Exact Mathematical Model for Dual-Pump Distributed Fiber Raman Amplifier

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***Abstract*— This paper presents a derivation of an analytic expression for characterizing the evolution of signal and noise photon numbers along the active fiber of a forward-dual-pump Raman amplifier with equal signal and pump loss coefficients.**

***Index Terms*— Raman fiber amplifier, distributed fiber Raman amplifier, multipump Raman fiber amplifier.**

I. INTRODUCTION

The transmission of many wavelength division multiplexed (WDM) channels over high speed/long-haul systems can be limited by variety of impairments including the limited bandwidth of optical amplifiers and their noise performance [1]. One of the most promising technologies that can significantly overcome these impairments is optical amplification by stimulated Raman scattering (SRS) [2]. This is mainly attributed to the fact that Raman gain is non-resonant, broadband and reasonably spectrally flat over a wide wavelength range [2, 3]. As standard installed fiber has a reasonably strong Raman scattering response at standard telecommunication windows, it is also possible to use already installed fiber as a gain medium for SRS based distributed amplification. Since SRS process is non resonant, it has been shown that the bandwidth of Raman amplifiers can be further extended and arbitrary gain profile can be synthesized using multiple pumping wavelengths [4, 5]. Moreover, as a result of low noise performance of Raman amplifiers compared with conventional Erbium doped fiber amplifiers (EDFAs), average optical channel power required to achieve an acceptable signal to noise ratio along installed system fiber can be reduced considerably [6]. This reduction in channel power directly leads to the reduction in nonlinear impairments, thus improving the overall system performance.

Therefore, much research effort has been spent on designing efficient Raman amplifiers for long-haul WDM systems. There are several publications on analytical characterization of forward and backward pumped Raman amplifiers [7, 8, 9]. Dakss and Melman [7] have derived analytical expressions for signal and noise power evolution along the fiber of a forward pumped Raman amplifier. However, their results are applicable only for fibers with equal signal and pump loss coefficients. Chinn [8] has derived analytical expressions for signal and noise-figure evolution along the fiber of backward-pumped Raman amplifiers with un-equal signal and pump loss coefficients. Though, similar studies have not been done for forward-pumped Raman amplifiers.

In this paper, an analytical expression is derived for signal photon number function through distributed fiber Raman amplifier (DFRA). This DFRA is forward – dual pumped for equal signal and pump loss coefficients by using effective values for pump power, pump loss coefficient and pump wavelength.

II. THEORETICAL MODEL

1) SINGLE PUMP

Figure 1 shows a schematic diagram of the forward pumped fiber Raman amplifier. The signal propagation length of the fiber, z , is measured from the end of the pump.

Fig. 1. A schematic diagram of forward pumped Raman amplifier.

Without loss of generality, one considers the single pumping case. Extension of the results to dual pumping is qualtitatively considered in Sec. II.2.

In the case of single pumping scheme, the evolution of signal and noise photon number, $n(z)$, and pump photon number, $n_p(z)$, along the fiber can be characterized by the differential equations [7, 8,13]

$$\frac{dn(z)}{dz} = \gamma n_p(z)(n(z)+1) - \alpha_s n(z) \tag{1}$$

$$\frac{dn_p(z)}{dz} = -\gamma n_p(z)(n(z)+1) - \alpha_p n_p(z) \tag{2}$$

where γ is the Raman gain coefficient, α_s is the signal loss coefficient and α_p is the pump loss coefficient. The experimental evidence for the adequacy and validity of Eqs. (1) and (2) for describing Raman amplification in optical fibers is found in [11].

