25th NATIONAL RADIO SCIENCE CONFERENCE (NRSC 2008)

March 18-20, 2008, Faculty of Engineering, Tanta Univ., Egypt

Dispersion Compensation Using Linearly Chirped Polymer Fiber Bragg Grating

Ishak l. Naguib[#](issackands@yahoo.com), Moustafa H. Aly*(drmosaly@gmail.com) and Ali M. Okaz[#] (Ali_okaz@yahoo.com)

 # Faculty of Engineering, University of Alexandria, Alexandria, Egypt.

*** College of Engineering and Technology, Arab Academy for Science &Technology & Maritime Transport, Alexandria, Egypt, Member of the Optical Society of America (OSA).**

Abstract A polymer fiber Bragg grating is modeled and investigated as a dispersion compensator using linearly chirped grating at different strain values. The obtained dispersion for the grating cancels the fiber dispersion, resulting in dispersion compensation in the fiber channel. The study includes the effect of the modulation depth on the grating reflectivity and the apodization profile effect on the maximum allowed length of the fiber Bragg grating.

Keywords**:** linearly chirped fiber Bragg grating, polymer optical fiber (POF), polymethylmethacrylate (PMMA), index modulation depth, full width half maximum bandwidth (FWHM).

1. Introduction

Silica-based optical fiber Bragg gratings are important in optical signal processing, but they have the disadvantage of being difficult to tune. Glass is relatively stiff material with large Young's modulus (~7.245×10^{10} N/m^2) and a small thermal expansion coefficient, so that the Bragg wavelength of silica gratings cannot be tuned easily by their mechanical or thermal methods. Now, researchers have developed polymer optical fiber (POF) Bragg gratings that can fulfill most of the functions of their silica counterparts, plus greater tunability [1]. In the early 1980s, scientists developed a POF based mainly on polymethylmethacrylate (PMMA). PMMA fibers have attenuation around 150 dB/km, freeze at temperatures below -20 ^{o}C and soften at temperatures higher than 120 ^{o}C. Thus, their applications are confined to a protected environment. The nonlinear coefficients of POFs are many times larger that of silica. In addition, the Young's modulus of a POF is about 30 times less than that of silica which translates to a mechanical tunability for polymer-based Bragg gratings that is about 30 times larger than a Bragg grating in silica fibers. Similarly, the thermal strain coefficient in a POF is 1.48 pm/$\mu\varepsilon$, compared with 1.15 pm/$\mu\varepsilon$ for silica. Linearly chirped POFs are used for dispersion compensation, wavelength division multiplexing (WDM), pulse shaping filters, distributed feedback lasers, and sensors [2].

The chromatic dispersion of optical fibers degrades the transmission characteristics of optical communication systems. This can be avoided in conventional transmission systems by setting the signal carrier wavelength to the zero dispersion wavelength. However, the signal carrier wavelength sometimes has to be set in a region far from the zero dispersion wavelength. In such case, a dispersion equalizer is needed [3]. Chirped Bragg gratings play an important role in dispersion compensation. One can introduce a chirp into a grating by using temperature gradient, strain gradient, a refractive index gradient or by varying the period of the grating. The advantages of these techniques are: i) The possibility of using a phase mask or interferometric setup, ii) The capability of producing different chirp profiles, and iii) The possibility of tuning the chirped grating. The chirping techniques based on a strain gradient along the fiber offer a good controllability. Linearly chirped POFs introduce a very large range of dispersion (from 2,400 ps/nm at a strain 0.02% to 110 ps/nm at a strain 0.4%) [1]. This range introduces the ability to compensate the same dispersion range produced in the fiber with opposite sign. In this paper, a study is introduced for some parameters of linearly chirped POF Bragg grating which can affect its performance as a dispersion compensator.

