Academy of Scientific Research and Technology
27th National Radio Science Conference
Faculty of Electronic Engineering, Menoufia Univ., Menouf, Egypt
16-18 March 2010

Optimum Conditions for a High Bit Rate RZ Soliton Train in EDFA with Nonadiabatic Amplification

Mohamed F. Hassan[1] (ezzfahmy@aast.edu), Hossam Zoweil[2](zoweil@gmail.com), Moustafa H. Aly[3] (mosaly@aast.edu) and Ahmed E.El-Samahy[1](a_samahy@hotmail.com)

1 Faculty of Science, University of Alexandria, Alexandria, Egypt.
2 Advanced Technology and New Materials Research Institute, Mubarak City for Scientific Research and Technology Applications, Alexandria, Egypt.
3 College of Engineering and Technology, Arab Academy for Science, Technology and Maritime Transport Alexandria, Egypt, Member of OSA.

Abstract

In this paper, the optimum conditions to propagate RZ-train of solitons down EDFA amplifier at high bit rate and gain are studied. The suggested model is based on the wave equation of the carrier envelope. The two energy level system is used to study the evolution of train of solitons through the erbium doped fiber amplifier (EDFA). The polarization induced term representing the effect of doping atoms on the propagating electric field is obtained by solving Maxwell-Bloch equations [1]. By adding the induced polarization to the nonlinear Schrödinger equation (NSE), the split-step Fourier method is used to solve NSE in the EDFA. The higher doping level used makes the dopants respond not so fast that the induced polarization follows the optical field nonadiabatically. Trains of different soliton width, time slot and doping level are used to obtain the optimum conditions for maximum bit rate and EDFA gain.

I. Introduction

Two distinct modulation formats can be used to generate a digital bit stream. The NRZ format is commonly used because the signal bandwidth is about 50% smaller compared with that of the RZ format. However, the NRZ format cannot be used when solitons are used as information bits. The reason is easily understood by noting that the pulse width must be a small fraction of the bit slot to ensure that the neighboring solitons are well separated [2-4]. Mathematically, the numerical soliton solution is valid only when it occupies the entire time window ($-\infty < \tau < \infty$). It remains approximately valid for a train of solitons only when individual solitons are well isolated. This requirement can be used to relate the soliton bit width T_0 to the bit rate B as [2]

$$\mathbf{B} = \frac{1}{T_B} = \frac{1}{2q_0 T_0}, \tag{1}$$

where $2q_0 = T_B/T_0$ is the separation between neighboring solitons in normalized units.

II. Mathematical Model

The propagation of the optical pulse through EDFA requires using Maxwell-Bloch equations for the two energy system [1]

$$\partial P = -(1/T_2)P + i(\varpi - \omega)P + \left(\frac{p^2}{\hbar}\right) A\, N, \tag{2}$$

$$\partial N=(N_0-N)(1/T_1)-\frac{1}{2}(AP^*+A^*\ P)/\hbar . \quad (3)$$

$N=N_2-N_1$ is the population inversion density with an initial value N_0 , $\overline{\omega}$ is the atomic transition frequency, P is the partial polarization produced by the doping atoms T_1 and T_2 are the population and polarization relaxation times, p is the dipole moment of the doping atom. The system of Eqs. (2) and (3) is solved numerically to obtain the partial polarization produced by the doping atoms.

The equation that describes the propagation of the soliton in the fiber is the NLS [5]. By using T measured in a frame of reference moving with the pulse at the group velocity v_g $(T=t-z/v_g)$ and if fiber losses are ignored, the NSE takes the form

$$i\frac{\partial A}{\partial z}=\frac{\beta_2}{2}\frac{\partial^2 A}{\partial T^2}-\gamma|A|^2 A . \quad (4)$$

If the nonlinear polarization due to excitation of doping atoms, obtained by solving Eqs. (2) and (3), is added to Eq. (4), one obtains the equation which describes the evolution of the optical pulse through the EDFA [6]

$$\frac{\partial A}{\partial z}+\frac{\beta_2}{2}\frac{\partial^2 A}{\partial T^2}=i\gamma|A|^2 A+\left(\frac{\mu_o\omega^2}{2\beta}\right)P , \quad (5)$$

where μ_o is the free space permeability and β is the propagation constant, ω is the carrier frequency.

