Error Performance of Free Space Optical MIMO Systems in Weak, Moderate, and Severe Atmospheric Turbulence Channels

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***Abstract*— Atmospheric turbulence causes degradation in the performance of free space optical (FSO) transmission. This turbulence is referred to as scintillation. To mitigate this effect, a multiple input multiple output (MIMO) system is employed. This paper investigates and compares the performance of FSO MIMO systems in different atmospheric turbulence conditions such as weak, moderate, and severe when binary pulse position modulation (BPPM) is employed. In particular, single input multiple output (SIMO) system using BPPM technique is investigated with equal gain combining (EGC), selection combining (SC), and maximal ratio combining (MRC) diversity schemes. Moreover, the probability of error performance is evaluated using Monte Carlo simulations assuming different atmospheric turbulence channels.**

***Index Terms*— Pulse position modulation (PPM), multiple-input-multiple-output (MIMO) systems, maximal ratio combining (MRC), selection combining (SC), equal gain combining (EGC), and probability of error.**

I. Introduction

Free space optical (FSO) communication has been spread in the last years. This is because of the very high data rates it can provide, which is on the order of gigabits per second [1]. FSO laser beams cannot be detected with spectrum analyzers or radio frequency (RF) meters. Since the laser beams generated by FSO systems are narrow and invisible, this makes them harder to find and even harder to intercept and crack. Also, it is immune to noise, interference, and jamming from other sources. FSO requires no radio frequency spectrum licensing that is translated into unlimited bandwidth, ease, speed and low deployment cost. FSO transmits invisible, eye-safe light beams from one "telescope" to another using low power. Each telescope consists of an optical transceiver with a transmitter and a receiver to provide full-duplex capability. Each optical wireless unit uses an optical source plus a lens or telescope that transmits light through the air to another lens receiving the information. So, it is very easy to reposition the system and change its place [2]. FSO systems can function over distances of several kilometers. Though, it requires a clear line of sight between the source and the destination and enough transmitter power.

As the laser beam is so narrow, it requires accurate pointing. It also needs a tracking mechanism to overcome the buildings sway. Also, the atmosphere consists of very small particles and molecules whose sizes are comparable to the carrier wavelength, which in turn results in various effects that the beam is subjected to. Typically, these effects are not known in the radio frequency (RF) systems. One of such effects is the scintillation process, which causes random fluctuations in the received irradiance of the optical beam. This effect is typically equivalent to fading in RF systems.

Scintillation takes place as a result of heating of the earth's surface, which results in the rise of thermal air masses. These masses are then combined together forming regions with different densities and sizes, which cause a difference in the refractive index that varies with time [1]. Moreover, these regions cause fluctuations in the irradiance of the received laser beam. Many studies have been carried out to analyze the scintillation effect and to describe its model. The distribution of random fluctuations depends on the optical turbulence strength. The scintillation index (SI) is considered the measure of the turbulence strength. For weak turbulence, the SI range is $SI \leq 0.3$. For moderate turbulence the SI range is given by $0.3 < SI < 5$. For severe turbulence the SI is $SI \geq 5$ [3]. In case of weak and moderate turbulence, it is lognormally distributed [1]. As the atmospheric turbulence increases, the lognormal model begins to deviate. Rayleigh distribution is considered the best scenario for MIMO in case of strong turbulence [4,5].

This paper mainly focuses on the mitigation of the scintillation effect (which will be referred to as fading) through the use of multiple lasers at the transmitter and multiple apertures at the receiver.

FSO MIMO channel assuming *Q-ary* a PPM scheme has been studied in [1,2] assuming non-ideal photodetection. In this work, first, BPPM is employed assuming non-ideal photodetection with EGC, SC, and MRC diversity techniques employed at the receiver in weak, moderate, and severe turbulence.

The rest of this paper is organized as follows. Section II describes the system model. Section III, presents the probability of error in case of no atmospheric turbulence using receive diversity. Section IV, the average probability of error performance is presented for SIMO system in case of BPPM with EGC, SC, and MRC diversity techniques in case of weak atmospheric turbulence. Furthermore, Section V presents the average probability of error performance using the same diversity techniques for the case of moderate turbulence. Then, section VI introduces the mitigation of severe atmospheric effect using the same diversity techniques. This is followed by conclusion in Section VII.

II. SYSTEM MODEL AND ASSUMPTIONS

FSO MIMO system usually comprises M lasers at the transmitter and N aperture receivers, as shown in Fig. 1.

