



Arab Academy for Science and Technology & Maritime Transport
College of Engineering and Technology
Department of Basic and Applied Sciences

- M** - *Rules of differentiation*
- *Trigonometric functions and their derivatives*
- *Inverse trigonometric functions and their derivatives*
- A** - *Logarithmic function and its derivative*
- *Exponential function and its derivative*
- *Derivatives of Hyperbolic and inverse Hyperbolic functions*
- T** - *Parametric function and its derivative*
- *Implicit function and its derivative*
- *The n^{th} derivative*
- H** - *L'Hôpital's rule (The limit of a function)*
- *Maclaurin's expansions*
- *Partial differentiation*
- *Curve sketching*
- 1** - *Physical application (velocity and acceleration)*
- *Conic sections (Parabola – Ellipse)*
- *Software application*

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Syllabus for Mathematics 1 (BA123)
Text Book: Calculus,
Sherman K. Stein & Anthony Barcellos
Program title: All Programs
Coordinator: Dr. Nasser El-Maghraby (R. 222)

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2	Trigonometric functions and their derivatives	Sheet 2	
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7	Parametric differentiation and Implicit differentiation	Sheet 7	
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9	The limit of a function, L'Hôpital's rule	Sheet 9	Sheet 6– Sheet 8
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12	Curve sketching: Critical points, maximum and minimum points, inflection points.	Sheet 12	Sheet 9–Sheet 11
13	Physical application: velocity and acceleration	Sheet 12	
14	Conic section : parabola Equation, vertex, focus, directrix, eccentricity, graph	Sheet 13	
15	Conic section : ellipse Equation, axes, foci, directrices, eccentricity, graph.	Sheet 13	Sheet 12– Sheet 13

Sheet 1 : Basic Differentiation Rules

$y = y(x)$	$y' = dy/dx = y'(x)$
1. $y = k$,k: constant	$y' = 0$
2. $y = x^k$,k is constant	$y' = kx^{k-1}$
3. $y = f(x) \pm g(x)$	$y' = f'(x) \pm g'(x)$
4. $y = kf(x)$,k is constant	$y' = kf'(x)$
5. $y = f(x)g(x)$	$y' = f(x)g'(x) + f'(x)g(x)$
6. $y = [f(x)]^k$,k is constant	$y' = k[f(x)]^{k-1}f'(x)$
7. $y = f(x)/g(x)$	$y' = [f'(x)g(x) - f(x)g'(x)]/g^2(x)$

Lecture Examples

a. Find dy/dx for each of the following

1) $y = x^4 - 3x^{-2} + 15x + 10$

2) $y = (\sqrt{x} - 1)^7$

3) $y = \frac{1}{(x^6 - 2)^5}$

4) $y = (x^3 - 1)^5 (2 + 3x^{-4})^7$

5) $y = \frac{x^3 - 1}{x^3 + 1}$

6) $y = \left(\frac{x^2 - 3}{x^{-4} + 2} \right)^{4/3}$

b. Find d^2y/dx^2 for each of the following

1) $y = x^7 - \frac{2}{x^3} + x^{-5} + 16x + 5$

2) $y = (2 - x^3)^8$

c. If $y = (x + \sqrt{x^2 - 1})^4$, Show that $y' = \frac{4y}{\sqrt{x^2 - 1}}$

Classroom Exercises

d. Find dy/dx for each of the following

$$1) y = \frac{3}{x} - \frac{4}{x^2} + 6x^5 + 7$$

$$3) y = \left(\frac{1-x^4}{1+x^4} \right)^{3/2}$$

$$5) y = \sqrt{\frac{x-1}{x+1}}$$

$$2) y = (x^4 - 1)^6$$

$$4) y = \sqrt{x^3 - 1} (1 - 3x)^5$$

$$6) y = \left(1 - \frac{1}{\sqrt{x}} \right)^{-4/3}$$

e. Find d^2y/dx^2 for each of the following

$$1) y = (x^3 - 1)^6$$

$$2) y = \frac{x^2 - 1}{x^2 + 1}$$

Homework

f. Find dy/dx for each of the following

$$1) y = \sqrt{x^5 - 4}$$

$$3) y = (x^2 + 4)^6 (1 - 2x)^7$$

$$5) y = (\sqrt{1+x^2})^5 \sqrt[3]{x^4 - 1}$$

$$2) y = x^{-3} (1+x^4)^5$$

$$4) y = \sqrt{x} (1 - \sqrt{x})^6$$

$$6) y = \left(\frac{x^2 - 4}{x^2 + 2} \right)^{7/2}$$

g. Find d^2y/dx^2 for each of the following

$$1) y = (x^{3/2} - 1)^4$$

$$2) y = \frac{x^2 - 1}{\sqrt{x + 1}}$$

Sheet 2 : Trigonometric Functions and their Derivatives

$$\cos^2 u + \sin^2 u = 1$$

$$1 + \tan^2 u = \sec^2 u$$

$$\cot^2 u + 1 = \operatorname{cosec}^2 u$$

$$\sin 2u = 2 \sin u \cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u$$

$$= 2 \cos^2 u - 1$$

$$= 1 - 2 \sin^2 u$$

$$y = \cos u \longrightarrow y' = -u' \sin u$$

$$y = \sin u \longrightarrow y' = u' \cos u$$

$$y = \tan u \longrightarrow y' = u' \sec^2 u$$

$$y = \cot u \longrightarrow y' = -u' \operatorname{cosec}^2 u$$

$$y = \sec u \longrightarrow y' = u' \sec u \tan u$$

$$y = \operatorname{cosec} u \longrightarrow y' = -u' \operatorname{cosec} u \cot u$$