Under normal small signal operating conditions of Raman amplifiers, depletion of pump can be ignored with high accuracy, leading to the following expression for pump evolution along the fiber [7, 8]

$$n_p(z) = n_p(0)e^{-\alpha_p z} \tag{3}$$

By solving Eqs. (1) and (2), one could find an approximate solution for signal photon number as [13]

$$n(z) = G_s(z)n(0) + (\alpha_p - \alpha_s)\sqrt{H_p(z)H_s(z)}\Omega(z) + H_s(z)u_0 Ei(u_0) - H_p(z)u_0 Ei(u(z)) \quad (4)$$

where $H_p(z)=\exp(-\alpha_p z - u(z))$, $H_s(z)=\exp(-\alpha_s z - u(z))$ and Raman gain signal from the left of the fiber is $G_s(z)=\exp(u_0)H_s(z)$ and the term $\Omega(z)$ represent the scattering noise distribution

$$\Omega(z) = \frac{\alpha_p}{2}u_0 z^2 - u_0(c + \ln(u_0))z - \frac{u_0^{(1+\ln(u_0))}}{\alpha_p}\left(1 - \frac{u^2(z)}{u_0^2}\right) \quad (5)$$

The numerical solution of Eqs. (1) and under un-depleted pump approximation is used to test the validity of analytical solutions derived. Photon numbers are converted into optical powers by using the mapping $\gamma n_P(0) \rightarrow g.R_m P_P$, where $g.R_m$ is the modal Raman gain and P_P is the input pump power [8]. The following parameters are used in all the numerical calculations unless explicitly specified otherwise: $g.R_m = 0.67\ W^{-1}km^{-1}$, α_s=0.22 dB/km and P_p = 800 mW.

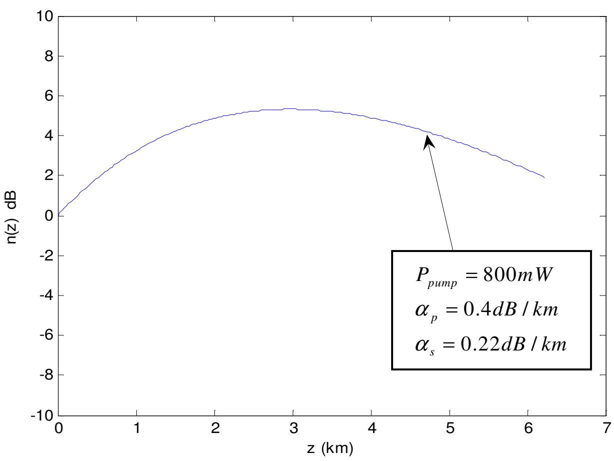

Fig. 2. Photon number, $n(z)$, against propagating distance (z).

The noise figure of Raman amplifier, NF(z), at a distance z from input can then be written as [8]

$$NF(z) = \frac{1 + 2(n(z) - G_s(z)n(0))}{G_s(z)} \quad (6)$$

Substitution for n(z) and $G_s(z)$ in Eq. (6) gives NF(z) as

$$NF(z) = [1 + 2(\alpha_p - \alpha_s)\sqrt{H_p(z)H_s(z)}\Omega(z) + 2(H_s(z)u_0 Ei(u_0) - H_p(z)u_0 Ei(u(z))]/[\exp(u_0)H_s(z)] \quad (7)$$

It is clear from Fig. 3 that NF(z) approaches 2.0 (i.e. 3 dB) for large Raman gain, which amounts to large pump powers (i.e. large u_0) and long amplifiers (i.e. $z \approx 1/\alpha_P$).

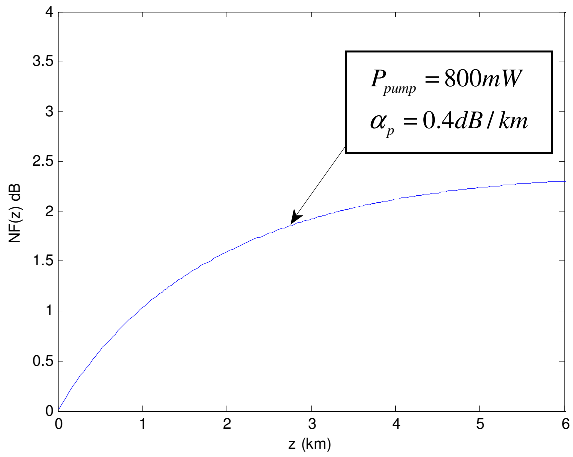

Fig. 4. Noise figure against propagating distance for 0.4 dB/km pump loss coefficient.