Equation (5) can be normalized using

$$\tilde{A}=\frac{A}{\sqrt{p_o}}, \quad \tilde{T}=\frac{T}{T_o}, \quad \tilde{z}=\frac{z}{L_D} \quad \text{and } \tilde{P}=\frac{P}{N_o p} , \quad (6)$$

where N_o is the doping level of Erbium and P is the dipole moment of erbium [7]. Based on Ref. [5], one introduces

$$L_D=\frac{T_0^2}{|\beta_2|}, \quad L_{NL}=\frac{1}{\gamma P_0}, \quad N^2=\frac{L_D}{L_{NL}}=\frac{\gamma P_0 T_0^2}{|\beta_2|},$$

where N is the order of the soliton.

The term $(\mu_o\omega^2/2\beta)$ is applied to link normalized $\tilde{A}$ and $\tilde{P}$. Then, one can write

$$\frac{\partial \tilde{A}}{\partial \tilde{z}}+\frac{\beta_2}{2}\frac{\partial^2 \tilde{A}}{\partial \tilde{T}^2}=i\gamma|\tilde{A}|^2 \tilde{A}+\left(\frac{\mu_o\omega^2}{2\beta}\right)\tilde{P} . \quad (7)$$

III. Numerical Solution

The propagation equation of the carrier envelope, A, Eq. (7), is a nonlinear partial differential equation that does not generally lend itself to analytic solutions except for some specific cases in which the inverse scattering method can be employed [8]. A numerical approach is therefore necessary for understanding the nonlinear effects in fibers. The method that has been used extensively to solve the pulse propagation problem in nonlinear dispersive media is the split-step Fourier method [9, 10].

The relative speed of this method compared with most finite-difference methods can be attributed in part to the use of the Fast Fourier Transform (FFT) algorithm [11]. Here, we describe the split-step Fourier method and its modification for the pulse propagation problem in EDFA. To understand the philosophy behind the split-step Fourier method, it is useful to write Eq. (7) in the form

$$\frac{\partial\tilde{A}}{\partial\tilde{z}}=\left(\hat{D}+\hat{N}_{NL}\right)\tilde{A}+\tilde{P}_{NL}\ , \tag{8}$$

where $\hat{D}$ is a differential operator that accounts for dispersion and absorption in a linear medium and $\hat{N}$ is a nonlinear operator that governs the effect of fiber nonlinearities on pulse propagation. These operators are given by

$$\hat{D}=\frac{i\beta_2}{2}\frac{\partial^2}{\partial T^2}\ , \tag{9}$$

$$\hat{N}=i\gamma(|A|^2), \tag{10}$$

$$\hat{P}=\frac{\mu_o\omega^2}{2\beta}. \tag{11}$$

Comparing Eq. (7) with Eq. (8), both operators $\hat{D}$ and $\hat{N}_{NL}$ can be easily obtained. $\tilde{P}_{NL}$, corresponding to the second term on the right hand side in Eq. (7), accounts for the effect of nonlinear polarization due to excitation of doping atoms.

Numerically, the envelope after a distance h, can be represented by

$$\tilde{A}\left(\tilde{z}+h,\tilde{T}\right)=\exp\left(\frac{h}{2}\hat{D}\right)\exp\left[\int_{\tilde{z}}^{\tilde{z}+h}N(\tilde{z}')d\tilde{z}'\right]\left(\exp\left[\frac{h}{2}\hat{D}\right]\tilde{A}(\tilde{z},\tilde{T})+h\tilde{P}_{NL}\right). \tag{12}$$

IV. Results and Discussion

A single-mode step index fiber is used with an effective area $A_{eff} = 20\ \mu m^2$, the carrier wavelength $\lambda = 1.55\ \mu m$, corresponding to the region of anomalous dispersion and minimum loss. The erbium dipole moment $p = 8.339\times10^{-32}$ C.m, the dispersion parameter $\beta_2 = -20\times10^{-27}\ s^2.m^{-1}$ (at 1.55 μm), the nonlinear index coefficient $n_2 = 2.3\times10^{-20}\ m^2.W^{-1}$ and the nonlinear parameter $\gamma = 5.3\ W^{-1}.km^{-1}$[7].