Lecture Examples

a. Find dy/dx for each of the following

1) $y = \sin x^3$

2) $y = (1 + \cos^3 x) \cot^2 2x$

3) $y = x^3 \cos x^2 - 2 \cot x^{-3}$

4) $y = \frac{x \sin 2x}{1 - \cos^2 3x}$

5) $y = \sec^3 \sqrt{4x^2 + 1}$

6) $y = \frac{\sin(x-1)}{x-1}$

b. Find d^2y/dx^2 for each of the following

1) $y = (1 - \cos^2 x)^{-3/2}$

2) $y = x \sec x$

c. If $y = a \sin ct + b \cos ct$, where a, b and c are constants, prove that $y'' = -c^2 y$

Classroom Exercises

d. Find dy/dx for each of the following

1) $y = \tan^2(\cos^3 x^2)$

2) $y = \sqrt{x^2 + 1} \cos^3 \sqrt{x^2 - 1}$

3) $y = x^2 \sec^3 x - 4 \cot^2 x^3$

4) $y = \frac{1 - \sin 2x}{1 + \cos 2x}$

5) $y = \sqrt{\tan^2 x + x \cos^3 x}$

6) $y = \sqrt{x-1} \sin \sqrt{x-1}$

e. Find d^2y/dx^2 for each of the following

$$1) y = x^4 (\cos 2x)$$

$$2) y = \frac{\sin x}{x}$$

Homework

f. Find dy/dx for each of the following

$$1) y = \sec^3 \sqrt{\cos x}$$

$$2) y = \cot(\sqrt{x} \tan \sqrt{x})$$

$$3) y = x^{3/2} \cot x^3$$

$$4) y = \operatorname{cosec}^4 \sqrt{x^2 - 1}$$

$$5) y = (1 - \sin \sqrt{x})^3 \cos \sqrt{x}$$

$$6) y = \sqrt{x} \operatorname{cosec} \sqrt{x}$$

g. Find d^2y/dx^2 for each of the following

$$1) y = x \tan x^3$$

$$2) y = \sin^3 x$$

Sheet 3: Inverse Trigonometric Functions and their Derivatives

$$y = \sin^{-1} u \rightarrow y' = \frac{u'}{\sqrt{1-u^2}}$$

$$y = \cos^{-1} u \rightarrow y' = \frac{-u'}{\sqrt{1-u^2}}$$

$$y = \tan^{-1} u \rightarrow y' = \frac{u'}{1+u^2}$$

$$y = \cot^{-1} u \rightarrow y' = \frac{-u'}{1+u^2}$$

$$y = \sec^{-1} u \rightarrow y' = \frac{u'}{|u|\sqrt{u^2-1}}$$

$$y = \operatorname{cosec}^{-1} u \rightarrow y' = \frac{-u'}{|u|\sqrt{u^2-1}}$$

Lecture Examples

a. Find dy/dx for each of the following

$$1) y = \cos^{-1} \sqrt{x}$$

$$2) y = (x^2 + 4) \operatorname{cosec}^{-1} 2x$$

$$3) y = x^3 (1 - \sec^{-1} x)$$

$$4) y = x^3 \sin^{-1} \sqrt{x} - 2 \cot^{-1} x^2$$

$$5) y = \tan^{-1} \left(\frac{x-1}{x+1} \right)$$

$$6) y = \frac{1 - \sin^{-1} x}{\cos^{-1} x}$$

b. If $y = \tan(\cos^{-1}x)$, Prove that $y' = \frac{-(y^2+1)}{\sqrt{1-x^2}}$

Classroom Exercises

c. Find dy/dx for each of the following

1) $y = \sqrt{x} \tan^{-1} \sqrt{x}$

2) $y = \cot^{-1} \left(\frac{\cos 3x}{1 + \sin 3x} \right)$

3) $y = x^3 \sec^{-1} x^2$

4) $y = \frac{\cos^{-1} x}{1 - \sin^{-1} x}$

5) $y = \sqrt{x^2 - 1} \sin^{-1} x - x \cos^{-1} x$

6) $y = \tan^{-1}(\cos x) + \cot^{-1}(\sin x)$

d. If $y = \cos(2\sin^{-1}x)$, prove that $(1-x^2)(y')^2 = 4(1-y^2)$

Homework

e. Find dy/dx for each of the following

1) $y = \sin^{-1} x^3$

2) $y = \cot^{-1}(\cos 2x)$

3) $y = \cot^{-1} \left(\frac{x-4}{x+4} \right)$

4) $y = \frac{\tan^{-1} x}{1 - \cos^{-1} \sqrt{x}}$

5) $y = x^2 \cos ec^{-1} \sqrt{x} - 3x \sin^{-1} x$

6) $y = \sqrt[3]{x} \sec^{-1} \left(\frac{x}{4} \right)$

f. Prove that $\frac{d}{dx} \left(\tan^{-1} \left(\frac{x-1}{x+1} \right) \right) = \frac{d}{dx} (\tan^{-1}(x))$

