

# Improved Performance of M-ary PPM in Different Free-Space Optical Channels due to Reed Solomon Code Using APD

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**Abstract**— Atmospheric turbulence induced fading is one of the main impairments affecting the operation of free-space optical (FSO) communication systems. In this paper, the bit error rate (BER) of M-ary pulse position modulation (M-ary PPM) of direct-detection and avalanche photodiode (APD) based is analyzed. Both log-normal and negative exponential fading channels are evaluated. The investigation discusses how the BER performance is affected by the atmospheric conditions and other parameters such as the forward error correction using Reed Solomon (RS) codes and increasing Modulation level. Results strongly indicate that, RS-coded M-ary PPM are well performing for the FSO links as it reduces the average power required per bit to achieve a BER below  $10^{-9}$  in both turbulence channels.

**Index Terms**— Free Space Optics (FSO), M-ary Pulse Position Modulation (M-ary PPM), Reed Solomon (RS) codes, Log Normal Channel, Negative Exponential Channel, Avalanche Photodiode (APD).

## 1 INTRODUCTION

Free space optical (FSO) systems have been widely deployed for inter-satellite and deep-space communications. In recent years, however, because of its numerous advantages over radio-frequency (RF) technology such as extremely high bandwidth, license-free and interference immunity, FSO has attracted considerable attention for a variety of applications, e.g. last mile connectivity, optical-fiber backup and enterprise connectivity. In such kind of applications, FSO systems basically utilize atmosphere as transmission medium rather than the free space. So, the performance of FSO link is inherently affected by atmospheric conditions. Among these conditions, atmospheric turbulence has the most significant effect. It causes random fluctuations at the received signal intensity, i.e., channel fading, which leads to an increase in the bit error rate (BER) of the optical link [1].

Current FSO communication systems employ intensity modulation with direct detection (IM/DD) and use light emitting diodes (LED) or laser diodes as transmitters and PIN photodiode or avalanche photodetectors (APD) as receivers. These devices modulate and detect solely the intensity of the carrier and not its phase. Furthermore, biological safety reasons constrain the average radiated optical power, thereby constraining the average signal amplitude. The most reported modulation technique used

for FSO is the on-off keying (OOK) which offers bandwidth efficiency but lacks power efficiency. Binary level signaling though is the simplest and most common modulation scheme for the optical intensity channel and offers low power efficiency and high bandwidth efficiency. Power efficiency as well as the improved system performance can be achieved by adopting pulse position modulation (PPM) schemes. M-ary PPM achieves high power efficiency at the expense of reduced bandwidth efficiency compared with other modulation schemes. The optimal PPM order is high, since a higher order modulation creates the higher peak power needed to overcome the weak average power. M-ary PPM has been previously suggested as a suitable modulation scheme for FSO systems [2]. The IrDA specification for the 4 Mbps short distance wireless infrared links specifies a 4-PPM modulation scheme [3].

Reed Solomon (RS) codes are a class of block codes that operate on symbols rather than bits. So, RS codes can correct both random bit errors and burst symbol errors. Moreover, their hard decoding algorithm can be easily implemented even at a high operation speed. International Telecommunication Union-Telecommunication (ITU-T) standard forward error correction (FEC) scheme based on RS (255,239) codes has been widely used in 10 Gbps practical optical fiber transmission systems [4]. Kiasaleh derived upper bounds on the BER of M-ary PPM over log-normal and exponential distributed channels, when an APD is used [5].

In this paper, Kiasaleh expressions are used to compare the effect of the BER of average photons per PPM bit with the former channels without coding and with RS (255,207) coding. The remainder of the paper is organized as follows. The models of FSO channels are presented in Section 2. Based on the theory presented, a numerical

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analysis of the M-ary PPM is carried out in Section 3. This is followed by the main conclusions in Section 4.

## 2 MODELS OF FSO CHANNELS

### 2.1 Log-Normal Channel

The log-normal channel is classified as “weak turbulence”, which is characterized by a scintillation index less than 0.75. In general, the scintillation index is a complicated function of the beam parameters, propagation distance, heights of the transmitter and receiver, and the fluctuations in the index of refraction. In fact, the main source of scintillation is due to fluctuations (due to temperature variations) in the index of refraction, which is commonly known as optical turbulence. The log-normal model is also valid for propagation distances less than 100 m [5].