Based on the described model, a computer program is constructed using the MATLAB-7 to solve Eq. (7) numerically to get the slowly wave envelope and to calculate the gain for the evolution of the optical RZ- train of solitons through the EDFA. Each step in z-direction is taken to be 1/50 of the dispersion length L_D, and each step in the time domain is taken to be 1/16 of the initial soliton width T. The time axis is divided into 256 parts to meet the use of fast Fourier transform (FFT).

Figure 1 shows the evolution of a RZ-train of solitons, each of pulse width 0.2 ps and a time slot 10 ps between pulses in an EDFA of a doping level 50×10^{23} atoms/m^3, input power 47.44 W and a dispersion length 2 m. This data gives a bit rate 250 Gbps for RZ transmission. The threshold time slot is 10 ps, after which the solitons of the train start to interact nonadiabatically as amplitude modulation for slowly varying wave envelope. The maximum obtained output power, at these conditions, is 85 times the initial input power, leading to high gain (19.29 dB). The train of solitons propagates down the fiber as fundamental solitons at z = 0 and then changes into higher order soliton of N = 2 after z = L_D.

Figure 2 shows the frequency domain which reveals that the trailing edge frequency components of the first soliton of the train interfere constructively with leading edge frequency components of the next one; this leads to a peak at the middle [12]. But, after a normalized distance of propagation z = 0.9, the rapid generation of these components stops this type of useful interference which helps in pulse compression [13].

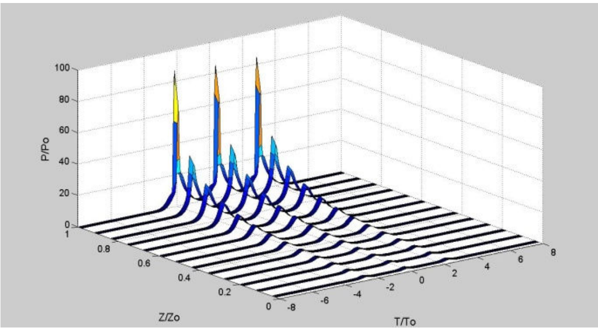

Figure 1 The evolution for RZ-train of solitons of width 0.2 ps down an EDFA of length 2 m, doping level 50×10^{23} atoms/m^3, and β_2 =-20×10^{-27} s^2.m^{-1} at λ=1.55 μm. The time slot between the pulses = 10 ps and the bit rate = 250 Gbps (time domain).

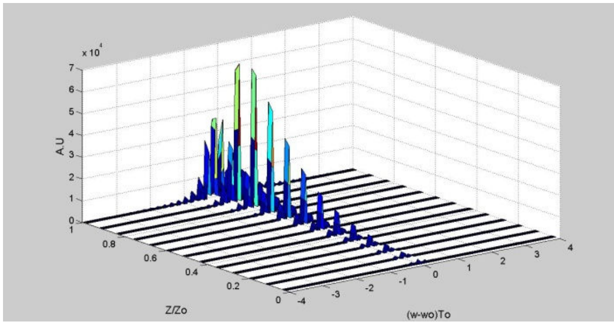

Figure 2 The evolution for RZ-train of solitons of width 0.2 ps down an EDFA of length 2 m, doping level 50×10^{23} atoms/m^3, and β_2 =-20×10^{-27} s^2.m^{-1} at λ=1.55 μm. The time slot between the pulses = 10 ps and the bit rate = 250 Gbps (frequency domain).

Figures 3 and 4 show what happens if the slot is smaller than the threshold time slot, the solitons of the train start to interact.

Figure 5 shows the evolution of a RZ-train of solitons with pulse width 0.3 ps and a time slot between pulses of 8 ps in an EDFA with a doping level 45×10^{23} atoms/m^3 at an input power 21.08 W and a dispersion length 4.5 m. Nonadiabatically, the maximum obtained output power, at these conditions, is 120 times the initial input power, leading to higher gain (20.79 dB). The time slot used makes the bit rate 208.3 Gbps for RZ transmission. The solitons of the train propagate down the fiber as fundamental solitons at z = 0 and then change into higher order solitons of N = 8 after z = L_D. As explained before, the threshold time slot is 8 ps, after which, the solitons of the train start to interact as amplitude modulation for slowly varying wave envelope.