Sheet 4 : Logarithmic Function and Its Derivatives

$\ln b = a \Leftrightarrow b = e^a$, $e = 2.71828 \dots, b \geq 0$

1. $\ln(1) = 0, \ln(0) = -\infty, \ln(e) = 1$

2. $\ln(ab) = \ln a + \ln b$

3. $\ln(a^n) = n \ln a$

$y = \ln u \rightarrow y' = \frac{u'}{u}$

4. $\ln(e^n) = n$

5. $\ln(a/b) = \ln a - \ln b$

Lecture Examples

a. Find dy/dx for each of the following

1) $y = x^3 \ln x$

2) $y = \ln(x^{-4}(x^5 - 2)^6)$

3) $y^x = x^y$

4) $y = \ln \left[\frac{x^3(1-x^2)^4}{\sin x(x-1)^5} \right]^{7/2}$

5) $y = \sin x^2 - 3x \cos x - x^x$

6) $(\sin x)^{\cos y} = (\sin y)^{\cos x}$

b. If $y = \ln(\sec x + \tan x)$, Prove that $y'' = \sec x \tan x$.

Classroom Exercises

c. Find dy/dx for each of the following

1) $y = (\ln x)^3$

2) $y = \ln \left(\frac{x^3 - 1}{x^2 + 1} \right)$

3) $y = \sqrt[4]{\frac{(1-x)^3 \tan^{-1} x}{x^x \sec x^3}}$

4) $y^{5/2} = x^{\ln x}$

5) $y = \frac{x^x (2 - \sin x)^{x^2}}{x^{\cos x} (1 - 2 \ln x)^5}$

6) $y = x \cos \sqrt{x} - x^{\sec x}$

d. If $y = \cos(\ln x) + \sin(\ln x)$

prove that $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$

Homework

e. Find dy/dx for each of the following

1) $y = \ln(1 - \ln x)$

2) $y = \ln \left[\frac{(1 - x^2)^5 (2 - \sin^{-1} x)}{(1 - \ln x)^2 (3 - \cos x)} \right]$

3) $y = \frac{(x-1)^3 (1 - \sin x)^4}{x^x (2 - \cos x)^2}$

4) $\sqrt{y} = \frac{x^5 \tan^{-1} x}{(1+x)^3 \sqrt{x}}$

5) $y = x^{\sin x}$

6) $y = (\ln(\sin x))^{\cos x}$

Sheet 5 : Exponential Function and Its Derivative

1. $e^a e^b = e^{a+b}$

2. $e^a / e^b = e^{a-b}$

3. $(e^a)^b = e^{ab}$

$y = e^u \rightarrow y' = u' e^u$

4. $e^{\ln a} = a$

Lecture Examples

a. Find dy/dx for each of the following

1) $y = e^{\sin^{-1} x}$

2) $y = e^{\tan^{-1} \sin x}$

3) $y = \cos^3 e^{x^2}$

4) $y = \cos^{-1}(1 - e^{-x})$

5) $y = \operatorname{cosec}^{-1} e^x - x^4 e^{\cot x}$

6) $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

b. If $y = \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}$, show that $y' = \frac{8}{(e^{2x} + e^{-2x})^2}$.

c. If $y = \tan^{-1} \ln e^{\tan \sqrt{x}}$, show that $yy' = 1/2$

Classroom Exercises

d. Find dy/dx for each of the following

1) $y = x^3 e^{x^5 - 3}$

2) $y = e^{\ln \sin^{-1}(\sin x^3)}$

3) $y = \sqrt{e^{\cos^{-1} x}}$

4) $y = \ln \left[\frac{e^{x^2} \sin x^3}{(1 - e^x)(2 - x)} \right]^6$

e. If $y = \ln(\cos x)$, show that $y'' + e^{-2y} = 0$

f. Find d^2y/dx^2 for each of the following

1) $y = e^{\sin x}$

2) $y = \cos e^{3x}$

Homework

g. Find dy/dx for each of the following

1) $y = e^{x^3}$

2) $y = e^{\cos^{-1}(\sin x)}$

3) $y = \ln \left[\frac{1 + e^{2x}}{(1 - e^{-2x})^3} \right]$

4) $y = x^5 \sec e^{-x}$

h. If $y = ae^{-2x} + be^{3x}$ where a and b are constant

Show that

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0.$$

i. Find d^2y/dx^2 for each of the following

$$1) y = e^{-4x}$$

$$2) y = \ln(e^{2x} - 4)$$

Sheet 6 : Derivatives of Hyperbolic Functions and Their Inverse

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\operatorname{cosech} x = \frac{2}{e^x - e^{-x}}$$

