Differentiation

1. Basic Rules

- $\frac{d}{dx} x^n = nx^{n-1}$
- $\frac{d}{dx} k = 0$
- $\frac{d}{dx}(uv) = uv' + u'v$
- $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - v'u}{v^2}$
- $\frac{d}{dx} u^n = nu^{n-1}u'$
- $\frac{d}{dx}\cos(u) = -\sin(u).u'$
- $\frac{d}{dx}\sin(u) = \cos(u).u'$
- $\frac{d}{dx}\tan(u) = \sec^2(u).u'$
- $\frac{d}{dx}\cot(u) = -\cos ec^2(u).u'$
- $\frac{d}{dx}\sec(u) = \sec(u)\tan(u).u'$
- $\frac{d}{dx}\cos ec(u) = -\cos ec(u)\cot(u).u'$

2. Equation of tangent and normal

- m = slope of tangent = dy / dx at (x_1,y_1)
- Equation of tangent y = m x + c
- Equation of normal y = -(1/m) x + c

3. Velocity and Acceleration

- S = distance
- Velocity = $\frac{d}{dt}(S)$ Acceleration = $\frac{d_2}{dt^2}(S)$, where t is the time.

Sheet (8)

Differentiation

Lecture

Find $\dfrac{dy}{dx}$ **for :**

1) $y = x^2(1+x^2)+\sqrt[3]{x}$
2) $y=\sqrt[3]{x^2}+\dfrac{3}{\sqrt{x}}+x^3$
3) $y=\sqrt{x}(3-x^2)+(6x^5-1)^3$
4) $y=\dfrac{\sqrt{x}}{(1+x^2)}$
5) $y = \tan x + \sin x$
6) $y = x^2 \sin x + (1+x^2)^4$
7) $y = \sec x + \cos x$

Section

Find $\dfrac{dy}{dx}$ **for :**

1) $y = x^3(1+x^2) + (x^2+1)^2$
2) $y = x^3 \cos x + (1-x^2)^3$
3) $y=\dfrac{x^3}{(1-x^2)^2}+\tan x$

4) $y = \dfrac{x^2}{(x^2 + \cos x)}$

5) $y = \cos x + \sin x$

6) $y = x\sqrt{5 - x^2} + (3 - 5x^2)^4$

7) $y = \dfrac{\sqrt{x}}{(3 - \cos x)^2} + x^2 \sin x$

Home

Find $\frac{dy}{dx}$ for :

1. $y = x^2 + \sqrt[3]{x} + \sqrt[4]{x^3}$
2. $y = \sqrt{x}(1 + 3x^2)^3 - \sin x$
3. $y = \dfrac{1 - \cos x}{1 + \sqrt{x}}$
4. $y = \dfrac{x^2}{(1 + \sqrt[3]{x})^3}$
5. $y = x^2 - \dfrac{1}{\sqrt{1 + 3x}}$
6. $y = \tan x + \cos x$
7. $y = \cos x + \dfrac{\sqrt{x}}{(1 + \cos x)}$

Sheet (9)

Application for differentiation

Lecture

1. Find the equation of the tangent and normal lines to the curve

   $y = x^2 - \dfrac{1}{\sqrt{1+3x}}$ at (1, 0.5)

2. The distance of a particle is given by S = $2t^3 + 3t + 6$ in m where t is time in sec. Find the velocity and acceleration at t = 10 sec.

3. The distance of a train is given by S = $t^3 - 4t^2 + 3t + 15$ in km. And t time in hr. Find the train velocity while moving by decreasing acceleration 2 km/hr^2.

Section

1. Find the equation of the tangent and normal lines to the curve

   $y = x^2(1+x^2) - \dfrac{x}{1+\sqrt{x}}$ at point (1, 1.5)

2. The distance of a train is given by S = $3t^2 + 14t + 5$ in km. where t is time in hours. Find the velocity and acceleration when the train moves a distance of 10 km.

3. Find the acceleration of a car arriving to rest if the distance moved by the car is given by S = $3t^3 + 12t^2 - 9t + 17$ in km and t is the time in hours.

Home

1. Find the equation of the normal and tangent lines to the curve

$$y = \frac{x}{1+\sqrt{x}} - \sqrt{x^2+3} \quad \text{at } (1, -1.5)$$

2. If the distance of a particle moving is given by $S = 3t^2 + 6t + 5$ in km, where is the time in hours, find acceleration and velocity after the particle moves a distance 14 km.

