Arab Academy for Science and Technology & Maritime Transport
College of Engineering and Technology
Department of Basic and Applied Sciences

MATH 1

- *Rules of differentiation*
- *Trigonometric functions and their derivatives*
- *Inverse trigonometric functions and their derivatives*
- *Logarithmic function and its derivative*
- *Exponential function and its derivative*
- *Derivatives of Hyperbolic and inverse Hyperbolic functions*
- *Parametric function and its derivative*
- *Implicit function and its derivative*
- *The n^{th} derivative*
- *L'Hôpital's rule (The limit of a function)*
- *Maclaurin's expansions*
- *Partial differentiation*
- *Curve sketching*
- *Physical application (velocity and acceleration)*
- *Conic sections (Parabola – Ellipse)*
- *Software application*

Arab Academy for Science and Technology
College of Engineering and Technology
Department of Basic and Applied Sciences

Syllabus for Mathematics 1 (BA123)
Text Book: Calculus,
Sherman K. Stein & Anthony Barcellos
Program title: All Programs
Coordinator: Dr. Nasser El-Maghraby (R. 222)

Week	Topic	Ex. In Class	Assignment
1	Rate of change and rules of differentiation	**Sheet 1**	
2	Trigonometric functions and their derivatives	**Sheet 2**	
3	Inverse trigonometric functions and their derivatives	**Sheet 3**	
4	Logarithmic function and its derivative	**Sheet 4**	**Sheet 1- Sheet 2**
5	Exponential function and its derivative	**Sheet 5**	
6	Derivatives of Hyperbolic and inverse Hyperbolic functions	**Sheet 6**	**Sheet 4- Sheet 5**

7	Parametric differentiation and Implicit differentiation	**Sheet 7**	
8	The n^{th} derivative	**Sheet 8**	
9	The limit of a function, L’Hôpital’s rule	**Sheet 9**	**Sheet 6– Sheet 8**
10	Partial differentiation	**Sheet 10**	
11	Maclaurin’s expansions	**Sheet 11**	
12	Curve sketching: Critical points, maximum and minimum points, inflection points.	**Sheet 12**	**Sheet 9–Sheet 11**
13	Physical application: velocity and acceleration	**Sheet 12**	
14	Conic section : parabola Equation, vertex, focus, directrix, eccentricity, graph	**Sheet 13**	
15	Conic section : ellipse Equation, axes, foci, directrices, eccentricity, graph.	**Sheet 13**	**Sheet 12– Sheet 13**

Sheet 1 : Basic Differentiation Rules

	$y = y(x)$		$y' = dy/dx = y'(x)$
1.	$y = k$	,k: constant	$y' = 0$
2.	$y = x^k$	,k is constant	$y' = kx^{k-1}$
3.	$y = f(x) \pm g(x)$		$y' = f'(x) \pm g'(x)$
4.	$y = kf(x)$	,k is constant	$y' = kf'(x)$
5.	$y = f(x)\,g(x)$		$y' = f(x)g'(x) + f'(x)g(x)$
6.	$y = [f(x)]^k$	,k is constant	$y' = k[f(x)]^{k-1} f'(x)$
7.	$y = f(x)/g(x)$		$y' = [f'(x)g(x) - f(x)g'(x)]/g^2(x)$

Lecture Examples

a. Find dy/dx for each of the following

1) $y = x^4 - 3x^{-2} + 15x + 10$ **2)** $y = (\sqrt{x} - 1)^7$

3) $y = \dfrac{1}{(x^6 - 2)^5}$ **4)** $y = (x^3 - 1)^5 (2 + 3x^{-4})^7$

5) $y = \dfrac{x^3 - 1}{x^3 + 1}$ **6)** $y = \left(\dfrac{x^2 - 3}{x^{-4} + 2}\right)^{4/3}$

b. Find d^2y/dx^2 for each of the following

1) $y = x^7 - \dfrac{2}{x^3} + x^{-5} + 16x + 5$ **2)** $y = (2 - x^3)^8$

c. If $y = (x + \sqrt{x^2 - 1})^4$**, Show that** $y' = \dfrac{4y}{\sqrt{x^2 - 1}}$