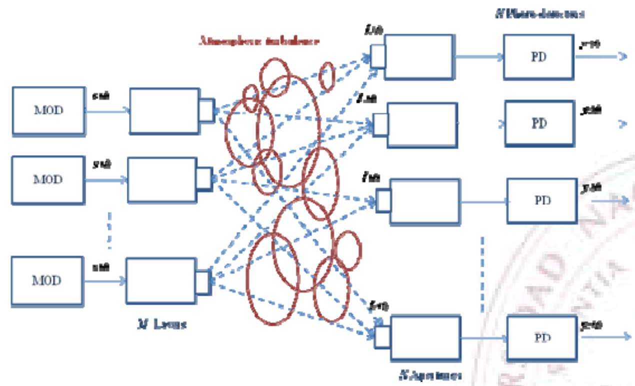

Fig.1 FSO block diagram [1].

It is worth noting that the total transmitted optical power is constant regardless of the number of lasers. The irradiances of the laser beams are added constructively at each receiving aperture (as the irradiance is optical power) [1].

$$I_n(t) = As(t)\sum_{m=1}^{M} h_{m,n}(t) \tag{1}$$

where, A denotes the received irradiance in the absence of scintillation, $s(t)$ is the transmitted irradiance and $h_{m,n}(t) \geq 0$ is the irradiance fading coefficient due to scintillation between the m'th laser and n'th aperture. The separation distance between lasers is assumed to be large enough to assure that the fading paths $h_{m,n}(t)$ for m =1,…, M and n = 1,…., N are independent and identically distributed (i.i.d.). In our model, a non-ideal photodetection (PD) is employed, such that shot noise and thermal noise processes are well approximated by a Gaussian distribution [1]. As non-ideal PD is assumed, then the PD will have a current that is directly proportional to the received irradiance [4,5]

$$y_n(t) = \rho I_n(t) + w_n(t) \tag{2}$$

where ρ denotes the responsivity of the photodetector and $w_n(t)$ represents the zero mean signal independent additive white Gaussian noise (AWGN) process with two-sided power spectral density.

We further assume a one laser source at the transmitter and N apertures at the receiver. In the following, BPPM is employed and the symbol time is assumed T_s. For Q symbols, T_s is divided into Q equal time slots of width T_p, (i.e. $T_p = T_s/Q$). The l'th symbol $X[l] = q$, where $q \in \{1,……, Q\}$ is being sent as a rectangular pulse. Then, the transmitted signal for viewing L frames and the presence of generally M transmitting lasers will be [1]

$$s(t) = \frac{1}{M}\sum_{l=0}^{L-1} rect\,(t - lT_s - (X[l]-1)T_p) \tag{3}$$

In general, if M lasers are used at the transmitter, then the transmitted power is constant and it does not depend on the number of transmitting lasers. Also, the M lasers are assumed to be separated with a large distance, which is sufficient to consider the fading paths to be totally independent from each other. We first assume a SIMO system, and the fading coefficients $h_{n,m}(t)$ are Rayleigh distributed, which corresponds to severe atmospheric turbulence.

Since non-ideal PD is being assumed, then by recalling Eq. (2), and letting $E_s = (\rho A)^2\, T_p$, which denotes the received symbol energy with the use of matched filter at each detector, and with an integrate-and-dump (ID) filter, then, the output of the ID for the n'th aperture for the l'th symbol period for the transmitted symbol $X[l] = j$ will be [1].

$$z_{n,q}[l] = \begin{cases} \sqrt{E_s}\left(\frac{1}{M}\sum_{m=1}^{M} h_{m,n}[l]\right) + w_{n,q}[l], & q = j \\ w_{n,q}[l] & q \neq j \end{cases} \tag{4}$$

III. AVERAGE PROBABILITY OF ERROR FOR NO ATMOSPHERIC TURBULENCE

When there is no atmospheric turbulence, only the thermal and shot noises are present since we assumed non-ideal PD. These noises are represented by Gaussian distribution with mean μ = 0, and $\sigma 2 = N_0/2$ W/Hz. The receive diversity is given by [6]:

$$P_b = \frac{1}{2}erfc\left(\sqrt{\frac{NE_b}{N_o}}\right) \tag{5}$$

where $erfc(.)$ is the complementary error function which is defined by $erfc(x) = \int_x^{\infty} \frac{2}{\pi} e^{-t^2} dt$ and E_b is the bit energy.

Figure (2) compares the theoretical probability of error performance to the simulated performance using Monte Carlo simulations in case of no turbulence.