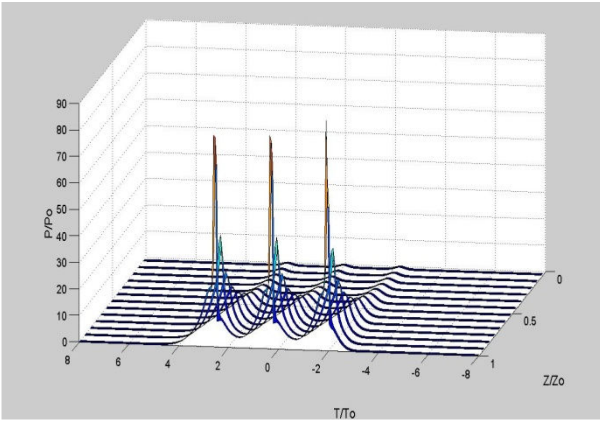

Figure 3 The evolution for RZ-train of solitons of width 0.2 ps down an EDFA of length 2 m, doping level 50×10^{23} atoms/m^3, and β_2 =-20×10^{-27} s^2.m^{-1} at λ=1.55 µm. The time slot between the pulses=9 ps (time domain -back view).

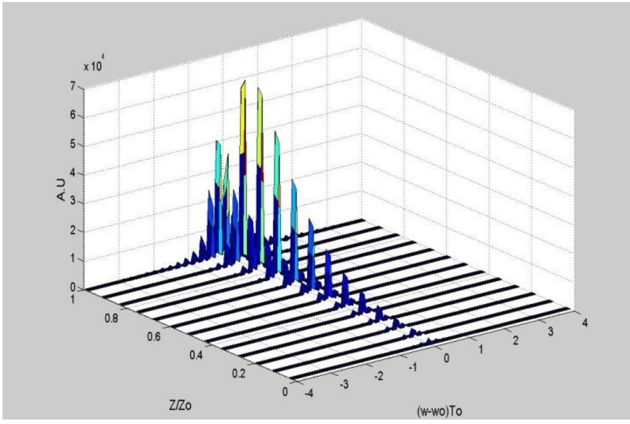

Figure 4 The evolution for RZ-train of solitons of width 0.2 ps down an EDFA of length 2 m, doping level 50×10^{23} atoms/m^3, and β_2 =-20×10^{-27} s^2.m^{-1} at λ=1.55 µm. The time slot between the pulses=9 ps (frequency domain).

Figure 6 shows the frequency domain which reveals that the trailing edge frequency components of the first soliton of the train interfere constructively with leading edge frequency components of the next one, this leads to a peak at the middle as explained in Fig. 2.

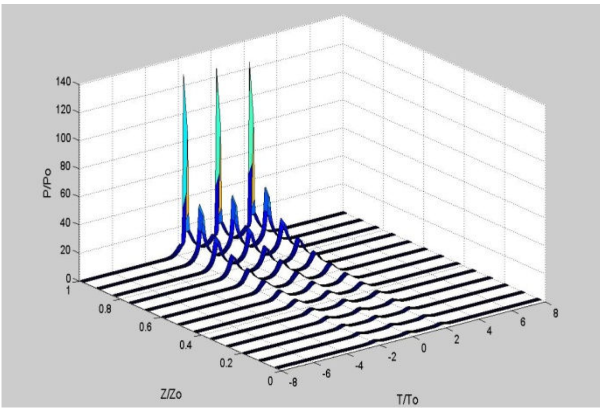

Figure 5 The evolution for RZ-train of solitons of width 0.3 ps down an EDFA of length 4.5 m, doping level 45×10^{23} atoms/m^3, and β_2 =-20×10^{-27} s^2.m^{-1} at λ=1.55 µm. The time slot between the pulses = 8 ps and the bit rate =208.3 Gbps (time domain).