Classroom Exercises

d. Find dy/dx for each of the following

1) $y = \frac{3}{x} - \frac{4}{x^2} + 6x^5 + 7$

2) $y = (x^4 - 1)^6$

3) $y = \left(\frac{1 - x^4}{1 + x^4}\right)^{3/2}$

4) $y = \sqrt{x^3 - 1}\,(1 - 3x)^5$

5) $y = \sqrt{\frac{x - 1}{x + 1}}$

6) $y = \left(1 - \frac{1}{\sqrt{x}}\right)^{-4/3}$

e. Find d^{2}y/dx^2 for each of the following

1) $y = (x^3 - 1)^6$

2) $y = \frac{x^2 - 1}{x^2 + 1}$

Homework

f. Find dy/dx for each of the following

1) $y = \sqrt{x^5 - 4}$

2) $y = x^{-3}(1 + x^4)^5$

3) $y = (x^2 + 4)^6 (1 - 2x)^7$

4) $y = \sqrt{x}\,(1 - \sqrt{x})^6$

5) $y = (\sqrt{1 + x^2})^5 \sqrt[3]{x^4 - 1}$

6) $y = \left(\frac{x^2 - 4}{x^2 + 2}\right)^{7/2}$

g. Find d^{2}y/dx^2 for each of the following

1) $y = (x^{3/2} - 1)^4$

2) $y = \frac{x^2 - 1}{\sqrt{x + 1}}$

Sheet 2 : Trigonometric Functions and their Derivatives

$\cos^2 u + \sin^2 u = 1$

$1 + \tan^2 u = \sec^2 u$

$\cot^2 u + 1 = \operatorname{cosec}^2 u$

$\sin 2u = 2 \sin u \cos u$

$\cos 2u = \cos^2 u - \sin^2 u$

$\quad = 2\cos^2 u - 1$

$\quad = 1 - 2\sin^2 u$

$y = \cos u \longrightarrow y' = -u' \sin u$

$y = \sin u \longrightarrow y' = u' \cos u$

$y = \tan u \longrightarrow y' = u' \sec^2 u$

$y = \cot u \longrightarrow y' = -u' \operatorname{cosec}^2 u$

$y = \sec u \longrightarrow y' = u' \sec u \tan u$

$y = \operatorname{cosec} u \longrightarrow y' = -u' \operatorname{cosec} u \cot u$

Lecture Examples

a. Find dy/dx for each of the following

1) $y = \sin x^3$

2) $y = (1 + \cos^3 x)\cot^2 2x$

3) $y = x^3 \cos x^2 - 2\cot x^{-3}$

4) $y = \dfrac{x \sin 2x}{1 - \cos^2 3x}$

5) $y = \sec^3 \sqrt{4x^2 + 1}$

6) $y = \dfrac{\sin(x-1)}{x-1}$

b. Find d^2y/dx^2 for each of the following

1) $y = (1 - \cos^2 x)^{-3/2}$

2) $y = x \sec x$

c. If y = a sin ct + b cos ct , where a,b and c are constants, prove that y" = -c^{2}y

Classroom Exercises

d. Find dy/dx for each of the following

1) $y = \tan^2(\cos^3 x^2)$

2) $y = \sqrt{x^2+1}\cos^3\sqrt{x^2-1}$

3) $y = x^2 \sec^3 x - 4\cot^2 x^3$

4) $y = \dfrac{1 - \sin 2x}{1 + \cos 2x}$

5) $y = \sqrt{\tan^2 x + x\cos^3 x}$

6) $y = \sqrt{x-1}\sin\sqrt{x-1}$

e. Find d^2y/dx^2 for each of the following

1) $y = x^4(\cos 2x)$

2) $y = \dfrac{\sin x}{x}$

Homework

f. Find dy/dx for each of the following

1) $y = \sec^3 \sqrt{\cos x}$

2) $y = \cot(\sqrt{x}\,\tan\sqrt{x})$

3) $y = x^{3/2} \cot x^3$

4) $y = \operatorname{cosec}^4 \sqrt{x^2 - 1}$

5) $y = (1 - \sin\sqrt{x})^3 \cos\sqrt{x}$

6) $y = \sqrt{x}\,\operatorname{cosec}\sqrt{x}$

g. Find d²y/dx² for each of the following

1) $y = x \tan x^3$

2) $y = \sin^3 x$

Sheet 3: Inverse Trigonometric Functions and their Derivatives

$$y = \sin^{-1} u \rightarrow y' = \frac{u'}{\sqrt{1-u^2}} \qquad y = \cos^{-1} u \rightarrow y' = \frac{-u'}{\sqrt{1-u^2}}$$

$$y = \tan^{-1} u \rightarrow y' = \frac{u'}{1+u^2} \qquad y = \cot^{-1} u \rightarrow y' = \frac{-u'}{1+u^2}$$

$$y = \sec^{-1} u \rightarrow y' = \frac{u'}{|u|\sqrt{u^2-1}} \qquad y = \operatorname{cosec}^{-1} u \rightarrow y' = \frac{-u'}{|u|\sqrt{u^2-1}}$$