As can be seen, the theoretical and simulated curves are in good agreement. When N = 1, the probability of error reaches the value of 10^{-4} at an approximate signal to noise ratio (SNR) = 11.5 dB. Whereas, when N = 2, the probability of error curve intersects with the SNR axis (at bit-error-rate (BER) = 10^{-4}) at SNR = 8.4 dB. While, at N = 4 the achieved SNR at the same BER is 5.5 dB. At N = 8, the corresponding value of SNR is 2.2 dB.

In the following sections, the probability of error will be evaluated using Monte Carlo simulations in case of weak, moderate, and severe turbulences. Monte Carlo simulations are carried out using MATLAB. In our simulations, we generate a random binary sequence of 1's and 0's. Then, this stream is pulse position modulated according to Eq. (3).

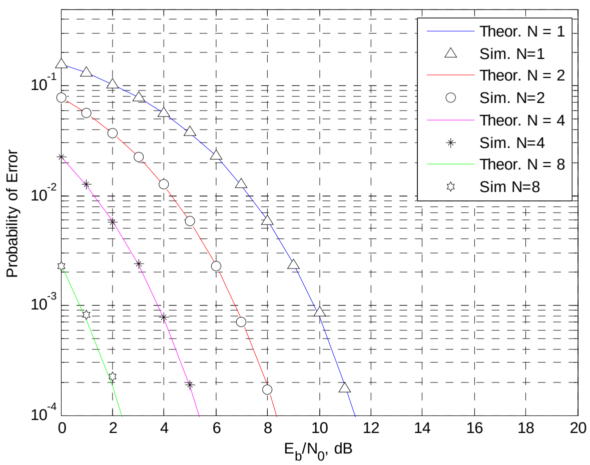

Fig.2 Probability of error in case of no-turbulence using receive diversity.

The symbols are then multiplied passed through the lognormal channel in case of weak and moderate turbulences with the appropriate SI values for each case, and a Rayleigh channel in case of severe turbulence. Finally, AWGN is added to the sequence after applying the effect of scintillation for each case. At the receiver, the received signal is detected using different diversity techniques. In particular, the received symbols are accumulated from all receive paths, and processed for each receive diversity technique. In further details, hard decision decoding is performed, and bit errors are counted for each technique in the assumed turbulence strength. These steps are repeated for various values of E_b/N_0, and the simulated BER is plotted versus the corresponding E_b/N_0 for each case.

IV. AVERAGE PROBABILITY OF ERROR PERFORMANCE FOR WEAK TURBULENCE

As stated in previous sections, SI is smaller than 0.3 for weak turbulence.

The weak turbulence channel is represented by lognormal distribution [1]:

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{\frac{-(\ln x-\mu)^2}{2\sigma^2}} \qquad (5)$$

where μ is the mean of the distribution and σ^2 is the variance.

When EGC is employed, the output of the ID is averaged over the number of receiving apertures [5]. To evaluate the average probability of error performance in case of BPPM in the presence of a weak turbulence, Monte Carlo simulations were carried out, and the results are shown in Fig.3. As can be seen, the performance improves by increasing the number of receiving apertures. At $N = 1$, the probability of error reaches 10^{-4} at SNR = 19 dB. For $N = 2$, it reaches the same BER at SNR = 14 dB. The performance is enhanced at $N = 4$ by approximately 4 dB. Also, the performance at $N = 8$ is improved by nearly 3.5 dB as compared to $N = 4$. This shows that the performance is improved by increasing the number of receiving apertures. However, this rate of improvement obviously decreases as the number of apertures is increased to higher values. Intuitively, at higher number of apertures, the rate of improvement is expected to be negligible, essentially if we take into consideration the higher complexity and cost required when increasing the number of receive apertures.

To further evaluate the probability of error performance in case of BPPM in the presence of weak turbulence using SC diversity technique, Monte Carlo simulations were also carried out, and the results are depicted in Fig.4.

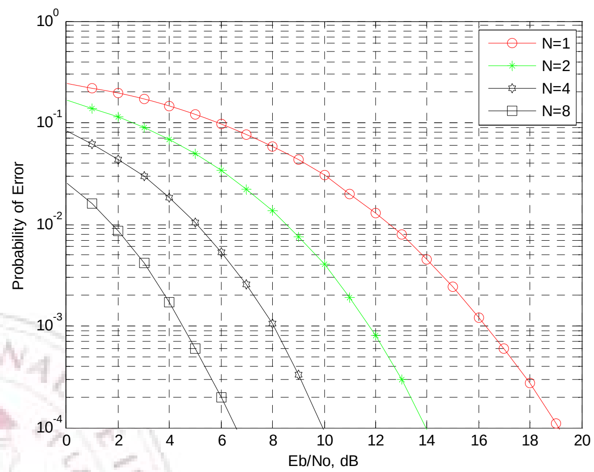

Fig.3 Probability of error in weak turbulence using EGC diversity technique for various values of N.