2. Operational Principle

We now introduce the analysis of Bragg gratings fabricated in etched fibers to produce different kinds of chirp profiles [1]. Consider a uniform grating in an optical fiber of radius r. If the fiber is subjected to a tension F, both the grating pitch, Λ, and the change in the effective refractive index, n_{eff}, cause a change in the Bragg wavelength, λ_{Bo}, according to [4]

$$\frac{\Delta\lambda_B(z)}{\lambda_{Bo}} = (1-\mathrm{P}_e)\frac{F}{\pi E r^2} \quad , \tag{1}$$

where E is Young's modulus of the polymer material (3.1×10^9 for PMMA) and P_e is the effective photoelastic constant of the fiber (~ 0.04 for PMMA).

If the grating is written in a fiber with non uniform diameter, different parts of the grating will have different Bragg wavelengths. The chirp function f (z/L) and fiber radius profile r (z) are related as to each other as follows [4]:

$$r(z) = \frac{r(0)r(L)}{[(r(0)^2 - r(L)^2)f(\frac{z}{L}) + r(L)^2]^{0.5}} \quad , \tag{2}$$

where r(0) is the fiber core radius at z=0.0, r(L) is the core radius at the grating end (z=L), R(z) is the core radius at any position z and L is the grating length.

For example, if f(z/L) =z/L this produces a linear chirp, represented by

$$\lambda_B(\frac{z}{L}) = \lambda_B(0) + \Delta\lambda_B \frac{z}{L} \quad , \tag{3}$$

where $\Delta\lambda_B(z)$ is the total chirp of the grating and $\lambda_B(o)$ is the grating wavelength at z=0.0.

Then, the Bragg wavelength distribution of this chirped grating is λ_B (z) given by

$$\lambda_B(z) = \lambda_B(o) + \Delta\lambda_B f(z). \tag{4}$$

where $\Delta\lambda_B$ is the total chirp value and f(z) is the chirp function.

To simulate the output reflectivity of a chirped fiber grating, we consider the scattering method [2]. Suppose an arbitrary grating is divided into N short sections in which the grating are assumed to be uniform. Each section is considered to be a two–port network characterized by a length L_i, an index n_i, a period Λ_i and a coupling coefficient k_i. This can be described by the transmission matrix T^i which can be defined as [2]:

$$\begin{bmatrix} b_i^{(2)} \\ a_i^{(2)} \end{bmatrix} = \begin{pmatrix} T_{11}{}^i \, T_{12}{}^i \\ T_{21}{}^i \, T_{22}{}^i \end{pmatrix} \begin{bmatrix} a_i^{(1)} \\ b_i^{(1)} \end{bmatrix}, \quad (5)$$

where $a_i^{(1)}$ and $b_i^{(1)}$ are, respectively, the input signal and output signal at port (1) and $a_i^{(2)}$ and $b_i^{(2)}$ are the input signal and output signal at port (2).

The transmission matrix of the whole system is then

$$T = \prod_{i=1}^{N} T^i = \begin{pmatrix} T_{11} \, T_{12} \\ T_{21} \, T_{22} \end{pmatrix}. \quad (6)$$

where:

$$T_{11}{}^i = S_{21}{}^i - S_{22}{}^i S_{11}{}^i / S_{12}{}^i; \quad (7\text{-a})$$

$$T_{12}{}^i = S_{22}{}^i / S_{12}{}^i; \quad (7\text{-b})$$

$$T_{21}{}^i = -S_{11}{}^i / S_{12}{}^i; \quad (7\text{-c})$$

and

$$T_{22}{}^i = 1 / S_{12}{}^i. \quad (7\text{-d})$$

with S_{kk} the reflection coefficient at port K and S_{kl} the transmitted signals between port K and L. These coefficients are defined as [2]

$$S_{11} = S_{22} = k \frac{\exp(-\mu l) - \exp(\mu l)}{(\mu - j\Delta\beta)\exp(-\mu l) + (\mu + j\Delta\beta)\exp(\mu l)}, \quad (8\text{-a})$$

and

$$S_{12} = S_{21} = k \frac{(2\mu)}{(\mu - j\Delta\beta)\exp(-\mu l) + (\mu + j\Delta\beta)\exp(\mu l)}. \quad (8\text{-b})$$