Lecture Examples

a. Find dy/dx for each of the following

1) $y = \cos^{-1}\sqrt{x}$

2) $y = (x^2 + 4)\operatorname{cosec}^{-1} 2x$

3) $y = x^3(1 - \sec^{-1} x)$

4) $y = x^3 \sin^{-1}\sqrt{x} - 2\cot^{-1} x^2$

5) $y = \tan^{-1}\left(\dfrac{x-1}{x+1}\right)$

6) $y = \dfrac{1 - \sin^{-1} x}{\cos^{-1} x}$

b. If y = tan(cos $^{-1}$x), Prove that $y' = \dfrac{-(y^2+1)}{\sqrt{1-x^2}}$

Classroom Exercises

c. Find dy/dx for each of the following

1) $y = \sqrt{x}\,\tan^{-1}\sqrt{x}$

2) $y = \cot^{-1}\left(\dfrac{\cos 3x}{1+\sin 3x}\right)$

3) $y = x^3 \sec^{-1} x^2$

4) $y = \dfrac{\cos^{-1} x}{1-\sin^{-1} x}$

5) $y = \sqrt{x^2-1}\,\sin^{-1} x - x\cos^{-1} x$

6) $y = \tan^{-1}(\cos x) + \cot^{-1}(\sin x)$

d. If y = cos(2sin $^{-1}$x), prove that $(1-x^2)(y')^2 = 4(1-y^2)$

Homework

e. Find dy/dx for each of the following

1) $y = \sin^{-1} x^3$

2) $y = \cot^{-1}(\cos 2x)$

3) $y = \cot^{-1}\left(\dfrac{x-4}{x+4}\right)$

4) $y = \dfrac{\tan^{-1} x}{1-\cos^{-1}\sqrt{x}}$

5) $y = x^2 \operatorname{cosec}^{-1}\sqrt{x} - 3x\sin^{-1} x$

6) $y = \sqrt[3]{x}\,\sec^{-1}\left(\dfrac{x}{4}\right)$

f. Prove that $\dfrac{d}{dx}\left(\tan^{-1}\left(\dfrac{x-1}{x+1}\right)\right) = \dfrac{d}{dx}\left(\tan^{-1}(x)\right)$

Sheet 4 : Logarithmic Function and Its Derivatives

$\ln b = a \Leftrightarrow b = e^a$ **, e = 2.71828 .....,b ≥ 0**

1. **ln(1) = 0, ln(0) = -** ∞**, ln(*e*) = 1**
2. **ln(*ab*) = ln a + ln b**
3. **ln(a^n) = n ln a**

$$y = \ln u \rightarrow y' = \frac{u'}{u}$$

4. **ln (e^n) = n**

5. **ln *(a/b)* = ln *a* – ln *b***

Lecture Examples

a. Find dy/dx for each of the following

1) $y = x^3 \ln x$

2) $y = \ln(x^{-4}(x^5 - 2)^6)$

3) $y^x = x^y$

4) $y = \ln\left[\dfrac{x^3(1-x^2)^4}{\sin x\,(x-1)^5}\right]^{7/2}$

5) $y = \sin x^2 - 3x\cos x - x^x$

6) $(\sin x)^{\cos y} = (\sin y)^{\cos x}$

b. If y = ln(sec x + tan x)**, Prove that** y'' = sec x tan x **.**

Classroom Exercises

c. Find dy/dx for each of the following

1) $y = (\ln x)^3$

2) $y = \ln\left(\dfrac{x^3 - 1}{x^2 + 1}\right)$

3) $y = \sqrt[4]{\dfrac{(1-x)^3 \tan^{-1} x}{x^x \sec x^3}}$

4) $y^{5/2} = x^{\ln x}$

5) $y = \dfrac{x^x (2 - \sin x)^{x^2}}{x^{\cos x}(1 - 2lnx)^5}$

6) $y = x\cos\sqrt{x} - x^{\sec x}$

d. If y = cos(lnx) + sin(lnx)

prove that $x^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = 0$

Homework

e. Find dy/dx for each of the following

1) $y = \ln(1-\ln x)$

2) $y = \ln\left[\dfrac{(1-x^2)^5(2-\sin^{-1}x)}{(1-\ln x)^2(3-\cos x)}\right]$

3) $y = \dfrac{(x-1)^3(1-\sin x)^4}{x^x(2-\cos x)^2}$

4) $\sqrt{y} = \dfrac{x^5\tan^{-1}x}{(1+x)\sqrt[3]{x}}$

5) $y = x^{\sin x}$

6) $y = (\ln(\sin x))^{\cos x}$

Sheet 5 : Exponential Function and Its Derivative

1. $e^a e^b = e^{a+b}$
2. $e^a / e^b = e^{a-b}$
3. $(e^a)^b = e^{ab}$
4. $e^{\ln a} = a$

$y = e^u \rightarrow y' = u' e^u$

Lecture Examples

a. Find dy/dx for each of the following

1) $y = e^{\sin^{-1}x}$

2) $y = e^{\tan^{-1}\sin x}$

3) $y = \cos^3 e^{x^2}$

4) $y = \cos^{-1}(1-e^{-x})$

5. $y = \operatorname{cosec}^{-1}e^x - x^4 e^{\cot x}$

6. $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

b. If $y=\dfrac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$, show that $y'=\dfrac{8}{\left(e^{2x}+e^{-2x}\right)^{2}}$.

