high frequency. This is because it is difficult to satisfy the required impedance equality conditions during the whole simulation for the SPDRC equivalent circuit model.
- The proposed model takes about 79 s processing time which indicates that the complexity of the proposed model (CPU time) is less than that of the SPDRC model (90.6 s) and higher than that of constant RC model (56 s). Although the constant RC model takes less time, but, it is less accurate than the proposed model, especially in the high frequency regime.
Appendix A. Supplementary data

Supplementary data related to this article can be found at https://doi.org/10.1016/j.mejo.2018.02.003.

Appendix. ADS simulation

A. RC RTD model

To model the voltage dependent current source using ADS, a symbolically-defined device (SDD) is used as shown in Fig. A.1.

\[ I(v1) = \left( \frac{(A1^* v1^* B1)}{C0} + \frac{C1^* \exp(D1^* v1))}{C1} \right) F1 + \left( \frac{(A2^* \exp(B2^* v1))}{C2} + \frac{C2^* \exp(D2^* v1)}{C2} + E2 \right) F2 \]

where \( v1 \) defines the voltage across the SDD device.

B. Series/parallel double RC network

To simulate this model, the capacitor \( C_D \) is simulated as a two port SDD device, as shown in Fig. A.2.

\[ C_D = C_o - \tau_c \times G, \]

and \( G \) is defined as:

\[ G = \left( \frac{(A1^* B1^* (v1^* (B1 - 1))) + (C1^* D1^* \exp(D1^* v1))}{F3} + (A2^* B2^* \exp(B2^* v1)) + \frac{(C2^* D2^* \exp(D2^* v1))}{F4} \right) \]

The values of \( C_o \) and \( \tau_c \) are defined as variables in the simulator.

C. The proposed model \( C(V, F) \) and \( G(V, F) \)

The frequency dependent capacitor is implemented in the ADS using two ports SDD device, as shown in Fig. A.3, and is given by:

\[ C_{eq} = \frac{(\omega^2 L_0^2 C_D - L_o + C_D R_o^2)}{(R_o)^2 + (\omega L_o)^2} \]
The $I(v_1)$ in this model is a function of voltage and frequency and is implemented in the ADS using one port SDD device, as shown in Fig. A.3, and is given by:

$$I(v_1) = \text{const} \cdot \left[ \frac{\text{Area}}{2.96} \times \left( (C_1 \cdot (v_1+B_1)) + (C_1 \cdot \exp(D_1 \cdot v_1)) \right) + (C_2 \cdot \exp(D_2 \cdot v_1)) + (C_2 \cdot \exp(D_2 \cdot v_1)) + (2 \cdot (E_2 \cdot F_2)) \right]$$

where $\text{const} = 1/(1 + (\omega \cdot t \cdot n \cdot v_1^2))$

References