$$\operatorname{sech} x = \frac{2}{e^x + e^{-x}}$$

$$\operatorname{coth} x = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$\cosh^2 u - \sinh^2 u = 1$$

$$1 - \tanh^2 u = \operatorname{sech}^2 u$$

$$\operatorname{coth}^2 u - 1 = \operatorname{cosech}^2 u$$

$$\sinh 2u = 2 \sinh u \cosh u$$

$$\cosh 2u = \cosh^2 u + \sinh^2 u$$

$$\tanh 2u = 2 \tanh u / (1 + \tanh^2 u)$$

$$y = \sinh u \rightarrow y' = u' \cosh u$$

$$y = \sinh^{-1} u \rightarrow y' = \frac{u'}{\sqrt{u^2 + 1}}$$

$$y = \cosh u \rightarrow y' = u' \sinh u$$

$$y = \tanh u \rightarrow y' = u' \operatorname{sech}^2 u$$

$$y = \cosh^{-1} u \rightarrow y' = \frac{u'}{\sqrt{u^2 - 1}}$$

$$y = \operatorname{coth} u \rightarrow y' = -u' \operatorname{cosech}^2 u$$

$$y = \operatorname{sech} u \rightarrow y' = -u' \operatorname{sech} u \tanh u$$

$$y = \tanh^{-1} u \rightarrow y' = \frac{u'}{1 - u^2}, \quad |u| < 1$$

$$y = \operatorname{cosech} u \rightarrow y' = -u' \operatorname{cosech} u \operatorname{coth} u$$

Lecture Examples

a. Find dy/dx for each of the following

$$1) y = x^4 \cosh^2 x^3$$

$$2) y = \tanh(x \ln x)$$

$$3) y = e^{\operatorname{cosh}^{-1} x^2}$$

$$4) y = (\sin^{-1} \sqrt{x})(1 - \cosh^{-1} x^2)$$

$$5) \quad y = \sqrt[5]{x^3} \tanh^{-1} x^2$$

$$6) \quad y = \ln \left[\frac{(x+1)^2 e^{\operatorname{cosech} x}}{\sqrt{x^3 - 1}} \right]$$

b. Show that $\cosh^{-1} x = \ln(x \pm \sqrt{x^2 - 1})$

Classroom Exercises

c. Find dy/dx for each of the following

$$1) \quad y = \sinh x^3$$

$$2) \quad y = \tanh^{-1}(\sec h 2x)$$

$$3) \quad y = \sin \left(\cosh^{-1} \sqrt{x^2 + 1} \right)$$

$$4) \quad y = \sqrt{\cosh^{-1}(e^{-x/2})}$$

d. Show that

$$\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$$

e. Solve the following equations

$$1. \quad e^{\cosh^{-1} x} = 2$$

$$2. \quad \ln \left(\frac{1 + \tanh x}{1 - \tanh x} \right) = 5$$

Homework

f. Find dy/dx for each of the following

$$1. \quad y = x^2 \coth^3 \sqrt{x}$$

$$2. \quad y = x e^{\sinh^{-1} x}$$

$$3. \quad y = (1 - \ln \sec x) \cosh^{-1} \sqrt{x}$$

$$4. \quad y = \ln \sqrt{\tanh 3x}$$

$$5. \quad y = \sinh^{-1}(\sin 2x)$$

$$6. \quad y = \tanh^{-1} \sqrt{\sec x}$$

f. Show that

$$\tanh^{-1} x = \frac{1}{2} \ln \frac{1+x}{1-x}$$

g. Solve

$$5 \cosh x - 4 \sinh x = 3$$

Sheet 7 : Parametric and Implicit Differentiation

Lecture Examples

a. Find dy/dx for each of the following

1) $y = t \ln t$, $y = \ln t/t$ 2) $x = e^t \cosh t$, $y = e^t \sinh t$

b. Find d^2y/dx^2 for each of the following

1) $x = \cos ect$, $y = \cos 2t$ 2) $x = \sqrt{1-t^2}$, $y = \sin^{-1} t$

c. If $x = \cos \frac{t}{1+t}$, $y = \sin \frac{t}{1+t}$, show that $y^3 y'' + 1 = 0$

d. If $x = \frac{t+1}{t-1}$, $y = \left(\frac{t-1}{t+1}\right)^5$, show that $y'' = 30x^{-7}$

e. Find dy/dx for each of the following

1) $x^3 - 3x^2y^4 + 7y^2 = 10$ 2) $x + \cos^{-1} y = xy$

3) $\sin^{-1} x + \tan(xy) = 5$ 4) $y = e^{-x} + e^y$

Classroom Exercises

f. Find dy/dx for each of the following

1) $x = \frac{3t}{1+t^3}$, $y = \frac{3t}{1+t^3}$

2) $x = \sqrt{1-\sin \theta}$, $y = \sqrt{1+\cos \theta}$

g. Find d^2y/dx^2 for each of the following

1) $x = \cos \theta + \theta \sin \theta$, $y = \sin \theta - \theta \cos \theta$

