

Dispersion Compensation Using Far Off Resonance Chirped AFBG: Comparison between Two Chirping Techniques

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Abstract- A dispersion compensator is designed using far off resonance a chirped apodized fiber Bragg grating (AFBG) in transmission. A comparison for the results obtained by two chirping techniques has been accomplished using different apodization profiles at linearly chirped apodized far off resonance fiber Bragg gratings (FBGs). Chirping is made using the two beam interference fringe spacing and the ultraviolet (UV) phase mask techniques. Better results (higher bandwidths) are obtained by the UV phase mask technique, especially at the positive hyperbolic tangent apodization profile.

Keywords: Apodized chirped fiber Bragg grating, off resonance grating, dispersion and dispersion slope, interferometer, two beam technique, ultraviolet phase mask technique.

1. Introduction

In the last years, optical transmission at 1550 nm has become wide spread, since the introduction of erbium doped fiber amplifiers has brought the idea of high capacity optical networks operating around this wavelength. One of the ultimate transmission limitations in systems operating at 1550 nm is the chromatic dispersion. Much effort has been invested in obtaining dispersion compensation schemes for standard fibers already installed. Three techniques stand out over the rest, attracting the most attention: dispersion compensating fiber (DCF), optical phase conjugation (OPC) and fiber Bragg gratings. Despite, the advantage that it can compensate over the complete erbium bandwidth, DCF length and thus, applicability is limited due to its high germanium content and small core diameter, which lead to high optical nonlinearity and losses, while OPC is associated to a net shift of the central frequency of the signal, producing a significant waste of bandwidth. On the other hand, fiber gratings have attained a prominent role in optical communications, not only as dispersion compensation devices, but being susceptible of

an amazing number of applications, including band pass filtering, amplifier gain flattening, temperature and pressure sensing, frequency stabilization in lasers [1].

The FBG has emerged in the last years as a rapidly growing technology. Of a special interest are chirped fiber Bragg gratings (CFBGs), given their utility to modify the amplitude and phase of optical signals. In the early nineties, soon after they were firstly theoretically described and fabricated, straight forward applications for dense wavelength division multiplexing and dispersion compensation were suggested and implemented [1]. The most widespread methods for CFBG fabrication consist of adaptations of the two basic techniques that are used for the fabrication of uniform FBGs; the two beam interferometric technique and the UV phase mask technique. Both are based on the irradiation of patterns of UV light upon photosensitive optical fiber. The first one implies a complex and expensive writing setup, which has to ensure the stability of the interfering beams and feed the writing motors back with the information gathered by the interferometric position monitoring system. The UV phase mask technique, on the other hand, is a far more simple, stable and cost effective technique for fabrication [1].

For the improvement of the dispersion compensation characteristics of the CFBG, the apodization is applied for which the modulation associated with the presence of side lobes is eliminated [2]. For dispersion compensation in transmission, the apodized FBGs (AFBGs) can be spliced directly into the transmission link. This will avoid the losses due to bulk optical devices such as the circulator if the grating is operated in transmission, the interaction between the signal optical field and the grating is much weaker, and hence imperfections in the grating do not become impressed upon the signal field [3]. A previous work on the far off resonance

gratings has been investigated by J. E. Sipe et al. [4]. They gave an approximate analytic expression for the wave number of light, proving that operation at low wavelengths compared to the Bragg resonance wavelength of the grating results in lower values of the quadratic dispersion produced by grating compared to that produced at the resonance wavelength.

This paper discusses the use of linearly chirped apodized far off resonance FBGs, operated in transmission, as dispersion compensators using a far more flexible and controllable approach to chirped grating fabrication relying on the two beam interference fringe spacing technique and the double UV exposure phase mask technique. Different apodization profiles and design parameters are studied showing their effects on the performance of the compensator for optimum dispersion compensator design.

2. Theory

We use the expressions for the local wave number, $k(z)$, obtained by J. E. Sipe [4]

$$k(z) = \bar{k} \left[1 + a_0(z) - \frac{1}{2} a_0^2(z) + \sum_{m=1}^{\infty} a_m^2(z) \frac{\bar{k}^2}{(m^2 K^2 - \bar{k}^2)} + \dots \right] \quad (1)$$