c. If $y=\tan^{-1}\ln e^{\tan\sqrt{x}}$, show that yy’ = 1/ 2

Classroom Exercises

d. Find dy/dx for each of the following

1) $y=x^{3}e^{x^{5}-3}$

2) $y=e^{\ln\sin^{-1}(\sin x3)}$

3) $y=\sqrt{e^{\cos^{-1}x}}$

4) $y=\ln\left[\dfrac{e^{x^{2}}\sin x^{3}}{(1-e^{x})(2-x)}\right]^{6}$

e. If $y=\ln(\cos x)$**, show that** $y''+e^{-2y}=0$

f. Find d^2y/dx^2 **for each of the following**

1) $y=e^{\sin x}$

2) $y=\cos e^{3x}$

Homework

g. Find dy/dx for each of the following

1) $y=e^{x^{3}}$

2) $y=e^{\cos^{-1}(\sin x)}$

3) $y=\ln\left[\dfrac{1+e^{2x}}{(1-e^{-2x})^{3}}\right]$

4) $y=x^{5}\sec e^{-x}$

h. If $y=ae^{-2x}+be^{3x}$ **where a and b are constant**

Show that

$$\frac{d^{2}y}{dx^{2}}-\frac{dy}{dx}-6y=0.$$

i. Find d^2y/dx^2 **for each of the following**

1) $y = e^{-4x}$ **2)** $y = \ln(e^{2x} - 4)$

Sheet 6 : Derivatives of Hyperbolic Functions and Their Inverse

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2} \qquad \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\operatorname{cosech} x = \frac{2}{e^x - e^{-x}} \qquad \operatorname{sech} x = \frac{2}{e^x + e^{-x}} \qquad \coth x = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$\cosh^2 u - \sinh^2 u = 1$

$1 - \tanh^2 u = \sec h^2\, u$

$\coth^2 u - 1 = \operatorname{cosech}^2 u$

$\sinh 2u = 2\sinh u \cosh u$

$\cosh 2u = \cosh^2 u + \sinh^2 u$

$\tan 2u = 2\tanh u / (1 + \tanh^2 u)$

$y = \sinh u \rightarrow y' = u' \cosh u$

$y = \cosh u \mapsto y' = u' \sinh u$

$y = \tanh u \rightarrow y' = u' \sec h^2 u$

$y = \coth u \rightarrow y' = -u' \operatorname{cosech}^2 u$

$y = \operatorname{sech} u \mapsto y' = -u' \operatorname{sech} u \tanh u$

$y = \operatorname{cosech} u \rightarrow y' = -u' \operatorname{cosech} u \coth u$

$$y = \sinh^{-1} u \rightarrow y' = \frac{u'}{\sqrt{u^2 + 1}}$$

$$y = \cosh^{-1} u \rightarrow y' = \frac{u'}{\sqrt{u^2 - 1}}$$

$$y = \tanh^{-1} u \rightarrow y' = \frac{u'}{1 - u^2} \quad , \ |u| \prec 1$$

Lecture Examples

a. **Find dy/dx for each of the following**

1) $y = x^4 \cosh^2 x^3$

2) $y = \tanh(x \ln x)$

3) $y = e^{\cosh^{-1} x^2}$

4) $y = (\sin^{-1} \sqrt{x})(1 - \cosh^{-1} x^2)$

5) $y=\sqrt[5]{x^3}\tanh^{-1}x^2$

6) $y=\ln\left[\dfrac{(x+1)^2 e^{\cos echx}}{\sqrt{x^3-1}}\right]$

b. Show that $\cosh^{-1}x=\ln(x\pm\sqrt{x^2-1}$

Classroom Exercises

c. Find dy/dx for each of the following

1) $\mathrm{y}=\sinh \mathrm{x}^3$

2) $y=\tanh^{-1}(\sec h2x)$

3) $\mathrm{y}=\sin\left(\cosh^{-1}\sqrt{\mathrm{x}^2+1}\right)$

4) $y=\sqrt{\cosh^{-1}(e^{-x/2})}$

d. Show that

$\sinh^{-1}x=\ln(x+\sqrt{x^2+1})$

e. Solve the following equations

1. $\mathrm{e}^{\cosh^{-1}\mathrm{x}}=2$

2. $\ln\left(\dfrac{1+\tanh x}{1-\tanh x}\right)=5$

Homework

f. Find dy/dx for each of the following

1. $y=x^2\coth^3\sqrt{x}$

2. $y=xe^{\sinh^{-1}x}$

3. $y=(1-\ln\sec x)\cosh^{-1}\sqrt{x}$

4. $\mathrm{y}=\ln\sqrt{\tanh 3\mathrm{x}}$

5. $y=\sinh^{-1}(\sin 2x)$

6. $y=\tanh^{-1}\sqrt{\sec x}$

f. Show that

$$\tanh^{-1} x = \frac{1}{2}\ln\frac{1+x}{1-x}$$

g. Solve

$5\cosh x - 4\sinh x = 3$

Sheet 7 : Parametric and Implicit Differentiation

Lecture Examples

a. Find dy/dx for each of the following

1) $y = t\ln t$, $y = \ln t/t$ **2)** $x = e^{t}\cosh t$, $y = e^{t}\sinh t$

b. Find d^2y/dx^2 for each of the following

1) $x = \operatorname{cosec} t$, $y = \cos 2t$ **2)** $x = \sqrt{1-t^2}$, $y = \sin^{-1} t$