2) $x = \sqrt{t^4 - 1}$, $y = \sec^{-1} t^2$

h. If $x = \tan \frac{t-1}{1+t}$, $y = \sec \frac{t-1}{1+t}$, show that $y'' = y^{-3}$.

i. Find dy/dx for each of the following

1) $x^{-2}y^5 - 2xy^2 + 7x = 12$

2) $x + y^2 = e^{x/y}$

3) $\ln y = x + e^y$

4) $y^2 = \sin^3 2x + \cos^3 2y$

5) $x^{1+y} + y^{1+x} = 1$

Homework

j. If $x = \tan t - t$, $y = \tan^3 t$, Find y'' .

k. If $x = t + \frac{1}{t}$, $y = t^2 + \frac{1}{t^2}$, Show that $y'' = 2$.

l. If $x = \frac{t-1}{t+1}$, $y = \frac{t+1}{t-1}$, Show that $y'' = 2y^3$.

m. Find dy/dx for each of the following

1) $y^4 - 4x^3y^2 + 6x^2 = 7$

2) $\tan^{-1} y = x^2 + y^2$

3) $y = e^{(x+y)^3}$

4) $\cosh^{-1} \sec y = xy^3$

Sheet 8 : The n^{th} Derivative

Lecture Examples

Find the n^{th} derivative for the following

1) $y = \ln(1 + 3x)$

2) $y = \sin 2x$

3) $y = e^{-(1+x)}$

4) $y = \frac{2}{1-4x}$

Classroom Exercises

$$5) \quad x = \ln(2 - 4x)$$

$$7) \quad y = \sin^2 x$$

$$6) \quad y = \frac{2x + 1}{x - 1}$$

$$8) \quad y = \ln \frac{x + 1}{2x + 1}$$

Homework

$$9) \quad y = \ln(1 - 2x)$$

$$11) \quad y = \frac{1}{x - 3}$$

$$10) \quad y = \frac{1 - x}{1 + x}$$

$$12) \quad y = 2 \sin x + 3 \cos x$$

Sheet 9 : L'Hôpital's Rule

Lecture Examples

$$1) \quad \lim_{x \rightarrow \pi/2} \frac{2 \cos x}{2x - \pi}$$

$$3) \quad \lim_{x \rightarrow 0} \frac{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}{x}$$

$$5) \quad \lim_{x \rightarrow 0} \frac{x \cos x + \tan 2x}{x \sec x + \sin 4x}$$

$$7) \quad \lim_{x \rightarrow \infty} \left(\frac{x + 3}{x - 1} \right)$$

$$2) \quad \lim_{x \rightarrow 0} \frac{1 - \cosh x}{x^2}$$

$$4) \quad \lim_{x \rightarrow 1} (1 - x) \tan \frac{\pi x}{2}$$

$$6) \quad \lim_{\phi \rightarrow 0} (\cos \phi \csc \phi - \cot \phi)$$

$$8) \quad \lim_{x \leftarrow 0} (\cos x)^{1/x^2}$$

Classroom Exercises

$$1) \quad \lim_{x \rightarrow \pi} \frac{1 - \sin(x/2)}{\pi - x}$$

$$3) \quad \lim_{x \rightarrow 0} \frac{\tan x + \sec x - 1}{\tan x - \sec x + 1}$$

$$2) \quad \lim_{x \rightarrow 1} \frac{\cot(\pi x/2)}{1 - \sqrt{x}}$$

$$4) \quad \lim_{x \rightarrow 0} \frac{1 - \sqrt{\cos x}}{x^2}$$

$$5) \lim_{x \rightarrow \pi/2} (\sec x - \tan x)$$

$$6) \lim_{x \rightarrow 0} \frac{\sin 4x - x \cos x}{x \sec x - \tan 3x}$$

$$7) \lim_{x \rightarrow 0} (\cos x)^{1/x}$$

$$8) \lim_{x \rightarrow \infty} \left(\frac{x}{x+1} \right)^x$$

Homework

$$1) \lim_{x \rightarrow 1} \frac{1-x^2}{\sin \pi x}$$

$$2) \lim_{x \rightarrow 1} \frac{\sin(x^3 - 1)}{x - 1}$$

$$3) \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{\sqrt[3]{1+x} - 1}$$

$$4) \lim_{x \rightarrow 0} \frac{\sin 3x}{1 - \cos 4x}$$

$$5) \lim_{x \rightarrow 0} \frac{\sinh x}{x}$$

$$6) \lim_{x \rightarrow 0} \frac{\tanh x}{x}$$

$$7) \lim_{x \rightarrow 0} (1 + \sin x)^{1/x}$$

$$8) \lim_{x \rightarrow \infty} \left(\frac{x}{x+2} \right)^{x-2}$$

Sheet 10 : Partial Differentiation

Lecture Examples

a. Find the first partial derivatives for each of the following

$$1) z = (x^2 - y) \sin x^3$$

$$2) z = (\sin 2y)^x$$

b. If $z = \tan^{-1} \frac{y}{x}$ show that

$$1) x \frac{\partial z}{\partial y} - y \frac{\partial z}{\partial x} = 1$$

$$2) \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$$

c. If $z = f(x^2 + y^2)$ show that $x \frac{\partial z}{\partial y} - y \frac{\partial z}{\partial x} = 0$