with

$$\mu^2 = |k|^2 - \Delta\beta^2, \quad (8\text{-c})$$

$$\Delta\beta = \beta - \frac{\pi}{\Lambda}, \quad (8\text{-d})$$

$$k = j \frac{\pi \Delta n}{\lambda_B}, \quad (8\text{-e})$$

$$\beta = \frac{2 n_o \pi}{\lambda}, \quad (8\text{-f})$$

and

$$\lambda_B = 2n_{eff}\Lambda. \tag{8-g}$$

where Δβ is the detuning parameter, k is the local coupling coefficient, Δn is the index modulation depth and l is length of each section.

Once the matrix T is determined, the reflection coefficent (ρ) can be obtained using [2]

$$\rho(\lambda) = \frac{-T_{21}}{T_{22}}. \tag{9}$$

Consequently, the reflectivity, R, can be obtained from the definition

$$R = |\rho|^2. \tag{10}$$

The refractive index of the core of the Bragg is given by [6]

$$n = n_o[1 + \Delta n(z)\cos\{\frac{2\pi}{\Lambda}z + \Phi(z)\}], \tag{11}$$

where Λ is the design period of the Bragg ($\lambda_B=2n_{eff}\Lambda$), Φ(z) is a slowly varying function of z that represents the chirp parameter and Δn(z) is the amplitude of the index modulation along the grating (the modulation depth). The modulation depth is related to the apodization profile, h (z), by [5]

$$\Delta n(z) = \Delta n\ h(z). \tag{12}$$

An example of the apodization profiles is the raised sine, given by [5]

$$h(z) = \sin^2(\frac{\pi z}{L}); \quad 0 \le z \le L \tag{13}$$

The group delay time of the reflected light is defined by

$$\tau = -\frac{d\phi}{d\omega} = -\frac{\lambda^2}{2\pi c}\frac{d\phi}{d\lambda}. \tag{14}$$

where τ is the group delay, ϕ is the phase of reflection coefficient, c the free space speed of light and ω is the frequency.

Finally, the dispersion, D, is defined through the group delay by [7]

$$D = \frac{d\tau}{d\lambda}. \tag{15}$$

3. Results and Discussion

The described method is modeled through MATLAB to study the effect of different parameters on the grating reflectivity and fiber dispersion. The obtained results are discussed in the following sections.

3.1 Effect of the index modulation depth on reflectivity

For a linearly chirped POF, the applied strain affects the maximum reflectivity which occurs at a center wavelength, λ_B. The reflectivity is symmetric around the center wavelength with a full width half maximum bandwidth, FWHM. The linearly chirped POF is used with a tapered cladding with r(o) = 62.5 μm and r(L) = 50 μm, taking the parameters of uniform gratings as: λ_{Bo} =1500 nm, $\Delta n = 1.5\times10^{-4}$, grating length L = 4 cm, n_{eff} = 1.48, P_e= 0.04, and a raised sine profile for apodization. The obtained reflectivity is displayed in Figs. 1 and 2 showing the influence of both strain and Δn, respectively.

The maximum reflectivity decreases with the strain under at a small value of Δn ($=1.5\times10^{-4}$). The variation in the strain changes the average dispersion of the fiber grating and so introduces the grating tunability, Fig.1.

As noted from Fig. 2, the maximum reflectivity increases with Δn. Though, there exists a minimum value of Δn that gives a maximum reflectivity (= 1.0) at all values of strains. Figure 3 displays the maximum reflectivity Δn and shows that the best value of Δn for all strains begins from $5\text{x}10^{-4}$ to assure that the maximum reflectivity reaches a value = 1 for all strain values till 0.4%. As a conclusion, the modulation depth Δn should be greater than 5×10^{-4} to assure that the reflectivity reaches one for all strains. Also, the bandwidth increases with the modulation depth at the same strain value and, the value of Δn must be chosen greater than 6×10^{-4} to increase the bandwidth.