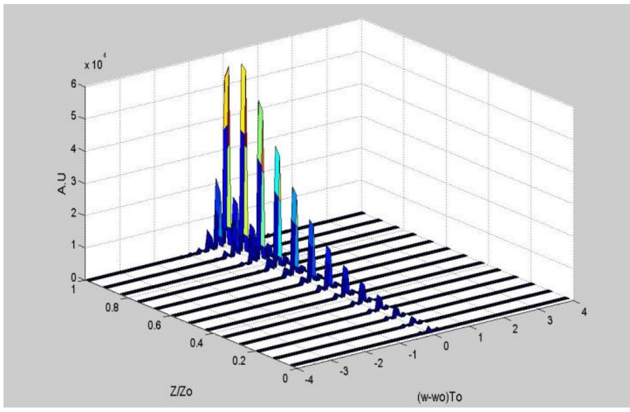

Figure 6 The evolution for RZ-train of solitons of width 0.3 ps down an EDFA of length 4.5 m, doping level 45×10^{23} atoms/m^3, and β_2 =-20×10^{-27} s^2.m^{-1} at λ=1.55 µm. The time slot between the pulses = 8 ps and the bit rate =208.3 Gbps (frequency domain).

By using the above conditions, the optimum situation is achieved to propagate the train of solitons with a maximum gain and a reasonable bit rate. Figure 7 shows what happens to the train of solitons of width 0.3 ps when the time slot between the solitons is changed to 7 ps. The situation is deviated away from the optimum conditions obtained in Fig. 5.

Figure 8 is the frequency domain obtained using a time slot 7 ps. Comparing Figs. 6 and 8, one notices that the frequency components due to the nonlinearity start to be generated at a higher bit rate if the time slot becomes 7 ps. This leads to amplitude modulation of higher order solitons.

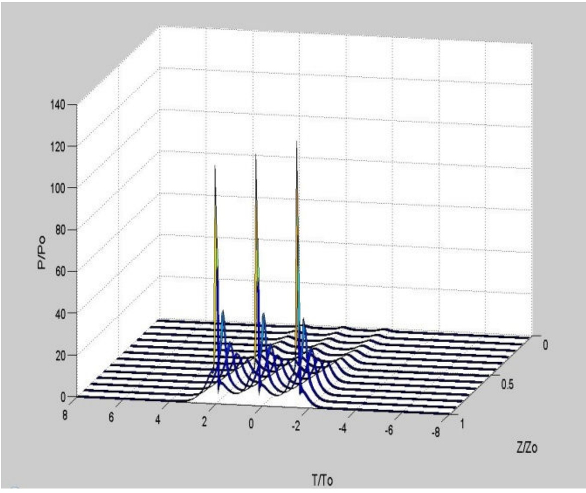

Figure 7 The evolution for RZ-train of solitons of width 0.3 ps, down an EDFA of length 4.5 m, doping level 45×10^{23} atoms/m^3 , and β_2 =-20×10^{-27} s^2.m^{-1} at λ=1.55 µm. The time slot between the pulses = 7 ps and the bit rate =208.3 Gbps (time domain- back view).

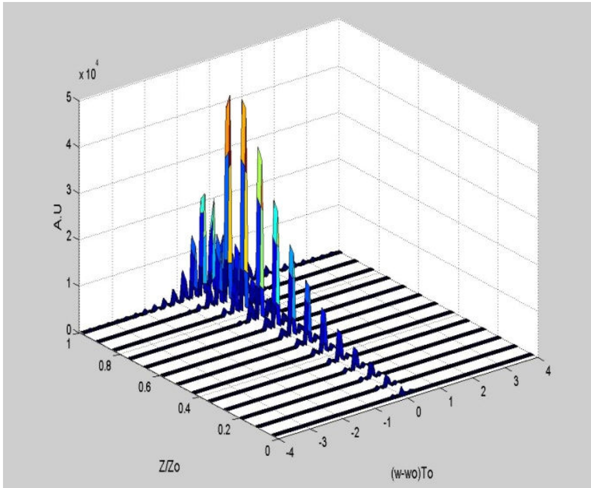

Figure 8 The evolution for RZ-train of solitons of width 0.3 ps, down an EDFA of length 4.5 m, doping level 45×10^{23} atoms/m^3 , and β_2 =-20×10^{-27} s^2.m^{-1} at λ=1.55 µm. The time slot between the pulses = 7 ps and the bit rate =208.3 Gbps (frequency domain).