Classroom Exercises

d. Find the first partial derivatives for each of the following

1) $z = y^2(x^4 - 1)^5 + 6y^2x$

2) $z = x^2 \sin \sqrt{x} + y \cos(xy)$

3) $z = \tan^{-1} \frac{y}{x}$

4) $z = e^{x/y} \tanh^{-1}(x^2 + y^2)$

e. If $z = \cot^{-1} \frac{y}{x}$ show that $\frac{\partial^2 z}{\partial y^2} + \frac{\partial^2 z}{\partial x^2} = 0$

f. If $z = \tan^{-1} \frac{x-1}{y-1}$ show that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$

Homework

g. Find the first partial derivatives for each of the following

1) $z = x^3 - 3x^2y^4 + y^2$

2) $z = (x + y) \sin(x - y)$

3) $z = e^x \ln \frac{x}{y}$

4) $z = (1 + \sin y)^{1 + \cos x}$

h. If $z = \ln(x^2 + y^2)$ show that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$

i. If $z = \cot^{-1} \frac{x}{y}$, show that

1) $y \frac{\partial z}{\partial x} - x \frac{\partial z}{\partial y} = -1$ and

2) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$

Sheet 11 : Maclaurin's Expansion

Maclaurin's Expansion:

$$f(x) = f(0) + f'(0) \frac{x}{1!} + f''(0) \frac{x^2}{2!} + f'''(0) \frac{x^3}{3!} + \dots + f^{(n)}(0) \frac{x^n}{n!} + \dots$$

Lecture Examples

a. Find Maclaurin's Expansion of each of the following :

(1) $f(x) = \sin 2x$

(2) $f(x) = \ln(2+3x)$, Find approximate value to $\ln(2.3)$.

b. Using Maclaurin's Expansion, show that

(1) $e^{-x} \cos x = 1 - x + \frac{1}{3}x^3 - \frac{1}{6}x^4 + \dots$

(2) $\frac{\cos x}{\sqrt{1+x}} = 1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3 + \dots$

Classroom Exercises

c. Find Maclaurin's Expansion for each of the following

1) $f(x) = \cos 3x$

2) $f(x) = \frac{1}{\sqrt{1+x}}$

3) $f(x) = e^{-3x}$

d. Using Maclaurin's Expansion show that :

1) $e^x \sin x = x + x^2 + \frac{x^3}{3} - \frac{x^5}{30} + \dots$

2) $\frac{e^x}{1-x} = 1 + 2x + \frac{5}{2}x^2 + \frac{8}{3}x^3 + \dots$

Homework

e. Find Maclaurin's Expansion for each of the following

1) $f(x) = \frac{1}{x+1}$

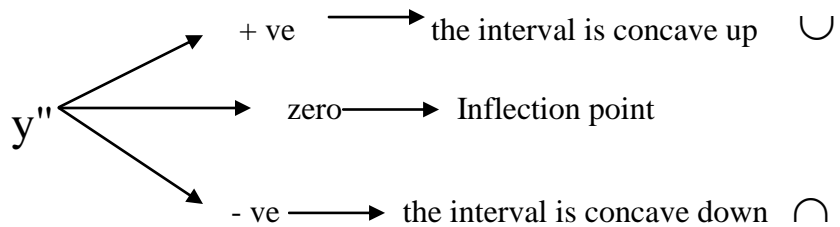
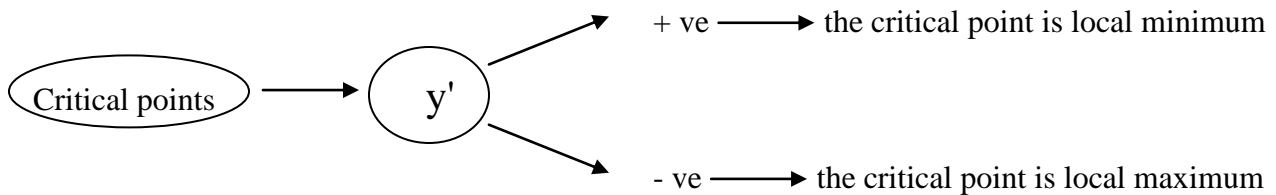
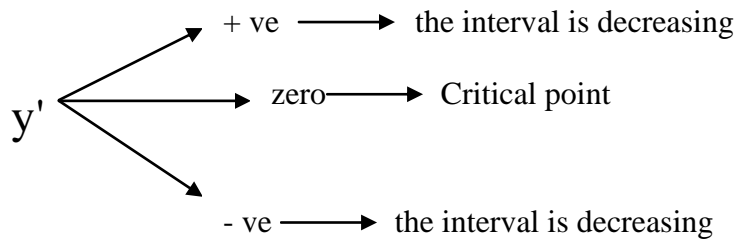
2) $f(x) = \cos 3x$

f. Using Maclaurin's Expansion, show that

(1) $\frac{e^{-x}}{1-x} = 1 + \frac{x^2}{2} + \frac{x^3}{3} + \dots$

(2) $\sinh x + \cosh x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \dots$