The bit error rate (BER) of an M-ary PPM in log-normal channel is given by [6]

$$P_b^M \leq \frac{M}{2\sqrt{\pi}} \sum_{i=-N, i \neq 0}^N w_i Q \left( \sqrt{\frac{e^{2(\sqrt{2}\sigma_k x_i + m_k)}}{Fe^{\sqrt{2}\sigma_k x_i + m_k} + K_n}} \right) \quad (1)$$

where M is the modulation level,  $w_i$  and  $x_i$  are the weight factors and the zeros of the Hermite polynomial [7].

The scintillation index ( $\sigma_{SI}^2$ ) as a function of the variance of the log-normal channel ( $\sigma_k^2$ ) is given by [5]

$$\sigma_{SI}^2 = e^{\sigma_k^2} - 1 \quad (2)$$

The average photons per PPM slot ( $E\{K_s\}$ ) are functions of the mean ( $m_k$ ) and the variance of the log-normal channel and have the form [5].

$$E\{K_s\} = e^{\left(\frac{\sigma_k^2 + m_k}{2}\right)} \quad (3)$$

The total noise photons per PPM slot,  $K_n$ , which results from background noise and thermal noise, is [5]

$$K_n = \frac{2\sigma_n^2}{(E\{g\}q)^2} + 2FK_b \quad (4)$$

where  $K_b$  is the average background noise photons per PPM slot,  $E\{g\}$  is the average gain of the APD and  $q$  is the electron charge.

The noise factor,  $F$ , of the APD is defined by [5]

$$F = 2 + \zeta E\{g\} \quad (5)$$

where  $\zeta$  is the ionization factor.

The variance,  $\sigma_n^2$ , of the thermal noise in a PPM slot is defined by [5]

$$\sigma_n^2 = \left( \frac{2KT T_{slot}}{R_L} \right) \quad (6)$$

where  $T$  is the effective absolute temperature of the receiver,  $K$  is Boltzmann constant,  $R_L$  is the APD load resistance and  $T_{slot}$  is the PPM slot duration which is related to the data rate,  $R_b$ , by [5]

$$T_{slot} = \frac{\log_2(M)}{MR_b} \quad (7)$$

In case of coding,  $R_b$  must be multiplied by  $(n/k)$ , where  $n$  is the codeword length and  $k$  is the message length.

The symbol error rate ( $P_{symbol}$ ) can be calculated from bit error rate ( $P_b$ ) as [8]

$$P_{symbol} = P_b \left( \frac{2(M-1)}{M} \right) \quad (8)$$

The probability of the uncorrectable symbol error ( $P_{ues}$ ) due to RS codes can be calculated by the formula [9, 10]

$$P_{ues} \leq \frac{1}{n} \sum_{i=t+1}^n i \binom{n}{k} P_q^i (1 - P_q)^{n-i} \quad (9)$$

where  $t = ((n-k)/2)$  is the symbol error correcting capability,  $P_q$  is the q-bit RS symbol error probability.

The BER after coding ( $P_{bc}$ ) is given by [9]

$$P_{bc} = P_{ues} \left( \frac{n+1}{2n} \right) \quad (10)$$

### 2.2 Negative Exponential Channel

The negative exponential channel is classified as “strong turbulence”, which is characterized by a scintillation index greater than 1. The negative exponential model is valid for propagation distances more than 100 m or several kilometers [5, 8].

The BER of the negative exponential channel,  $P_b^M$ , is given by [5].

$$P_b^M \leq \frac{M}{2} \sum_{i=-N, i \neq 0}^N w_i |x_i| Q \left( \frac{E\{K_s\} x_i^2}{\sqrt{FE\{K_s\} x_i^2 + K_n}} \right) \quad (11)$$

To get the BER after coding ( $P_{bc}$ ) due to negative exponential channel using RS (255,207), we apply the same procedure of the log-normal channel coding steps, which are

mentioned in (7), (8), (9), (10) and use this in (11).