The procedure is repeated for a pulse width 0.5 ps, a time slot 8 ps between pulses in an EDFA of a doping level 40×10^{23} atoms/m^3 at an input power 7.6 W and a dispersion length 12.5 m, Fig. 9. Again, nonadiabatically, the maximum obtained output power, at these conditions, is 95 times the initial input power, leading to high gain (19.77 dB). The time slot used makes the bit rate 125 Gbps for RZ transmission. The solitons of the train propagate down the fiber as fundamental solitons at z = 0 and then change into higher order solitons of N = 7 after z = 1L_D. The above conditions give less bit rate for RZ transmission. The study of this case confirms that the conditions used to obtaine Fig. 5 were the optimum. Figure 10 is the frequency domain of the above case.

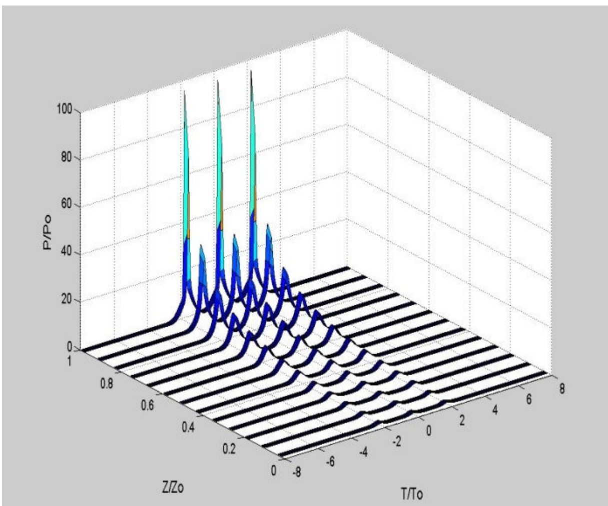

Figure 9 The evolution for RZ-train of solitons of width 0.5 ps down an EDFA of length 12.5 m, doping level 40×10^{23} atoms/m^3 , and β_2 =-20×10^{-27} s^2.m^{-1} at λ = 1.55 µm. The time slot between the pulses = 8 ps and bit rate =125 Gbps (time domain).

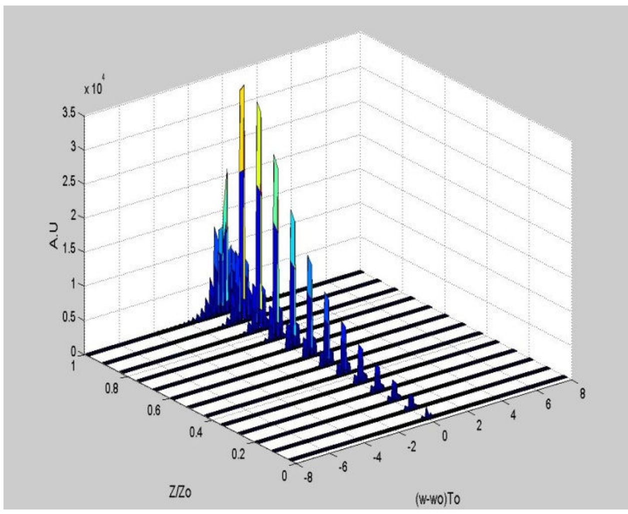

Figure 10 The evolution for RZ-train of solitons of width 0.5 ps down an EDFA of length 12.5 m, doping level 40×10^{23} atoms/m^3 , and β_2 =-20×10^{-27} s^2.m^{-1} at λ = 1.55 µm. The time slot between the pulses = 8 ps and bit rate =125 Gbps (frequency domain).

Figures 11 and 12 are the time and frequency solutions of a RZ-train of solitons of pulse width 0.5 ps and 7 ps between pulses. It is clear that the solitons of the train start to interact it is deviated away from the conditions in Fig 9. Also, the study of this case confirms that the case of Fig. 5 is the optimum.

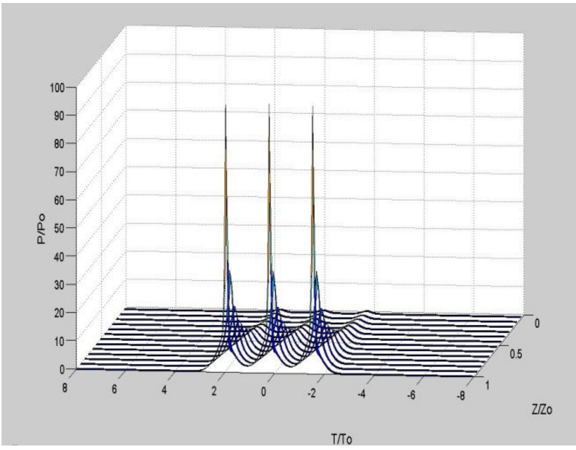

Figure 11 The evolution for RZ-train of solitons of width 0.5 ps, down an EDFA of length 12.5 m, doping level 40×10^{23} atoms/m^3, and β_2 =-20$\times$10^{-27} s^2.m^{-1} at λ = 1.55 µm. The time slot between the pulses = 7 ps (time domain-back view).