However, the wave number of the resulting plane wave solution is not \bar{k} , associated with the reference refractive index \bar{n} , but is changed due to the off resonant grating. This shift is given by (1). We will drop all higher order resonances. Also, ignoring the DC contribution to the grating's refractive index profile, we then find that the wave number shift for a grating with unit strength is given by

$$k(z) \equiv \frac{\bar{k}^{-2}}{(K^2 - \bar{k}^{-2})} \quad (2)$$

where

$$\bar{k} = \omega \bar{n} / c$$

and

$$K = 2\pi / \Lambda$$

The following is the mathematical model of the AFBG in transmission. The chirping effect on the grating performance as a dispersion compensator for different profiles is also studied. In order to model the operation of an AFBG in transmission, the "**effective medium method**" is used [5]. The refractive index, at any distance, z , of the AFBG along its length, L , is

$$n(z) = n_o \left[1 + \sigma(z) + 2h(z) \cos\left(\frac{2\pi}{\Lambda} z + 2\phi(z)\right) \right] \quad (3)$$

where, n_o is the background refractive index, Λ is the nominal Bragg grating pitch, σ is the variation in the average refractive index, h is the modulation amplitude (> 0) and ϕ is the grating phase.

Using (3) in Maxwell's equations and applying perturbation techniques, the grating can be represented as an "**effective medium**" with an effective refractive index, n_{eff} , an effective dielectric permittivity, ϵ , an effective magnetic permeability, μ , and an effective local impedance, Z .

A local detuning of a grating, $\delta(z, \Delta)$, is defined as [5]

$$\delta(z, \Delta) = \Delta + \frac{\pi}{\Lambda} \sigma(z) - \frac{d}{dz} \phi(z) \quad (4)$$

where

$$\Delta(k) = kn_o - \frac{\pi}{\Lambda} \quad (5)$$

with k the local wave number obtained by J. E. Sipe [4] of the incident light given in (2).

Both medium permittivity and permeability as functions of z are obtained as

$$\epsilon(z, \Delta) = \delta(z, \Delta) + \kappa(z) \quad (6)$$

and

$$\mu(z, \Delta) = \delta(z, \Delta) - \kappa(z) \quad (7)$$

where

$$\kappa(z) = h(z) \frac{\pi}{\Lambda} \quad (8)$$

Therefore, the effective refractive index, $n_{\text{eff}}(z, \Delta)$, is

$$n_{\text{eff}}(z, \Delta) = \sqrt{\epsilon\mu} = \sqrt{\delta^2(z, \Delta) - \kappa^2(z)} \quad (9)$$

and the effective local impedance, $Z(z, \Delta)$, is

$$Z^2(z, \Delta) = \frac{\mu}{\epsilon} = \frac{\delta(z, \Delta) - \kappa(z)}{\delta(z, \Delta) + \kappa(z)} \quad (10)$$

The dispersion, $d(k)$, and the dispersion slope, $d'(k)$, of the grating are given, respectively, by [5]

$$d(k) = -\frac{2\pi n_o^2}{\lambda^2 c} \frac{d^2}{d\Delta^2} \arg\{t(\Delta)\} \quad \text{ps/nm.} \quad (11)$$

and

$$d'(k) = \left(\frac{2\pi n_o}{\lambda}\right)^2 \frac{n_o}{c} \frac{d^3}{d\Delta^3} \arg\{t(\Delta)\} \quad \text{ps/nm}^2. \quad (12)$$

where the transmission coefficient has an amplitude, $t(\Delta)$, given by [1]

$$t(\Delta) = 4Z(0, \Delta) \left| \frac{Z(L, \Delta)}{Z(0, \Delta)} \right|^{1/2} \cdot \{1/[e^{i\varphi}(Z(0, \Delta) + 1) \cdot (Z(L, \Delta) + 1) - e^{-i\varphi}(Z(0, \Delta) - 1)(Z(L, \Delta) - 1)]\} \quad (13)$$

and a phase, $\varphi(z)$, given by [1]

$$\varphi(z) = \int_0^L n_{eff}(z, \Delta) dz \quad (14)$$

When AFBGs are used, the grating profile, $h(z)$, grows and decays continuously from and to zero value; therefore $\kappa(0) = \kappa(L) = 0$. Hence,

$$Z(0, \Delta) = Z(L, \Delta) = 1 \quad (15)$$

Therefore, the transmission coefficient can be rewritten in the form

$$t(\Delta) = e^{-i\varphi} \quad (16)$$

Well away from the reflection band edges, the argument of the transmission coefficient, $\arg\{t\}$, can be simplified to

$$\arg\{t(\Delta)\} = \text{sgn}(\Delta)\varphi(\Delta) \quad (17)$$

at its lowest order because n_{eff} is real, where $\text{sgn}(\Delta) = +1$ for $\Delta > 0$ and $= -1$ for $\Delta < 0$.