c. If $x = \cos\dfrac{t}{1+t}$, $y = \sin\dfrac{t}{1+t}$, **show that** $y^3 y'' + 1 = 0$

d. If $x = \dfrac{t+1}{t-1}$, $y = \left(\dfrac{t-1}{t+1}\right)^5$, **show that** $y'' = 30x^{-7}$

e. Find dy/dx for each of the following

1) $x^3 - 3x^2y^4 + 7y^2 = 10$ **2)** $x + \cos^{-1} y = xy$

3) $\sin^{-1} x + \tan(xy) = 5$ **4)** $y = e^{-x} + e^{y}$

Classroom Exercises

f. Find dy/dx for each of the following

1) $x = \dfrac{3t}{1+t^3}$, $y = \dfrac{3t}{1+t^3}$ 2) $x = \sqrt{1-\sin\theta}$, $y = \sqrt{1+\cos\theta}$

g. Find d^2y/dx^2 for each of the following

1) $x = \cos\theta + \theta\sin\theta$, $y = \sin\theta - \theta\cos\theta$ **2)** $x = \sqrt{t^4 - 1}$, $y = \sec^{-1} t^2$

h. If $x = \tan\frac{t-1}{1+t}$ **,** $y = \sec\frac{t-1}{1+t}$ **, show that** $y'' = y^{-3}$ **.**

i. Find dy/dx for each of the following

1) $x^{-2}y^5 - 2xy^2 + 7x = 12$ **2)** $x + y^2 = e^{x/y}$

3) $\ln y = x + e^y$ **4)** $y^2 = \sin^3 2x + \cos^3 2y$

5) $x^{1+y} + y^{1+x} = 1$

Homework

j. If $x = \tan t - t$, $y = \tan^3 t$, Find y''.

k. If $x = t + \frac{1}{t}$, $y = t^2 + \frac{1}{t^2}$, Show that y'' = 2.

l. If $x = \frac{t-1}{t+1}$, $y = \frac{t+1}{t-1}$, Show that $y'' = 2y^3$.

m. **Find dy/dx for each of the following**

1) $y^4 - 4x^3y^2 + 6x^2 = 7$ **2)** $\tan^{-1} y = x^2 + y^2$

3) $y = e^{(x+y)^3}$ **4)** $\cosh^{-1} \sec y = xy^3$

Sheet 8 : The n^th Derivative

Lecture Examples

Find the n^th derivative for the following

1) $y = \ln(1+3x)$ **2)** $y = \sin 2x$

3) $y = e^{-(1+x)}$ **4)** $y = \frac{2}{1-4x}$

Classroom Exercises

5) $x = \ln(2-4x)$

6) $y = \dfrac{2x+1}{x-1}$

7) $y = \sin^2 x$

8) $y = \ln\dfrac{x+1}{2x+1}$

Homework

9) $y = \ln(1-2x)$

10) $y = \dfrac{1-x}{1+x}$

11) $y = \dfrac{1}{x-3}$

12) $y = 2\sin x + 3\cos x$

<u>Sheet 9 : L'Hôpital's Rule</u>

Lecture Examples

1) $\lim\limits_{x\to\pi/2} \dfrac{2\cos x}{2x-\pi}$

2) $\lim\limits_{x\to o} \dfrac{1-\cosh x}{x^2}$

3) $\lim\limits_{x\to o} \dfrac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x}$