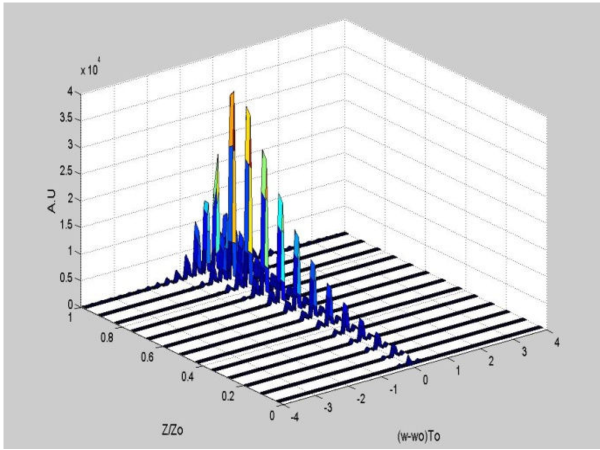

Figure 12 The evolution for RZ-train of solitons of width 0.5 ps, down an EDFA of length 12.5 m, doping level 40×10^{23} atoms/m^3, and β_2 =-20$\times$10^{-27} s^2.m^{-1} at λ = 1.55 µm. The time slot between the pulses = 7 ps (frequency domain).

Figure 13 shows a wonderful situation when the time slot between the solitons is the same as the width of the solitons which equal to 0.5 ps. The solitons of the train attract each other to form a single soliton of a higher amplification and order, where the energies of the outer solitons shed at the middle one. The formed soliton undergoes compression during its propagation downs the fiber [13] - [15]. Figure 14 shows the frequency domain of this case. It reveals that no frequency components due to nonlinearity of the medium are generated till a normalized distance z = 0.3. After that, these components start to generate at a slow rate.

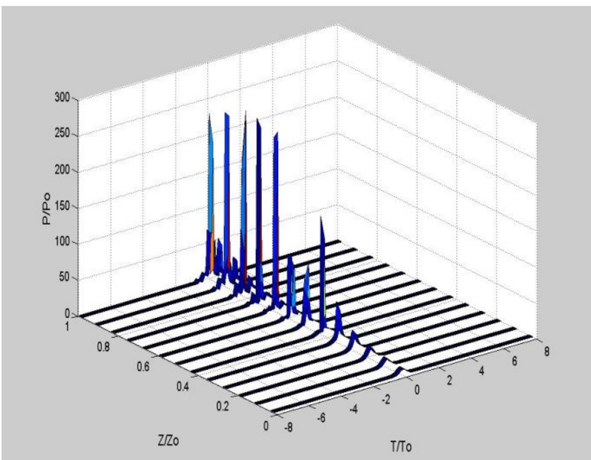

Figure 14 The evolution for RZ-train of solitons of width 0.5, down an EDFA of length 12.5 m, doping level 40×10^{23} atoms/m^3 , and β_2 =-20$\times$10^{-27} s^2.m^{-1} at λ = 1.55 µm. The time slot between the pulses =0.5 ps (frequency domain).

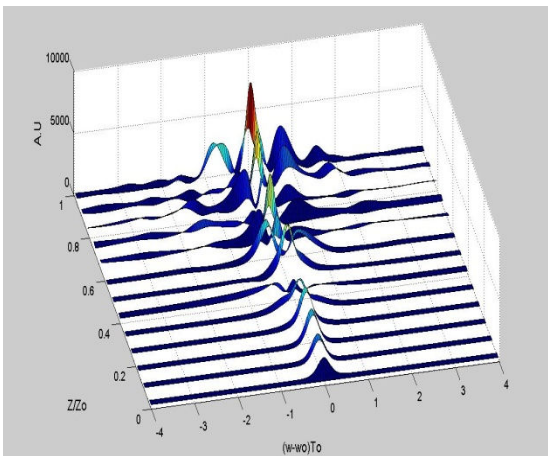

Figure 13 The evolution for RZ-train of solitons of width 0.5, down an EDFA of length 12.5 m, doping level 40×10^{23} atoms/m^3 , and β_2 =-20$\times$10^{-27} s^2.m^{-1} at λ = 1.55 µm. The time slot between the pulses =0.5 ps (time domain).