Sheet 12 : Differentiation applications



The equation of motion of any particle is given by:

- The displacement of the particle:

$$s = s(t)$$

- then, its velocity is given by:

$$v = \frac{ds}{dt}$$

- and its acceleration is given by:

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

Lecture Examples

i) In each of the following curves,

1) $y = x^2 - 4x + 3$

2) $y = -2x^2 + 12x + 7$

find

- The critical point.
- The intervals in which the curve is increasing and decreasing.
- The local maximum and minimum points.
- Sketch the curve.

ii) In each of the following curves,

1) $y = x^3 - 6x^2 + 10$

2) $y = (x^2 - 9)^2$

find

- The critical points.
- The intervals in which the curve is increasing and decreasing.
- The local maximum and minimum points.
- The inflection point.
- The concavity of the curve.
- Sketch the curve

iii) Find the velocity and the acceleration for each of the following

1) $s(t) = t^3 - 6t^2 + 7$

2) $s(t) = t^3 e^{t^4 - 1}$

Classroom Exercises

i) In each of the following curves,

1) $y = 2x^2 - 8x + 10$

2) $y = -3x^2 - 12x$

find

- The critical point.
- The intervals in which the curve is increasing and decreasing.
- The local maximum and minimum points.
- Sketch the curve.

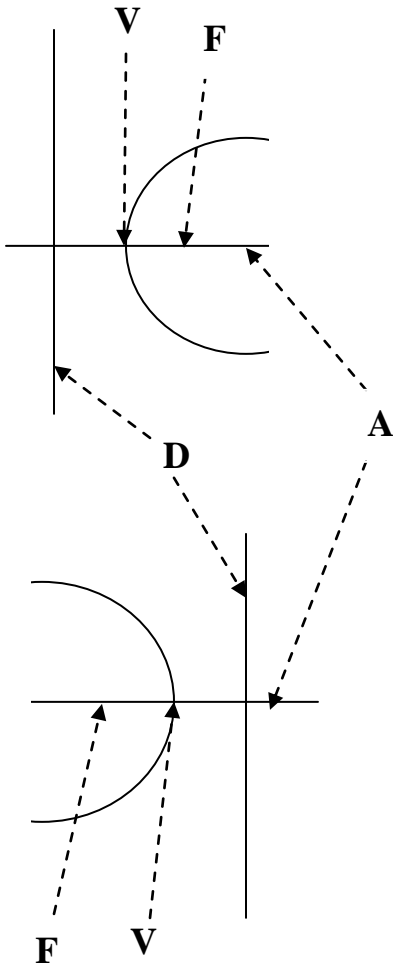
- ii) In each of the following curves,
- 1) $y = 6x^2 - x^3$ 2) $y = x^3 - 3x^2 - 9x$
- find
- The critical points.
 - The intervals in which the curve is increasing and decreasing.
 - The local maximum and minimum points.
 - The inflection point.
 - The concavity of the curve.
 - Sketch the curve.
- ii) Find the velocity and the acceleration for each of the following
- 1) $s(t) = \sin 5t - \cos 5t$ 2) $s(t) = t^6(1 - \ln t)^4$

Homework

- i) In each of the following curves,
- 1) $y = 12x - 3x^2$ 2) $y = 3x^2 - 6x$
- find
- The critical point.
 - The intervals in which the curve is increasing and decreasing.
 - The local maximum and minimum points.
 - Sketch the curve.
- iii) In each of the following curves,
- 1) $y = 2x^3 - 12x^2 + 18x$ 2) $y = x^3 - 9x^2 + 8$
- find
- The critical points.
 - The intervals in which the curve is increasing and decreasing.
 - The local maximum and minimum points.
 - The inflection point.
 - The concavity of the curve.
 - Sketch the curve.
- iv) Find the velocity and the acceleration for each of the following
- 1) $s(t) = t \sinh 3t + \cosh 3t$ 2) $s(t) = \frac{1 - e^{2t}}{e^{2t} - e^{-2t}}$

Sheet 13 : Conic Sections

The Parabola



$$(y - y_0)^2 = 4c(x - x_0)$$

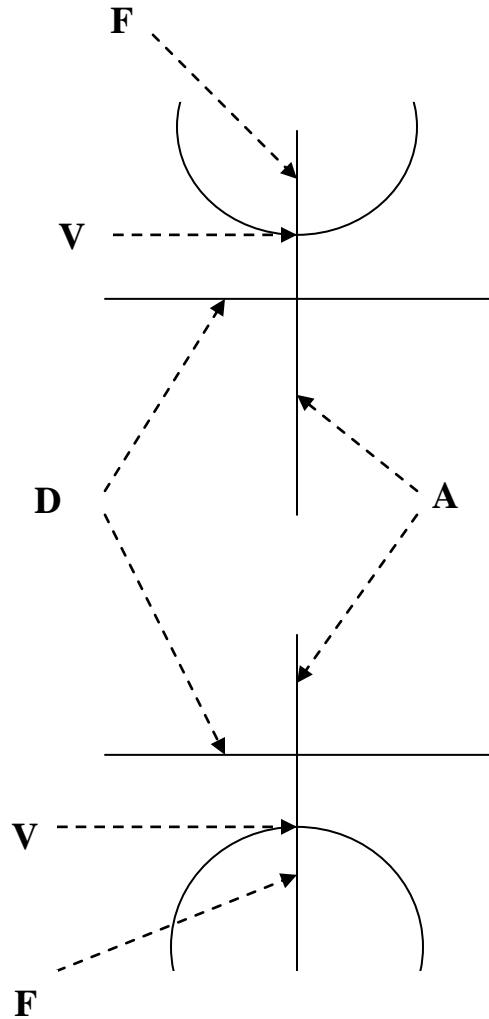
vertex (x_0, y_0)