This result is precise for AFBGs (i.e. $h(0) = h(L) = 0$ and is continuous for all z). The effect of apodization on the grating leads to express the dispersion and the dispersion slope of the compensator, respectively, as [1]

$$d(k) = -\frac{2\pi n_o^2}{\lambda^2 c} \int_0^L \frac{\kappa^2(z) dz}{[\delta^2(z, \Delta) - \kappa^2(z)]^{3/2}} \quad (18)$$

and

$$d'(k) = \left(\frac{2\pi n_o}{\lambda^2} \right)^2 \frac{3n_o}{c} \int_0^L \frac{\kappa^2(z) \delta(z, \Delta) dz}{[\delta^2(z, \Delta) - \kappa^2(z)]^{5/2}} \quad (19)$$

One of the most interesting Bragg grating structures with immediate applications in telecommunications is the chirped fiber Bragg grating (CFBG). This grating has a monotonically varying period [5]. A CFBG is considered, with constant variations in the average refractive index for the length of the grating ($\sigma(z) = 0$). The grating phase, $\phi(z)$ is assumed to change linearly with the grating length as

$$\phi(z) = \phi_0 + az \quad (20)$$

where ϕ_0 is the starting phase and a is the linear change in the grating phase. The change of phase along the grating will result in "a".

$$\frac{d}{dz} \phi(z) = a \quad (21)$$

The fringe spacing $\Lambda(z)$ along the fiber axis may be obtained from the geometry of the interfering beams. Here, in this paper, a single cylindrical

lens is used in just one arm of the interferometer. It clearly indicates an almost linear variation of the reflected wavelength with distance along the grating. The fringe spacing is given by [6]

$$\Lambda(z) = \frac{\Lambda_{pm}}{2} \left[1 - z \left(\frac{\alpha}{f - r} \right) \left(1 - \frac{\lambda_w^2}{\Lambda_{pm}^2} \right)^{-1/2} \right] \quad (22)$$

where Λ_{pm} is the phase period, λ_w is the writing beam wavelength, f focal length of the lens, r is the distance between the lens and the mask, α is the tilting angle of the grating with respect to the mask and z is the distance along the fiber. Equation (21) shows a linear variation of the grating period with distance z .

Here, in this paper, the linearly chirped apodized far off resonance FBG is considered to be chirped first by the two beam technique and then by using the UV phase mask technique. By definition, the grating wavelength is a function of the position z for both chirping techniques as [6] and [7]

$$\lambda_B = 2n_{eff} \Lambda(z) \quad (23)$$

Hence,

$$\delta(z, \Delta) = \left(\frac{\bar{k}^2}{K^2 - \bar{k}^2} \right) n_o - \frac{\pi}{\Lambda(z)} - a, \quad (\text{nm}^{-1}) \quad (24)$$

and

$$\kappa(z) = h(z) \frac{\pi}{\Lambda(z)}. \quad (\text{m}^{-1}) \quad (25)$$

The effect of the apodization profiles with the quadratic dispersion parameter for off resonance grating is investigated using eight different profiles: positive tanh, Cauchy, sine, Gauss, Hamming, sinc, raised sine and Blackman. These are respectively displayed in Fig. 1. The apodization functions correspond to well known window functions employed in filter design to suppress side lobes in the rejected band [2].

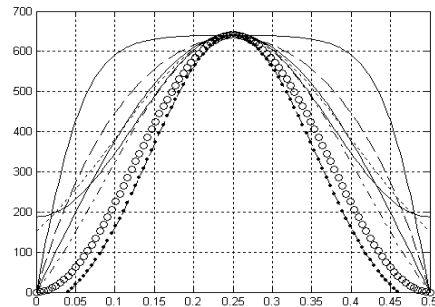


Fig. 1 Apodization profiles versus Bragg grating length.

The values of the parameter values which provide an efficient way to control the characteristics of the functions are chosen such that all the profiles have similar characteristics. This is composed with a flat region at the grating center and a constant slope decaying characteristics towards the grating edges.

The compensator performance can be measured using its bandwidth, $\Delta\lambda$, given by [5]

$$\Delta\lambda = \frac{d(k)}{d'(k)} \quad (26)$$

This represents the maximum bandwidth of a signal which can be compensated without the off resonance AFBG dispersion slope affecting the signal [5].