### 3 NUMERICAL RESULTS AND DISCUSSIONS

Based on the described model, the BER of  $10^{-9}$ , which is considered as a practical performance target for FSO link [11], is calculated for log-normal channel and negative exponential channel and the obtained results are displayed in Figs. 1-3. In these figures, the value of scintillation index ( $\sigma^2_{SI}$ ) is taken 0.3 for weak turbulence and 1 for strong turbulence. The values of other parameters are taken as: BER=2.4 Gbps,  $K_b = 10$  photons per PPM slot,  $RL=50 \Omega$ ,  $\zeta=0.028$ ,  $T=300^\circ K$ ,  $E\{g\}=150$ ,  $n=255$ ,  $k=207$ ,  $t=24$  symbols.

Variations of BER with the average number of photons received per PPM bit (logarithmic), which are equal  $\log_{10}(E\{K_s\}/\text{number of bits in PPM symbol})$ , are shown in the following figures. In discussions, all results of average photons per PPM bit are in numerical values.

In Fig. 1, binary PPM (BPPM) is used to compare between the effect of weak turbulence and strong turbulence without coding and with RS (255,207) coding.

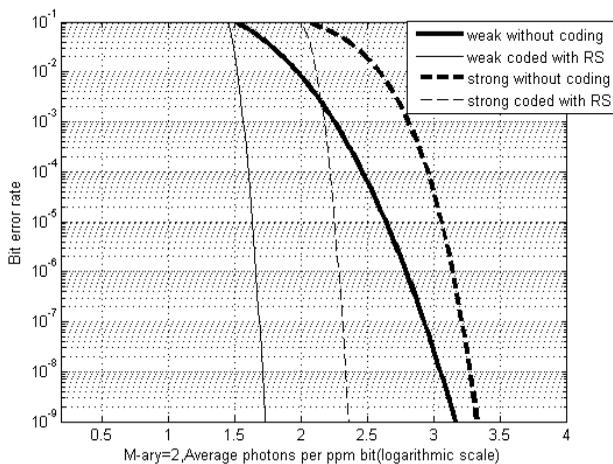


Fig.1. Comparison of non-coded BPPM in weak turbulence of 0.3 scintillation index with Reed Solomon (255,207) and in strong turbulence of 1.0 scintillation index.

At a BER of  $10^{-9}$ , the value of average photons per PPM bit in strong turbulence is found 2046 without coding and 219 with coding which gives an improvement of 9.7 dB. While the value of average photons per PPM bit in weak turbulence is found 1462 without coding and 52 with coding which gives an improvement of 14.49 dB. It also shows that, coded strong turbulence BPPM outperforms weak turbulence BPPM without coding at a BER less than  $10^{-3}$ .

In Fig. 2, 8-PPM is used to show the effect of multilevel modulation and to compare between the effect of weak turbulence and strong turbulence without coding and with RS (255,207) coding.

At a BER of  $10^{-9}$ , the value of average photons per PPM bit in strong turbulence is found 728 without coding and 141 with coding giving an improvement of 11.61 dB.

While the value of average photons per PPM bit in weak turbulence is found 543 without coding and 32 with coding which gives an improvement of 16.6 dB. It is shown that, coded strong turbulence 8-PPM outperforms weak turbulence 8-PPM without coding at BER less than  $10^{-4}$ .

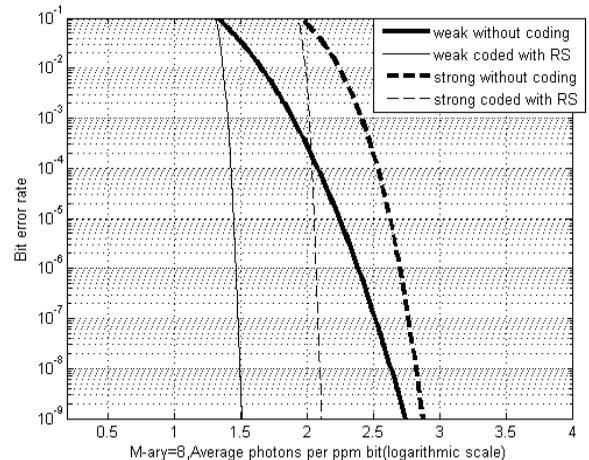


Fig. 2. Comparison of non-coded 8-PPM in weak turbulence of 0.3 scintillation index with Reed Solomon (255,207) and in strong turbulence of 1.0 scintillation index.