focus $(x_0 + c, y_0)$

axis $y = y_0$

directrix $x = x_0 - c$

The Ellipse



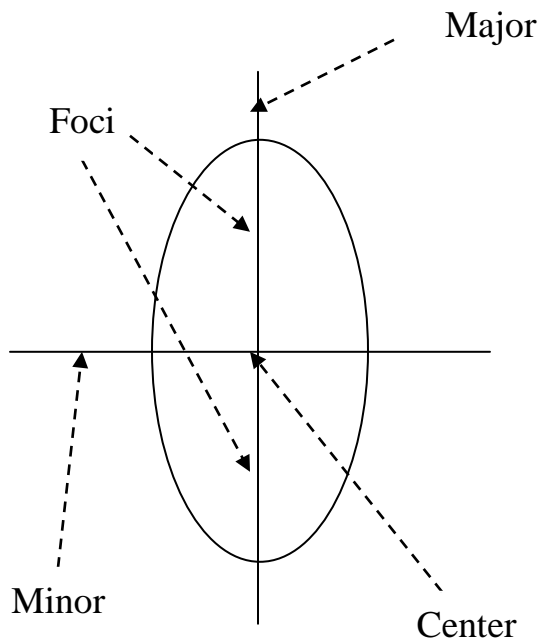
$$(x - x_0)^2 = 4c(y - y_0)$$

vertex (x_0, y_0)

focus $(x_0, y_0 + c)$

axis $x = x_0$

directrix $y = y_0 - c$



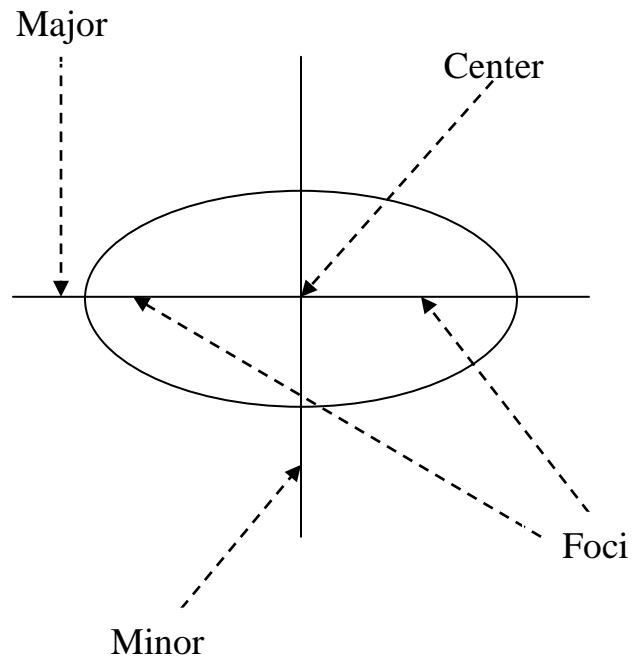
$$\frac{(x - x_0)^2}{b^2} + \frac{(y - y_0)^2}{a^2} = 1$$

Major (2a) $x = x_0$

Minor (2b) $y = y_0$

Center (x_0, y_0)

Foci $(x_0, y_0 \pm c)$



$$\frac{(x - x_0)^2}{a^2} + \frac{(y - y_0)^2}{b^2} = 1$$

Major (2a) $y = y_0$

Minor (2b) $x = x_0$

Center (x_0, y_0)

Foci $(x_0 \pm c, y_0)$

Lecture Examples

a. Discuss and sketch the following curves:

1) $x^2 + 2x - 4y - 3 = 0$

2) $2y^2 - 4x - 4y - 14 = 0$

3) $4x^2 + 9y^2 + 24x = 0$

4) $2x^2 + 9y^2 + 8x - 72y + 134 = 0$

Classroom Exercises

b. Discuss and sketch the following curves:

1) $x^2 + 10x + 4y + 13 = 0$

2) $y^2 - 4x - 4y + 12 = 0$

3) $x^2 + 4y^2 - 2x - 3 = 0$

4) $25x^2 + 16y^2 + 100x - 32y - 284 = 0$

Homework

c. Discuss and sketch the following curves:

1) $x^2 - 16y - 6x + 9 = 0$

2) $y^2 + 6x + 8x - 15 = 0$

3) $16x^2 + 4y^2 - 64x + 8y + 4 = 0$

4) $9x^2 + 16y^2 - 36x + 32y = 92$