3. Find the acceleration and distance moved by a car moving by constant velocity 30 km/hr if the car distance is given by $S = 3t^3 + 15t + 3$ where t in hrs. and S in km.

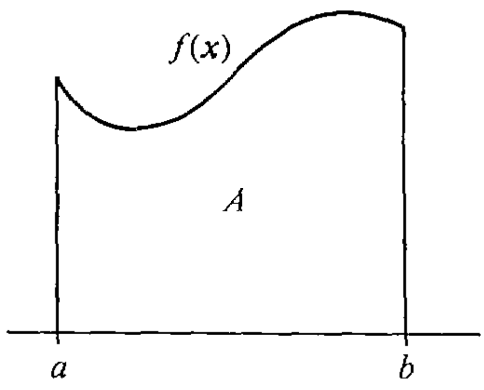
Integration

1. Basic Rules

- $\int (x'')dx = \frac{x^{n+1}}{n+1} + c \qquad n \neq -1$
- $\int kdx = kx + c$, k is constant
- $\int (x^n)dx = \frac{x^{n+1}}{n+1} + c \qquad n \neq -1$
- $\int_a^b$ f(x) dx = [F(x)] = F (b) – F (a) , where $\frac{d}{dx}F(x) = f(x)$
- $\int \sin(ax)dx = \frac{-\cos(ax)}{a} + c$
- $\int \cos(ax)dx = \frac{\sin(ax)}{a} + c$

2.Area under the Curve

Area A = $\int_a^b f(x)dx$



Sheet (10)

Integration

Lecture

Evaluate

1. $\int (3x^2 + 6x + 3)dx$
2. $\int \left\lfloor \sqrt[5]{x^2} + x(1 + x^2) \right\rfloor \; dx$
3. $\int (3x^2 + 3)^2 \; dx$
4. $\int_0^1 \left[(1 + 3x)^3 + \cos x \right] dx$
5. $\int_0^1 \frac{1}{\sqrt[4]{3x + 1}} \; dx$

Section

Evaluate

1. $\int (x^3 + 2x)^2 dx$
2. $\int \sqrt{x}(1 + x^2) \; dx$

3. $\int (\frac{\sqrt{x}+5}{x^2})\ dx$

4. $\int \frac{1}{\sqrt[3]{2+3x}}\ dx$

5. $\int_1^4 (x^3 + \frac{1}{\sqrt{x}} + 3x^2)\ dx$

6. $\int_0^1 \frac{dx}{\sqrt[3]{1-x}}$

7. $\int \cos(5x) dx$

Home

Evaluate

1. $\int (x^2 + 1)\ dx$
2. $\int (x^2 + 1)^2\ dx$
3. $\int (x + 1)^2\ dx$
4. $\int \left(\frac{1 + x^3}{x^2} \right)\ dx$
5. $\int \frac{dx}{\sqrt[4]{1-3x}}$
6. $\int_2^3 (x^3 + 2x + 1)\ dx$

Sheet (11)

Application for Integration

Evaluate the area under the following curves

Lecture

1. $y = 2x^2 + 3x - 1, \qquad x \in [0,2]$
2. $y = x^3, \qquad x \in [0,3]$
3. $y = 2\sin(x), \qquad x \in \left[0, \frac{\pi}{2}\right]$
4. $y = \cos\left(\frac{x}{2}\right), \qquad x \in [0, \pi]$

Section

1. $y = 2x^3 + x - 1, \qquad x \in [0,1]$
2. $y = x^2 - 1, \qquad x \in [0,2]$
3. $y = 2\sin(x) + x, \qquad x \in \left[0, \frac{\pi}{2}\right]$
4. $y = \cos(x), \qquad x \in \left[0, \frac{\pi}{2}\right]$

Home

1. $y = 2x^2 - 1, \qquad x \in [0,4]$
2. $y = x^3 - 5, \qquad x \in [0,3]$
3. $y = 4\sin(x), \qquad x \in \left[0, \frac{\pi}{2}\right]$
4. $y = x + \cos\left(\frac{x}{2}\right), \qquad x \in [0, \pi]$