In Fig. 3, 256-PPM is used for weak turbulence and strong turbulence without coding and with RS (255,207) coding.

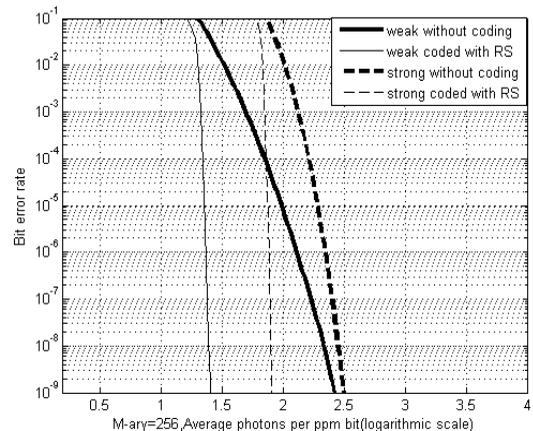


Fig. 3. Comparison of non-coded 256-PPM in weak turbulence of 0.3 scintillation index with Reed Solomon (255,207) and in strong turbulence of 1.0 scintillation index.

At a BER of  $10^{-9}$ , the value of average photons per PPM bit in strong turbulence is found 308 without coding and 80 with coding showing an improvement of 14.08 dB. While, in weak turbulence, the value of average photons per PPM bit is found 258 without coding and 25 with coding giving an improvement of 17.67 dB. It also shows that, coded strong turbulence 256-PPM outperforms weak turbulence 256-PPM coding less at BER less than  $10^{-4}$ .

The obtained results for coded and non-coded M-ary PPM are summarized and compared in Tables 1 and 2 for strong and weak turbulence, respectively.

TABLE 1

Average number of photons per ppm bit required achieving ber  $10^{-9}$  in strong turbulence.

M-ary PPM	Strong turbulence	Strong turbulence with RS code	Improvement due to RS code + multilevel modulation in Strong turbulence respect to non coded BPPM	
			%	dB
BPPM	2046	219	89.3	9.7
8-PPM	728	141	93	11.61
256-PPM	308	80	96	14.08

TABLE 2

Average number of photons per ppm bit required achieving ber  $10^{-9}$  in weak turbulence.

M-ary PPM	Weak turbulence	Weak turbulence with RS code	Improvement due to RS code + multilevel modulation in Weak turbulence respect to non coded BPPM	
			%	dB
BPPM	1462	52	96.4	14.49
8-PPM	543	32	97.8	16.6
256-PPM	258	25	98.3	17.67

All results obtained indicate that, RS codes have low performance in low photons per bit range. This is because the low photons per bit causes burst errors which are beyond the correction capability of RS codes. Under these conditions, spreading the errors by using a code matched interleaver and using concatenated codes will make the coding more effective [12].

## 4 CONCLUSION

In this paper, the performance of FSO system with M-ary PPM based on RS codes scheme has been numerically analyzed in weak and strong atmospheric turbulence. The results show that, the average number of photons per bit at  $10^{-9}$  BER has been improved by 14.08 dB (compared with the value of BPPM without coding) in strong turbulence and to 17.67 dB in weak turbulence using RS (255,207) + 256-PPM.

The results also show that, coded strong turbulence outperforms the non-coded weak turbulence without coding for BPPM, 8-PPM and 256-PPM at  $10^{-9}$  BER, which indicates a great improvement of the performance of the system tolerance for the intensity fluctuations induced by atmospheric turbulence. RS codes can be combated with matched interleaver concatenated coding to solve the problem in low photons per bit range.

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