3.2 Effect of fiber length on dispersion

At a certain strain value, the group delay increases with the grating length, Fig.4, and consequently the average dispersion increases, Fig.5. If the grating length increases more, the group delay will reach a nonlinear region. Figure 6 shows a sample result when L = 8 cm. Therefore, if the average dispersion increases with the POF grating length but under a certain limit which depends on the strain after which the group delay being nonlinear. The following table summarizes the obtained maximum grating length, L_{max}, at different strain values.

Strain %	L_{max} (cm)
0.1	4.3
0.2	4.2
0.3	4.5
0.4	4.7

Table 1 Maximum grating length of the POF at strain values for a raised sine apodization profile.

3.3 Effect of apodization on dispersion

The same procedure of the raised sine apodization profile is repeated to study the effects of other different apodization profiles. The main apodization profiles, h (z), are represented as [5-6]

1) sine profile:

$$h(z) = \sin(\frac{\pi z}{L}); \quad 0 \le z \le L \tag{16-a}$$

2) sinc profile:

$$h(z) = \frac{\sin(x)}{x}, \; x = \frac{2\pi(z-\frac{L}{2})}{L}; \quad 0 \le z \le L \tag{16-b}$$

3) positive-tanh profile:

$$h(z) = \tanh(\frac{8z}{L}); \quad 0 \le z \le \frac{L}{2} \tag{16-c}$$

$$= \tanh(\frac{8(L-z)}{L}); \quad \frac{L}{2} \le z \le L$$

4) Hamming profile:

$$h(z) = \frac{1 + H\cos(2\pi(z - \frac{L}{2})}{1+H}, \text{ H=0.9}; \ 0 \le z \le L \tag{16-d}$$

5) Blackman profile:

$$h(z) = \frac{1 + 1.19\cos(x) + 0.19\cos(2x)}{2.38}, \tag{16-e}$$

$$x = \frac{2\pi(z - \frac{L}{2})}{L}; \quad 0 \le z \le L$$

6) Bartlett profile:

$$h(z) = \frac{2z}{L}; \quad 0 \le z \le \frac{L}{2} \tag{16-f}$$

$$= -2(\frac{z}{L} - 1); \ \frac{L}{2} \le z \le L$$

7) Cauchy profile:

$$h(z) = \frac{1 - (\frac{2(z - L/2}{L})^2}{1 - (\frac{2C(z - L/2}{L})^2}, \ \text{C=0.5}; \quad 0 \le z \le L \tag{16-g}$$

The obtained results for the different apodization profiles showed their effects on the maximum reflectivity, the level of group delay ripples, and also on the sidelobes level of the reflected spectrum. Though, the study concentrates on the maximum grating length, L_{max}, of the POF after which the group delay becomes nonlinear. Tables 2-5 summarize the obtained results at different strain values for each apodization profile.

Apodization profile	L_{max} (cm)
Raised sine	4.3
Sine	3.1
Sinc	3.7
Tanh	3.3
Hamming	3.7
Blackman	3.6
Bartlett	3.1
Cauchy	3.2

Table 2 Maximum grating length of the POF at 0.1% strain for different apodization profiles.

Apodization profile	L_{max} (cm)
Raised sine	4.2
Sine	4
Sinc	3.9
Tanh	3.7
Hamming	4.1
Blackman	4.3
Bartlett	4.1
Cauchy	3.9

Table 3 Maximum grating length of the POF at 0.2% strain for different apodization profiles.

Apodization profile	L_{max} (cm)
Raised sine	4.5
Sine	4.1
Sinc	4.2
Tanh	4
Hamming	4.5
Blackman	4.7
Bartlett	4.3
Cauchy	4.1

Table 4 Maximum grating length of the POF at 0.3% strain for different apodization profiles.

Apodization profile	L_{max} (cm)
Raised sine	4.7
Sine	4.3
Sinc	4.4
Tanh	4
Hamming	4.6
Blackman	5
Bartlett	4.5
Cauchy	4.1

Table 5 Maximum grating length of the POF at 0.4% strain for different apodization profiles.