2) MULTIPLE PUMP

The described power series method can be easily extended to derive an approximate analytical method for multiple pumping case with same pumping wavelengths. Assume that there are *M* different laser beams pumping the DFRA, then, Eqs. (1) and (2) need to be modified to

$$\frac{dn(z)}{dz} = \left(\sum_{k=1}^{M}\gamma_k n_{pk}(z)\right)(n(z)+1) - \alpha_s n(z), i = 1,2,3,...M \quad (8)$$

$$\frac{dn_{pi}(z)}{dz} = -\gamma_i n_{pi}(z)\left(\sum_{\substack{k=1\\k\neq i}}^{M} n_{pk}(z) + n(z) + 1\right) - \alpha_{pi} n_{pi}(z), i = 1,2,...M \quad (9)$$

where $\gamma_k : k = 1, \cdots, M$ are the Raman gain coefficients and $\alpha_{Pk} : k = 1, \cdots, M$ are the pump loss coefficient for M pump wavelengths with photon numbers, $n_{Pk}(z)$. All the other parameters assume the same meaning as in Eqs. (1) and (2). Using an approximation similar to impulsive pump depletion approximation [12] for forward propagating pumps and seeking a solution for signal photon number, one could obtain an approximate analytical solution for multiple pump wavelengths. Such solutions are especially useful for seeding more accurate numerical methods for flattening gain spectrum of Raman amplifiers [10]. The exact details of the above two approaches are beyond the scope of this paper and published elsewhere.

3) DUAL FORWARD PUMP

In this section, a trial is done to find a solution for the dual forward pump DFRA giving the signal photon number and the pumps photon number by exact solution for the previous differential equation when M=2. Some assumptions are made: the loss coefficients of the signal and both pumps are equal, the initial number of photons for the two pumps are equal and the Raman propagation coefficients are also equal. Therefore, one can get

$$\frac{dn(z)}{dz} = \left(\sum_{k=1}^{M=2} \gamma_k n_{pk}(z)\right)(n(z)+1) - \alpha_s n(z) \tag{10}$$

$$\frac{dn_{pi}(z)}{dz} = -\gamma_i n_{pi}(z)\left(\sum_{\substack{k=1\\k\neq i}}^{M=2} n_{pk}(z) + n(z) + 1\right) - \alpha_{pi} n_{pi}(z),$$
$$i = 1,2 \tag{11}$$

Under normal small signal operating conditions of Raman amplifiers, depletion of pump can be ignored with high accuracy, leading to the following expression for pump evolution along the fiber $n_{pi}(z)$

$$n_{pi}(z) = n_{pi}(0)\exp[-\int_0^z (\gamma_i \sum_{\substack{k=1\\k\neq i}}^{2} n_{pk}(z) + \alpha_{pi})dz] \tag{12}$$

For the first and second pumps, one could find the formula of each one as

$$n_{p1}(z) = n_{p1}(0)\exp(-\alpha_{p1} z),$$
$$n_{p2}(z) = n_{p2}(0)\exp[\frac{\gamma_2 n_{p1}(0)}{\alpha_{p1}} (\exp(-\alpha_{p1} z - 1) - \alpha_{p2} z)] \tag{13}$$

Now, for the evolution of photon number for signal, the solution is found by the integrating factor μ, defined by

$$\mu = \exp[\int [\alpha_s - \sum_{\substack{k=1\\k\neq i}}^{M=2} npk(z)]dz] \tag{14}$$

Integrating, one can write

$$\mu = \exp\{(\alpha_s z + \frac{\gamma_1 n_{p1}(0)}{\alpha_{p1}}\exp(-\alpha_{p1} Z) + \frac{n_{p2}(0)}{n_{p1}(0)}\exp(\frac{\gamma_2 n_{p1}(0)}{\alpha_{p2}}\exp(-\alpha_{p2} Z))\} \tag{15}$$