4) $\lim\limits_{x\to 1} (1-x)\tan\dfrac{\pi x}{2}$

5) $\lim\limits_{x\to o} \dfrac{x\cos x+\tan 2x}{x\sec x+\sin 4x}$

6) $\lim\limits_{\phi\to o} (\operatorname{cosec}\phi - \cot\phi)$

7) $\lim\limits_{x\to\infty} \left(\dfrac{x+3}{x-1}\right)$

8) $\lim\limits_{x\leftarrow o} (\cos x)^{1/x^2}$

Classroom Exercises

1) $\lim\limits_{x\to\pi} \dfrac{1-\sin(x/2)}{\pi-x}$

2) $\lim\limits_{x\to 1} \dfrac{\cot(\pi x/2)}{1-\sqrt{x}}$

3) $\lim\limits_{x\to o} \dfrac{\tan x+\sec x-1}{\tan x-\sec x+1}$

4) $\lim\limits_{x\to 0} \dfrac{1-\sqrt{\cos x}}{x^2}$

5) $\lim_{x\to\pi/2}(\sec x-\tan x)$

6) $\lim_{x\to o}\dfrac{\sin 4x - x\cos x}{x\sec x-\tan 3x}$

7) $\lim_{x\to o}(\cos x)^{1/x}$

8) $\lim_{x\to\infty}\left(\dfrac{x}{x+1}\right)^{x}$

Homework

1) $\lim_{x\to 1}\dfrac{1-x^2}{\sin \pi x}$

2) $\lim_{x\to 1}\dfrac{\sin(x^3-1)}{x-1}$

3) $\lim_{x\to 0}\dfrac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1}$

4) $\lim_{x\to 0}\dfrac{\sin 3x}{1-\cos 4x}$

5) $\lim_{x\to o}\dfrac{\sinh x}{x}$

6) $\lim_{x\to 0}\dfrac{\tanh x}{x}$

7) $\lim_{x\to 0}(1+\sin x)^{1/x}$

8) $\lim_{x\to\infty}\left(\dfrac{x}{x+2}\right)^{x-2}$

Sheet 10 : Partial Differentiation

Lecture Examples

a. Find the first partial derivatives for each of the following

1) $z=(x^2-y)\sin x^3$

2) $z=(\sin 2y)^x$

b. If $z=\tan^{-1}\dfrac{y}{x}$ **show that**

1) $x\dfrac{\partial z}{\partial y}-y\dfrac{\partial z}{\partial x}=1$

2) $\dfrac{\partial^2 z}{\partial x^2}+\dfrac{\partial^2 z}{\partial x^2}=0$

c. If $z=f(x^2+y^2)$ **show that** $x\dfrac{\partial z}{\partial y}-y\dfrac{\partial z}{\partial x}=0$

Classroom Exercises

d. Find the first partial derivatives for each of the following

1) $z = y^2(x^4 - 1)^5 + 6y^2x$ **2)** $z = x^2 \sin\sqrt{x} + y\cos(xy)$

3) $z = \tan^{-1}\dfrac{y}{x}$ **4)** $z = e^{x/y}\tanh^{-1}(x^2 + y^2)$

e. If $z = \cot^{-1}\dfrac{y}{x}$ show that $\dfrac{\partial^2 z}{\partial y^2} + \dfrac{\partial^2 z}{\partial y^2} = 0$

f. If $z = \tan^{-1}\dfrac{x-1}{y-1}$ show that $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} = 0$

Homework

g. Find the first partial derivatives for each of the following

1) $z = x^3 - 3x^2y^4 + y^2$ **2)** $z = (x+y)\sin(x-y)$

3) $z = e^{\frac{y}{x}}\ln\dfrac{x}{y}$ **4)** $z = (1+\sin y)^{1+\cos x}$

h. If $z = \ln(x^2 + y^2)$ show that $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} = 0$

i. If $z = \cot^{-1}\dfrac{x}{y}$, show that

1) $y\dfrac{\partial z}{\partial x} - x\dfrac{\partial z}{\partial y} = -1$ and 2) $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} = 0$

Sheet 11 : Maclaurin's Expansion

Maclaurin's Expansion:

$$f(x) = f(0) + f'(0)\frac{x}{1!} + f''(0)\frac{x^2}{2!} + f'''(0)\frac{x^3}{3!}\cdots + f^{(n)}(0)\frac{(x)^n}{n!} + \cdots$$

Lecture Examples

a. Find **Maclaurin's Expansion** of each of the following :

(1) $f(x) = \sin 2x$

(2) $f(x) = \ln(2+3x)$, *Find approximate value to* $\ln(2.3)$.

b. Using Maclaurin's Expansion, show that

(1) $e^{-x}\cos x = 1 - x + \frac{1}{3}x^3 - \frac{1}{6}x^4 + \cdots$

(2) $\frac{\cos x}{\sqrt{1+x}} = 1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3 + \cdots$

Classroom Exercises

c. Find Maclaurin's Expansion for each of the following

1) $f(x) = \cos 3x$

2) $f(x) = \frac{1}{\sqrt{1+x}}$

3) $f(x) = e^{-3x}$

d. Using Maclaurin's Expansion show that :

1) $e^x \sin x = x + x^2 + \frac{x^3}{3} - \frac{x^5}{30} + \cdots$

2) $\frac{e^x}{1-x} = 1 + 2x + \frac{5}{2}x^2 + \frac{8}{3}x^3 + \cdots$

Homework

e. Find Maclaurin's Expansion for each of the following

1) $f(x) = \frac{1}{x+1}$

2) $f(x) = \cos 3x$

f. Using Maclaurin's Expansion, show that

(1) $\frac{e^{-x}}{1-x} = 1 + \frac{x^2}{2} + \frac{x^3}{3} + \cdots$

(2) $\sinh x + \cosh x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \cdots$

Sheet 12 : Differentiation applications

y'

- + ve $\longrightarrow$ the interval is decreasing
- zero $\longrightarrow$ Critical point
- - ve $\longrightarrow$ the interval is decreasing

Critical points $\longrightarrow$ y'

- + ve $\longrightarrow$ the critical point is local minimum
- - ve $\longrightarrow$ the critical point is local maximum

y''

- + ve $\longrightarrow$ the interval is concave up $\cup$
- zero $\longrightarrow$ Inflection point
- - ve $\longrightarrow$ the interval is concave down $\cap$

The equation of motion of any particle is given by:

- The displacement of the particle:

$$s = s(t)$$

- then, its velocity is given by:

$$v = \frac{ds}{dt}$$

- and its acceleration is given by:

$$a = \frac{dv}{dt} = \frac{d^2 s}{d t^2}$$

Lecture Examples

i) In each of the following curves,

1) $y = x^2 - 4x + 3$ 2) $y = -2x^2 + 12x + 7$

find

a. The critical point.
b. The intervals in which the curve is increasing and decreasing.
c. The local maximum and minimum points.
d. Sketch the curve.

ii) In each of the following curves,

1) $y = x^3 - 6x^2 + 10$ 2) $y = \left(x^2 - 9\right)^2$

find

a. The critical points.
b. The intervals in which the curve is increasing and decreasing.
c. The local maximum and minimum points.
d. The inflection point.
e. The concavity of the curve.
f. Sketch the curve

iii) Find the velocity and the acceleration for each of the following

1) $s(t) = t^3 - 6t^2 + 7$ 2) $s(t) = t^3 e^{t^4 - 1}$

Classroom Exercises

i) In each of the following curves,

1) $y = 2x^2 - 8x + 10$ 2) $y = -3x^2 - 12x$

find

a. The critical point.
b. The intervals in which the curve is increasing and decreasing.
c. The local maximum and minimum points.
d. Sketch the curve.

ii) In each of the following curves,

1) $y = 6x^2 - x^3$ 2) $y = x^3 - 3x^2 - 9x$

find

g. The critical points.
h. The intervals in which the curve is increasing and decreasing.
i. The local maximum and minimum points.
j. The inflection point.
k. The concavity of the curve.
l. Sketch the curve.

ii) Find the velocity and the acceleration for each of the following

1) $s(t) = \sin 5t - \cos 5t$ 2) $s(t) = t^6(1 - \ln t)^4$

Homework

i) In each of the following curves,

1) $y = 12x - 3x^2$ 2) $y = 3x^2 - 6x$

find

a. The critical point.
b. The intervals in which the curve is increasing and decreasing.
c. The local maximum and minimum points.
d. Sketch the curve.

iii) In each of the following curves,

1) $y = 2x^3 - 12x^2 + 18x$ 2) $y = x^3 - 9x^2 + 8$

find

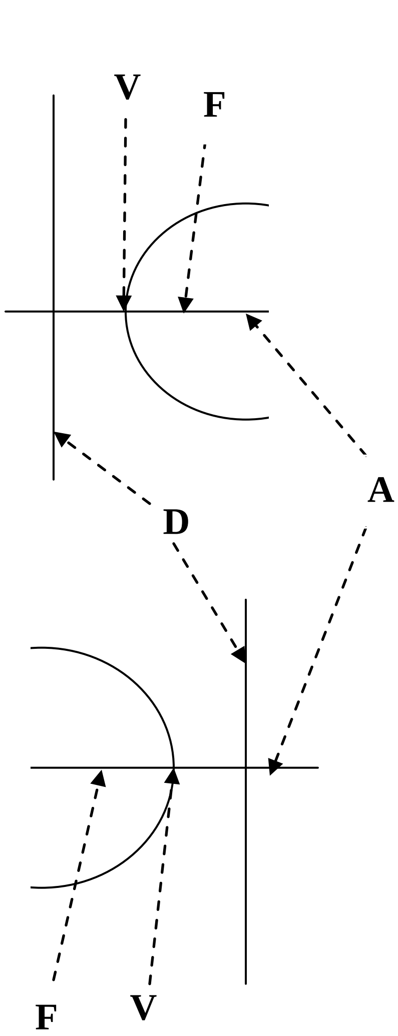
a. The critical points.
b. The intervals in which the curve is increasing and decreasing.
c. The local maximum and minimum points.
d. The inflection point.
e. The concavity of the curve.
f. Sketch the curve.