4. Conclusion

The present introduces a study on dispersion compensation using linearly chirped POF Bragg gratings made in tapered fibers. The index modulation depth affects reflectivity and FWHM bandwidth. The maximum allowed POF grating length to act as a dispersion compensator is evaluated for different apodization profiles at different strain values. It is found to be in the range 3.1-4.7 cm. The obtained dispersion for the grating cancels the fiber dispersion, resulting in dispersion compensation in the fiber channel.

5. References

[1] Park L. Chu, "Polymer Optical Fiber Bragg Gratings," Optics & Photon. News, vol. 16, no. 7/8, pp. 53-56, 2005.
[2] J. L. Cruz, L. Dong, S. Barcelos and L. Reekie,"Fiber Bragg Gratings With Various Chirp Profiles Made in Etched Tapers," Appl. Opt., vol. 35, no. 34, pp. 6781-6787, 1996.
[3] T. Komukai, T. Inui and M. Nakazawa, "The Design of Dispersion Equalizers Using Chirped Fiber Bragg Gratings," IEEE J. Qunatum Electron., vol. 36, no. 4, pp. 409-417, 2000.
[4] Hongbo Liu, Huiyong Liu, Gangding Peng and Trevor W. Whitbread, "Tunable Dispersion Using Linearly Chirped Polymer Optical Fiber Bragg Gratings With Fixed Center Wavelength," IEEE Photon. Technol. Lett., vol. 17, no. 2, pp.411-413, 2005.
[5] Karin Ennser , Mikhail N. Zervas, and Richard I. Laming, "Optimum of Apodized Linearly Chirped Fiber Grating for Optical Communications," IEEE J. Quantum Electron., vol. 34, no.5, pp. 770-778, 1998.
[6] Daniel Paster, Diego Ortega, Vicente Tatay and Javier Marti', "Design of Apodized Linearly Chirped Fiber Gratings for Dispersion Compensation," IEEE J. Lightwave Technol., vol. 14, no. 11, pp. 2581-2588, 1996.

[7] Andreas Othonos and Kyriacos Kalli, "Fiber Bragg Gratings: Fundamentals and Applications in Telecommunications and Sensing," 1st ed., Artech House, Boston, London, 2001.

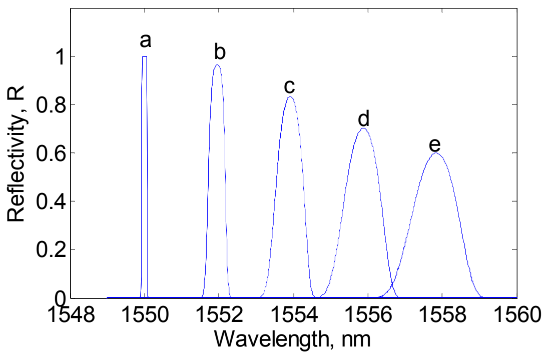

Fig.1 Reflectivity of linearly chirped POF at different strains with $\Delta n = 1.5 \times 10^{-4}$. (a) no strain (b) strain = 0.1% (c) strain = 0.2% (d) strain = 0.3% (e) strain = 0.4%.

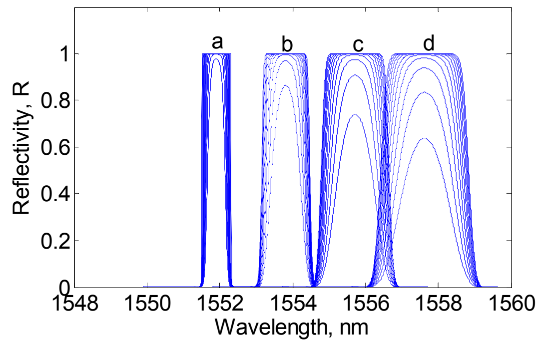

Fig.2 Reflectivity at different modulation index amplitudes. The values of Δn are 1.5, 2.5, 3.5, 4.5, 5.5, 6, 6.5×10^{-4} starting from inside to outside. The strain values are (a) 0.1% (b) 0.2% (c) 0.3% (d) 0.4%.