Another measure for the performance of the compensator is its eye closure penalty which, for chirp free signals, is given by [5]

$$P(\text{dB}) = 10 \log \left(\frac{1}{1 - \gamma \ell^2} \right) \quad (27)$$

where ℓ is the transmission link length and γ is directly proportional to the level of intersymbol interference and is given by [5]

$$\gamma = \left(\frac{\pi}{4} \right)^2 \frac{\lambda^4 D^2 B^4}{c^2} \quad (28)$$

where D is the fiber dispersion and B is the bit rate [5]. For a dispersion compensator, the dispersion, d , of the compensator is set to negate the impact of fiber dispersion. That is,

$$d(k) = -D\ell \quad (29)$$

Using (28), together with (18) and (27) in (26), yields the eye closure penalty for a chirped off resonance AFBG compensator as

$$P(\text{dB}) = 10. \log \left(\frac{1}{1 - \left(\frac{\pi}{4} \right)^2 \frac{\lambda^4 B^4}{c^2} d^2(k)} \right) \quad (30)$$

3. Numerical Results and Discussion

A computer simulation is performed for the described model for the off resonance AFBG at different apodization profiles. The two chirping techniques are considered at each apodization profile. The compensator bandwidth and its eye closure penalty are investigated and the obtained results are displayed in Figs. 2 through 4 to select profile that gives the best performance including higher the bandwidth at zero dB eye closure penalty.

Results displayed in Fig. 2 show that a chirped off resonance AFBG compensator to cancel the impact of fiber dispersion can be obtained using sine profile with a bandwidth of 4.19 nm and exact zero dB eye closure penalty at a grating length $L = 0.0275$ m using the two beam chirping technique.

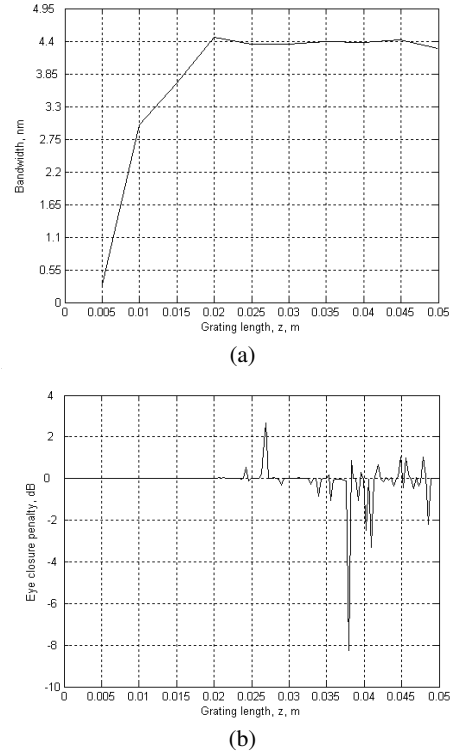
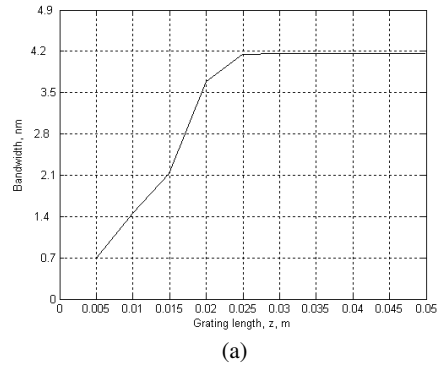
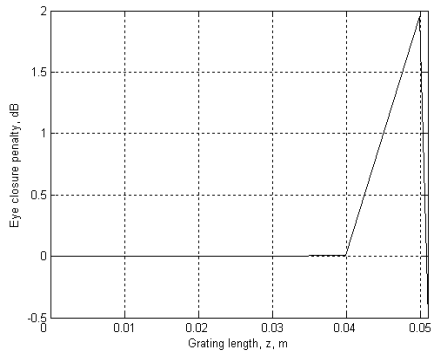


Fig. 2 Dispersion compensator bandwidth (a) and eye closure penalty (b) for the sine profile (Two Beam).

For the same profile, the UV phase mask technique achieves a noticeable improvement in Fig. 3 in both the bandwidth and the grating length as the widest bandwidth of 4.4 nm and exact zero dB eye closure penalty are gained at grating length $L = 0.02$ m.

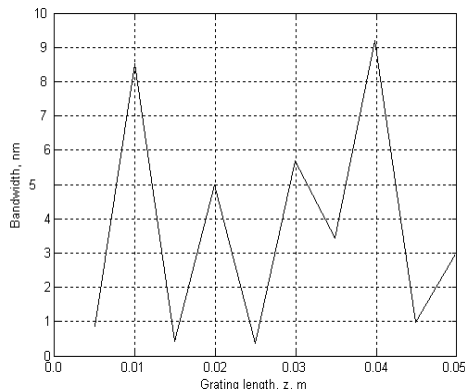




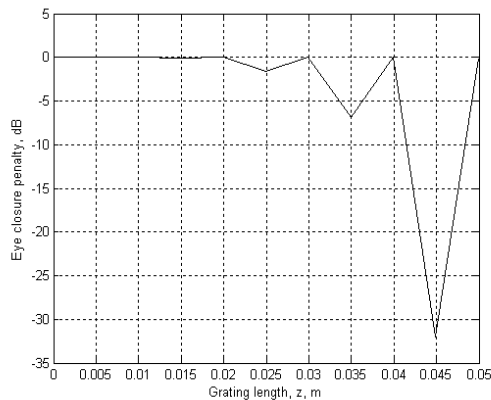
(b)