The solution of the integral factor may be complicated, so, for simplicity, one assumes that $\gamma_1 = \gamma_2 = \gamma$, $\alpha_s = \alpha_{p1} = \alpha_{p2} = \alpha$ and $n_{p1}(0) = n_{p2}(0) = n_p(0)$. This results in n(z) in the form

$$n(z) = \frac{1}{\mu}\int \mu \sum_{k=1}^{M=2} \gamma_k n_{pk}(z) + n(0) \tag{16}$$

$$n(z) = \frac{1}{\mu}\int [\mu\gamma n_{p1}(z) + \mu\gamma n_{p2}(z)]dz + n(0) \tag{17}$$

$$n(z) = \frac{1}{\mu}[\gamma[\int \mu n_{p1}(z)dz + \int \mu n_{p2}(z)dz]] + n(0)$$
$$\int \mu n_{p1}(z) = -\frac{n_p(0)}{\alpha}\{\frac{h(z)}{6} - \frac{65\ln(h(z)-1)}{24} + \frac{1437 - 538R + 307R^2}{24(9R^2 - 26R + 23)\ln(h(z)-R)}\}, \tag{18}$$

$$R = \sqrt{3h^2(z) - 13h^3(z) + 23(h(z) - 25)}$$

The integrations in Eq. (18) are solved as

$$\int \mu n_{p1}(z) = -\frac{n_p(0)}{\alpha}\{\frac{h(z)}{6} - \frac{65\ln(h(z)-1)}{24} + \frac{1437 - 538R + 307R^2}{24(9R^2 - 26R + 23)\ln(h(z)-R)}\},$$
$$R = \sqrt{3h^2(z) - 13h^3(z) + 23(h(z) - 25)} \tag{19}$$

where $h(z) = e^{A\exp(-\alpha z)}$ and $a = \gamma n_p(0)/\alpha$

$$\int \mu n_{p2}(z) =$$
$$\frac{-n_p(0)}{\alpha e^{-A}}\{\frac{h^2(z)}{12} + \frac{14\mathrm{h(z)}}{9} - \frac{65}{24\ln(\mathrm{h(z)}-1)} + 1/((7675\text{-}2750\mathrm{R} + 2377\mathrm{R}^2)/(9\mathrm{R}^2\text{-}26\mathrm{R}+23)\ln(\mathrm{h(z)}\text{-}\mathrm{R}), \tag{20}$$
$$\mathrm{R} = \sqrt{(3\mathrm{h(z)}^3\text{-}13\mathrm{h(z)}^2 + 23\mathrm{h(z)}\text{-}25)}$$

III. NUMERICAL SIMULATION

Based on the described model and using MATLAB ver. 7, calculations are performed to get the number signal photons. Figure 3 shows that the number of photon increases with fiber length when two identical forward pumps are used. This relation can be controlled using multipump or using different attenuation coefficients, different Raman gain and different pump power for the dual forward DFRA.

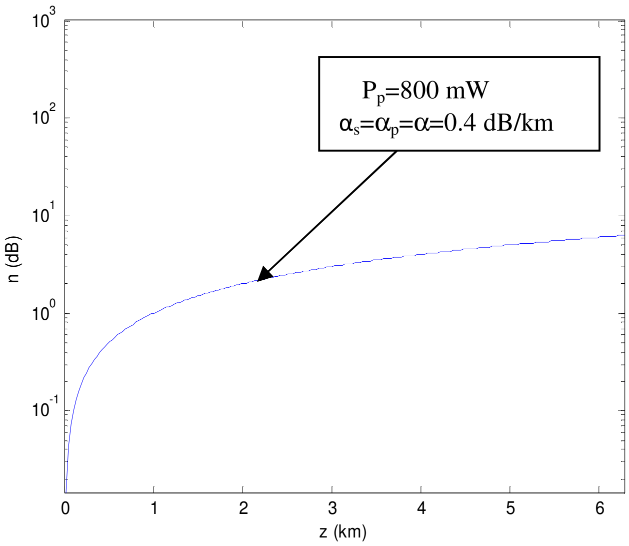

Fig.3 Relation between number of signal photons and propagating distance by using equally pumped signals.

IV. CONCLOSION

In this paper, a mathematical expression is derived for the signal photon number for a dual pump DFRA. The obtained results show a better performance (without decay) than the single pump. This performance can be controlled using different parameters for the two forward pumps.

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