iv) Find the velocity and the acceleration for each of the following

1) $s(t) = t\sinh 3t + \cosh 3t$ 2) $s(t) = \dfrac{1 - e^{2t}}{e^{2t} - e^{2t}}$

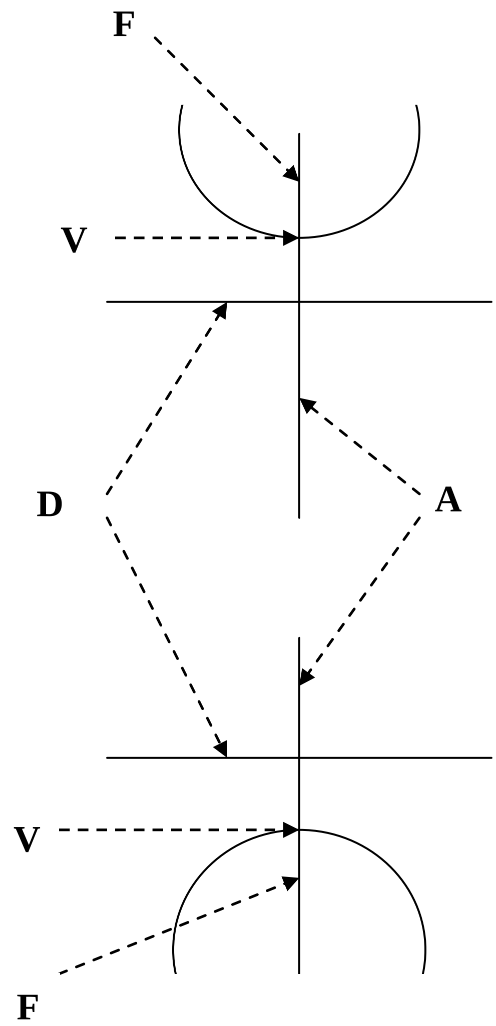
Sheet 13 : Conic Sections

The Parabola



$$(y-y_0)^2=4c(x-x_0)$$

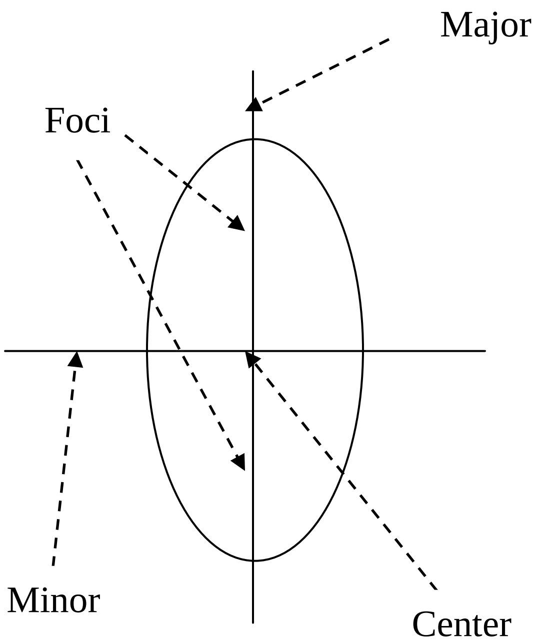
vertex	(x_0,y_0)
focus	(x_0+c,y_0)
axis	$y=y_0$
directrix	$x=x_0-c$

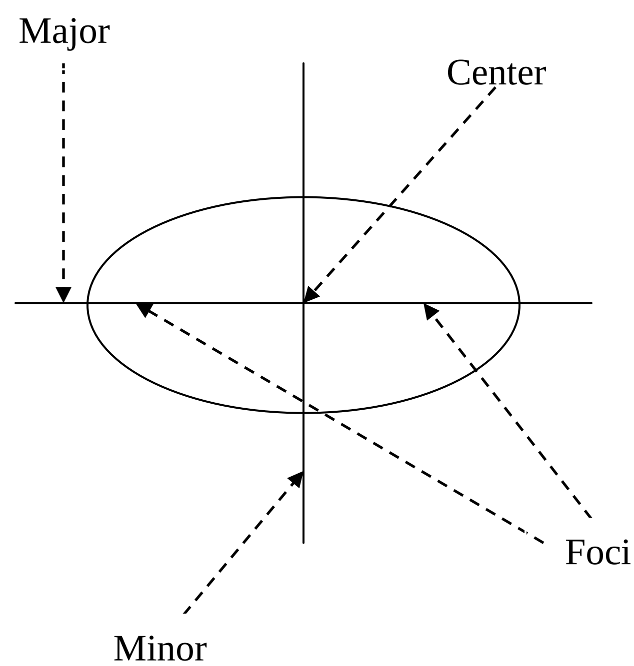


$$(x-x_0)^2=4c(y-y_0)$$

vertex	(x_0,y_0)
focus	(x_0,y_0+c)
axis	$x=x_0$
directrix	$y=y_0-c$

The Ellipse





$$\frac{(x-x_0)^2}{b^2}+\frac{(y-y_0)^2}{a^2}=1$$

Major $(2a)$	$x=x_0$
Minor $(2b)$	$y=y_0$
Center	(x_0,y_0)
Foci	$(x_0,y_0\pm c)$

$$\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}=1$$

Major $(2a)$	$y=y_0$
Minor $(2b)$	$x=x_0$
Center	(x_0,y_0)
Foci	$(x_0\pm c,y_0)$

Lecture Examples

a. Discuss and sketch the following curves:

1) $x^2 + 2x - 4y - 3 = 0$

2) $2y^2 - 4x - 4y - 14 = 0$

3) $4x^2 + 9y^2 + 24x = 0$

4) $2x^2 + 9y^2 + 8x - 72y + 134 = 0$

Classroom Exercises

b. Discuss and sketch the following curves:

1) $x^2 + 10x + 4y + 13 = 0$

2) $y^2 - 4x - 4y + 12 = 0$

3) $x^2 + 4y^2 - 2x - 3 = 0$

4) $25x^2 + 16y^2 + 100x - 32y - 284 = 0$

Homework

c. Discuss and sketch the following curves:

1) $x^2 - 16y - 6x + 9 = 0$

2) $y^2 + 6x + 8x - 15 = 0$

3) $16x^2 + 4y^2 - 64x + 8y + 4 = 0$

4) $9x^2 + 16y^2 - 36x + 32y = 92$