It is obvious from figure that at $N = 1$ the probability of error reaches 10^{-4} at SNR = 18 dB. At $N = 2$, it cuts the SNR axis (at BER = 10^{-4}) at 15.5 dB. The performance is improved at $N = 4$ than that at $N = 2$ by nearly 2 dB. The performance is further improved at $N = 8$ as compared to $N = 4$ by approximately 1.5 dB.

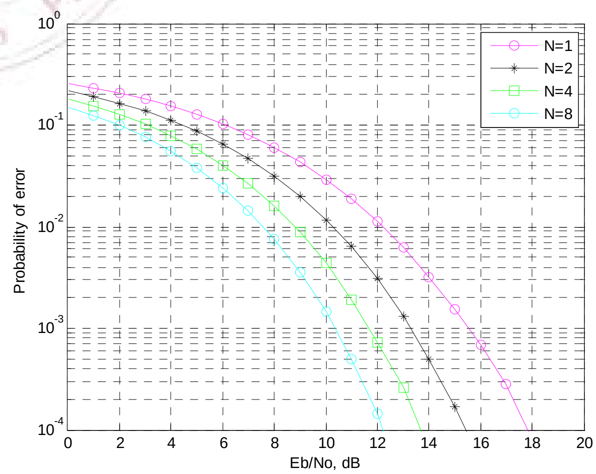

Fig.4 Probability of error in weak turbulence using SC diversity technique for various values of N.

When MRC is employed, the output of the combiner is a weighted sum of all branches. In particular, branches with high signal-to-noise-ratios are given weights higher than other branches [7, 8, 9]. To study the probability of error performance in case of BPPM in a weak turbulent channel using MRC diversity technique, Monte Carlo simulations were carried out, and results are as shown in Fig.5.

At $N = 1$ the probability of error reaches 10^{-4} at SNR 18 dB. At $N = 2$, it cuts the SNR axis at 13 dB for the same BER. The performance is improved at $N = 4$ than that at $N = 2$ by approximately 4 dB and is also improved at $N = 8$ as compared to $N = 4$ by nearly 3 dB. Again, this assures that the performance is improved by increasing the number of receiving apertures.

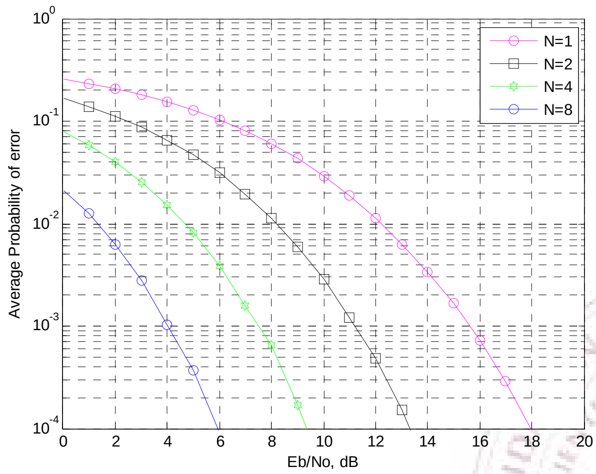

Fig.5 Probability of error in weak turbulence using MRC diversity technique for various values N.

V. AVERAGE PROBABILITY OF ERROR PERFORMANCE FOR MODERATE TURBULENCE

The SI range for moderate turbulence is also represented by lognormal distribution as weak turbulence,
and has 0.3< SI < 5. The same diversity techniques that were employed for weak turbulence are investigated for moderate turbulence.

Figure 6 shows the probability of error assuming EGC technique in moderate turbulent channel.

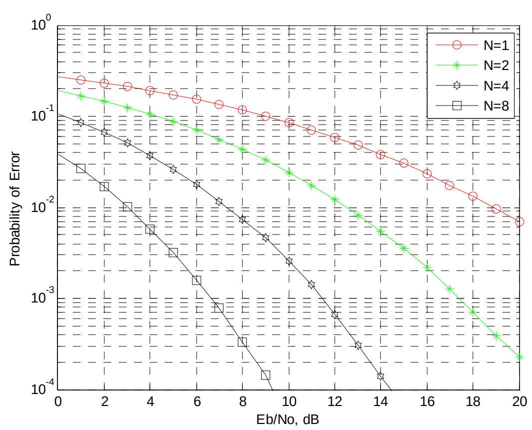

Fig.6 Probability of error in moderate turbulence using EGC diversity technique for various values of N.