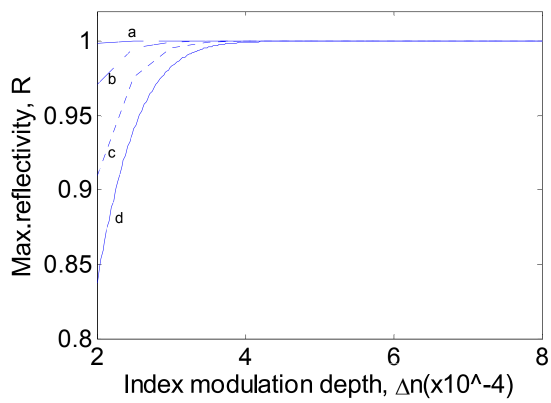

Fig.3 The maximum reflectivity against the modulation index depth under different strain, (a) strain 0.1%, (b) strain 0.2%, (c) strain 0.3%, (d) strain 0.4%.

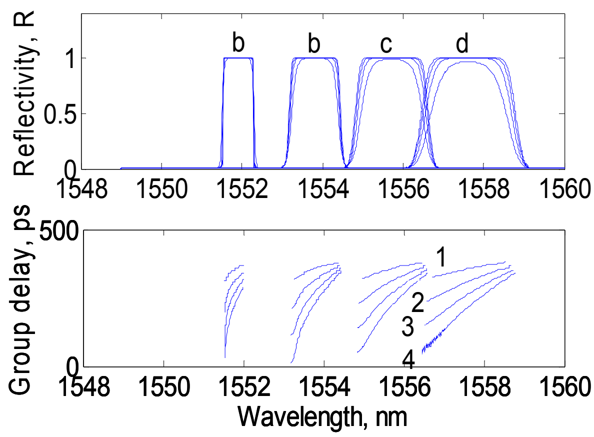

Fig.4 Effect of fiber length in group delay at different strain values. (a) strain = 0.1% (b) strain = 0.2% (c) strain = 0.3% (d) strain = 0.4% at for different grating lengths. (1) L = 1 cm (2) L = 2 cm (3) L = 3 cm (4) L = 4 cm.

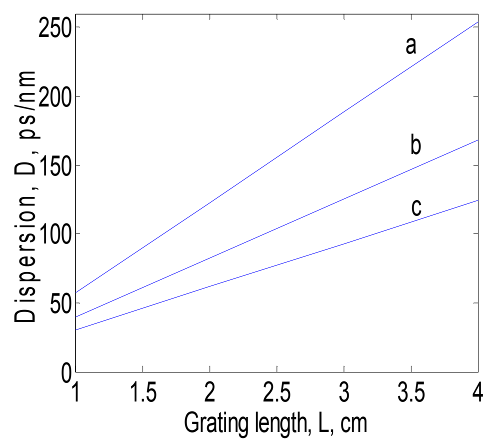

Fig.5 Effect of grating length in dispersion. (a) strain = 0.2% (b) strain = 0.3% (c) strain = 0.4%.

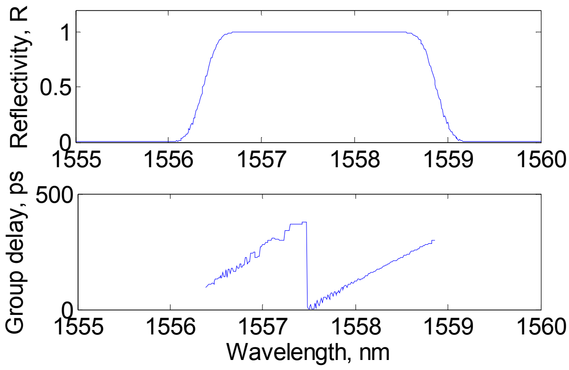

Fig.6 The group delay under a 0.4% strain with a grating length L = 8cm.