VI. Conclusion

The propagation of a RZ-train of solitons down an EDAD is studied. Trains of different solitons width, time slot and doping level are investigated to obtain the optimum conditions for maximum bit rate and gain. It is found that the train of soliton width 0.3 ps and a time slot 8 ps propagating in an EDAD of length 4.5 m, gives, nonadiabatically, a bit rate of 208.3 Gbps. At at a doping level 45×10^{23} atoms/m^3, the output power is 120 greater the initial power input, leading to a gain of 20.79 dB. It is shown that, if the optimum conditions are not realized, less gain and bit rate are obtained. A fascinating phenomenon is encountered, which is the transformation of the train of solitons into a single soliton of a high gain and order if the soliton width of the train is the same as the time slot. We suggest that this phenomenon can be used to obtain a soliton of very large gain and soliton compression.

VII. References

[1] Govind P. Agrawal, Fiber Optics Communication Systems, 3rd ed., Wiley-Interscience, New York, 2002.
[2] G. Keiser, Optical Fiber Communications, 3rd ed., Mc Grow-Hill, New York, 2000.
[3] A. Mecozzi, J. D. Moores, H. A. Haus and Y. Lai, "Soliton Transmission Control," Opt. Lett., vol. 16, p. 1841, 1991.
[4] A. Icsevgi and W. E Lamb, "Propagation of Light Pulses in a Laser Amplifier," Phy. Rev., vol. 185, no. 2, pp. 517-544, 1969.
[5] Govind P. Agrawal, Nonlinear Fiber Optics, 3rd ed., Academic Press, San Diego, California, 2001.
[6] V.E. Zakharov and A. B. Shabat, "Eigenvalues of the Scattering Problem for Two Separated Sech-Shaped Pulses," Sov. Phys. JETP, vol. 34, pp. 62-69, 1972.
[7] R.H. Hardin and F. D. Tappert, "Applications of the Split-Step Fourier Method to the Numerical Solution of Nonlinear and Variable Coefficient Wave Equation," SIAM Rev. Chronicle, vol. 15, p. 423, 1973.
[8] R.A. Fisher and W. K. Bischel, "Pulse Self-Dispersion in Self-Phase Modulation," Appl. Phys. Lett., vol. 23, p. 661, 1973.
[9] J. W. Cooley and J. W. Tukey, "An Algorithm for Machine Calculation for a Complex Fourier Series, Math. Comput., vol. 19, p. 297, 1965.
[10] M. H. Aly, M. S. Abouelwafa, M. Algammal, and H. K. Zoweil, "Erbium Doped Fiber Amplifier", Alex. Eng. Journal, vol. 35, no. 5, pp. D (71-77), 1996.
[11] Govind P. Agrawal, "Optical Pulse Propagation in Doped Fiber Amplifier," Phys. Rev. A, vol. 44, no. 11, pp.7493-7501, 1991.
[12] R.Gangwar, S. P. Singh and N. Singh, "Soliton Based Optical Communication", Progress in Electromagnetic Research, PIER, vol. 74, pp. 157-166, 2007.
[13] I.V. Mel'nikov, R.F. Nabiev and A.V. Nazarkin, "Coherent Amplification of Ultrashort solitons in Doped Fibers," Opt. Lett., vol. 15, no. 23, pp. 1348 – 1350, 1990.
[14] A. Mahalingam, K. Porsezian, MS. Mani, and A. Uthayakumar, " Propagation of Dispersion-Nonlinearity- managed Solitons in an inhomogeneous Erbium- doped Fiber System," J. Phys. A: Math. Theor., vol. 42, pp. 165101-165113, 2009.
[15] Yongshan C., and Fei L., "Evolutions of Matter-Wave bright Soliton with spatially modulated Nonlinearity," Chinese Opt. Lett., vol. 7, no. 15, pp. 834-837, 2009.