It is evident from figure that the performance improves by increasing the number of receiving apertures. At both $N = 1$ and $N = 2$, the probability of error reaches 10^{-4} at SNR greater than 20 dB. At $N = 4$, the BER reaches 10^{-4} at SNR = 15 dB. The performance gets better at $N = 8$ by nearly 6 dB. This proves that even in moderate turbulent channels the performance is improved by increasing the number of receiving apertures.

When the SC diversity technique is employed, the probability of error performance was also studied using Monte Carlo simulations, as shown in Fig.7 for moderate turbulent channels.

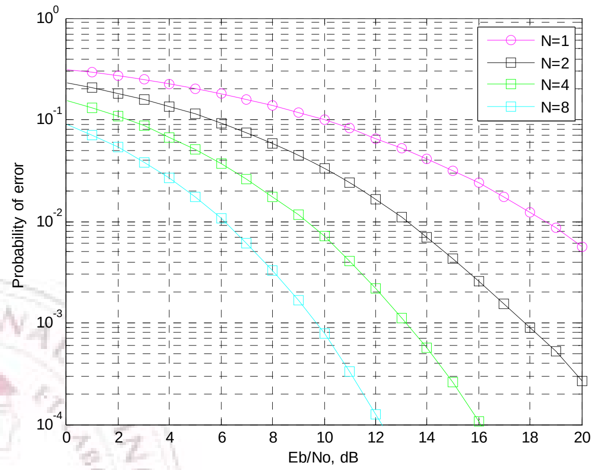

Fig.7 Probability of error in moderate turbulence using SC diversity technique for variuos values of N.

It is clear that at both $N = 1$ and $N = 2$ the probability of error reaches 10^{-4} at SNR more than 20 dB. At $N = 4$, it cuts the SNR axis at 16 dB. The performance is improved at $N = 8$ than that at $N = 4$ by nearly 4 dB. Again, the performance improves by increasing the number of receiving apertures at the receiver.

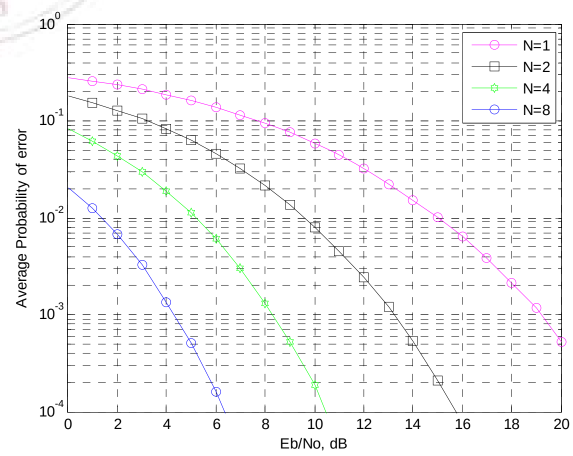

Fig.8 Probability of error in moderate turbulence using MRC diversity technique for various values of N.

Similarly, MRC diversity technique was employed for the case of moderate turbulence. The probability of error performance was also studied using Monte Carlo simulations as displayed in Fig.8. For $N = 2$ and BER = 10^{-4}, the approximate SNR is

16 dB. For the same BER and N = 4 and 8, the SNR is approximately equal to 10 and 6, respectively.

VI. AVERAGE PROBABILITY OF ERROR PERFORMANCE FOR SEVERE TURBULENCE

In this section, we study the performance of FSO MIMO systems in severe turbulence. Rayleigh distribution is considered the best model for representing the severe atmospheric turbulence [10]. The SI range is as stated before is SI ≥ 5. Like the cases of weak and moderate turbulences, the study of different diversity techniques will be carried out using Monte Carlo simulations. The resulting probability of error for the case of severe turbulence is shown in Fig.9 for the case of applying EGC technique.