Fig. 3 Dispersion compensator bandwidth (a) and eye closure penalty (b) for the sine profile (UV Phase Mask).

The corresponding results for the tanh apodization profile are illustrated, respectively, in Figs. 4 and 5, showing a higher bandwidth at zero eye closure penalties.

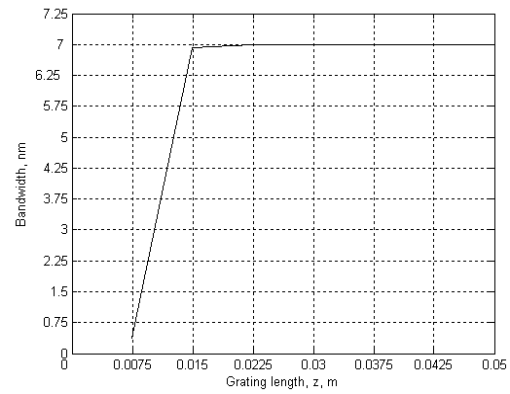


(a)

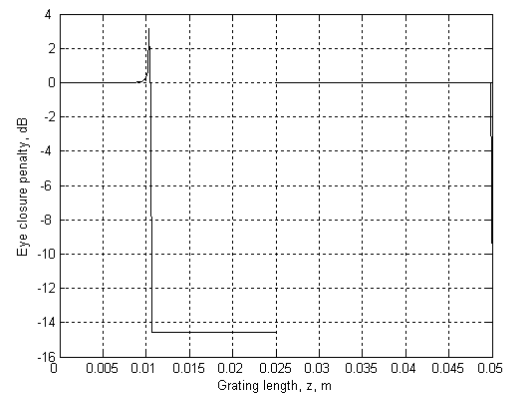


(b)

Fig. 4 Dispersion compensator bandwidth (a) and eye closure penalty (b) for the tanh profile (Two Beam).



(a)



(b)

Fig. 5 Dispersion compensator bandwidth (a) and eye closure penalty (b) for the tanh profile (UV Phase Mask).

The same procedure is repeated for the other apodization profiles. Similar results to previously displayed ones are obtained. The maximum bandwidth at exactly zero dB eye closure penalty is investigated. A complete comparison for the eight different apodization profiles bandwidth (at zero dB eye closure penalty) is illustrated in Table I. The positive tanh profile clearly shows the best compensator performance (highest bandwidth), especially when the UV phase mask technique is used.

4. Conclusion

The characteristics of the off resonance apodized linearly chirped fiber gratings written through the two beam fringe spacing technique and UV phase mask technique have been studied systematically. It is shown that optimum apodization profiles have a flat center region and apodized edges with continuously decreasing slopes. From the considered profiles, the **positive hyperbolic tangent profile** results in an overall superior performance.

Apodization Profile	Two Beam Technique		UV Phase Mask Technique	
	Compensator Bandwidth (nm)	Grating Length (m)	Compensator Bandwidth (nm)	Grating Length (m)
Sine	4.19	0.0275	4.40	0.020
Raised Sine	4.48	0.0255	4.95	0.028
Sinc	3.40	0.045	4.00	0.030
Positive tanh	7.00	0.025	9.10	0.039
Blackman	2.39	0.025	2.70	0.04
Gauss	4.00	0.025	4.45	0.035
Hamming	3.75	0.03	3.95	0.025
Cauchy	4.95	0.0285	7.10	0.025

Table I Dispersion compensator bandwidth and grating length for a chirped FBG at different apodization profiles, using the two beam and UV phase mask techniques.

As compared with the other apodization profiles, it provides highly linear time delay characteristics with minimum reduction in linear dispersion. This results in compensated fiber links of a maximum length and minimum transmission penalty, which has both optimum design requirements the maximum bandwidth of 9 nm and exactly zero dB eye closure penalty at a grating length of 0.004 m which is a very reasonable practical value for a grating length. Also, the UV phase mask technique has better results than that of the two beam technique for each profile; it is very distinguishable in the positive tanh apodization profile.

5. References

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