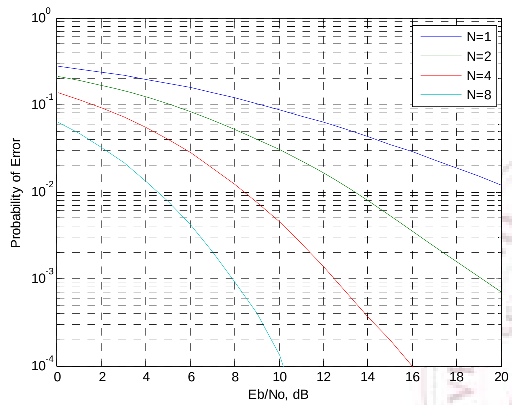

Fig. 9 Probability of error for EGC at $M=1$ in severe turbulent channel for various values of N.

As expected, the performance is also improved by increasing the number of receiving apertures. The probability of error in the case of N = 4 is decreased by 10 dB from that of N = 2. Also, the performance is further improved by 4 dB when N is increased to 8. Then, BER is investigated when SC diversity technique employed, and the results are shown in Fig.10.

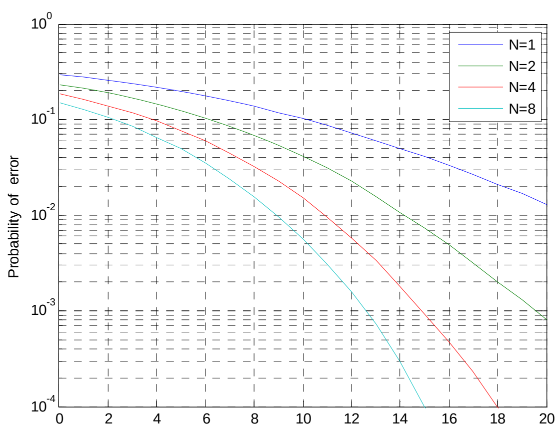

Fig .10 Probability of error of BPPM for SC technique at $M=1$ in severe turbulent channel for various of N.

Comparing the performance of probability of error by increasing the number of receiving apertures, the probability of error at N = 4 reaches 10^{-4} at SNR = 18 dB, and at N = 8 it reaches the same BER at SNR=15 dB. Thus, the performance improves by nearly 3 dB with increasing the number of receiving apertures from N = 4 to N = 8.

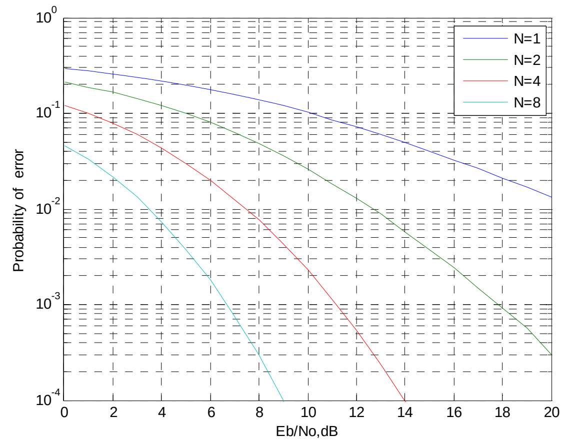

Fig.11 Probability of error of BPPM for MRC technique at $M=1$ in severe turbulent channel for various values of N.

Figure 11 shows that increasing the number of receiving lasers also improves the BER performance in severe turbulence when MRC combining is employed. At N = 1 and N = 2, the probability of error reach 10^{-4} at SNR > 20 dB. At N = 4, the probability of error reaches 10^{-4} at SNR = 14 dB. When N is euqal 8, the probability of error reaches 10^{-4} at SNR = 9 dB.

VII. CONCLUSIONS

The scintillation has a detrimental effect on the propagating laser beam due to the random fluctuations that take place in the propagating beam. As mentioned before, the turbulence strength is determined by the value SI, which consequently affects the distribution of turbulence. The turbulence may be weak, moderate, or severe. In case of weak and moderate turbulences, the amplitude distribution is lognormally distributed. While, in case of severe turbulence, it is Rayleigh distributed. MIMO systems are employed to mitigate the effect of turbulence. In this paper, SIMO systems were employed to mitigate the turbulence effect using different diversity techniques. Different diversity techniques were employed for weak, moderate, and severe turbulences. MRC diversity technique is considered the best diversity technique to be used. In case of weak turbulence, it achieves a better probability of error behavior than other diversity techniques such as EGC and SC. It is much better than EGC by nearly 1 dB and by nearly 3.5 dB than that of SC. In case of moderate turbulence, MRC showed a better behavior by nearly 5 dB compared to EGC and by 6 dB compared to SC. While, in case of severe turbulence, EGC probability of error behavior is worse than that of MRC by 1 dB and SC behavior is worse by 6 dB.

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