

Differentiation

1. Basic Rules

- $\frac{d}{dx} x^n = nx^{n-1}$
- $\frac{d}{dx} k = 0$
- $\frac{d}{dx} (uv) = uv' + u'v$
- $\frac{d}{dx} \left(\frac{u}{v}\right) = \frac{u'v - v'u}{v^2}$
- $\frac{d}{dx} u^n = nu^{n-1}u'$
- $\frac{d}{dx} \cos(u) = -\sin(u).u'$
- $\frac{d}{dx} \sin(u) = \cos(u).u'$
- $\frac{d}{dx} \tan(u) = \sec^2(u).u'$
- $\frac{d}{dx} \cot(u) = -\operatorname{cosec}^2(u).u'$
- $\frac{d}{dx} \sec(u) = \sec(u)\tan(u).u'$
- $\frac{d}{dx} \operatorname{cosec}(u) = -\operatorname{cosec}(u)\cot(u).u'$

2. Equation of tangent and normal

- $m = \text{slope of tangent} = dy / dx$ at (x_1, y_1)
- Equation of tangent $y = mx + c$
- Equation of normal $y = -(1/m)x + c$

3. Velocity and Acceleration

- $S = \text{distance}$
- Velocity = $\frac{d}{dt}(S)$ Acceleration = $\frac{d^2}{dt^2}(S)$, where t is the time.

Sheet (8)

Differentiation

Lecture

Find $\frac{dy}{dx}$ for :

1) $y = x^2(1+x^2) + \sqrt[3]{x}$

2) $y = \sqrt[3]{x^2} + \frac{3}{\sqrt{x}} + x^3$

3) $y = \sqrt{x}(3-x^2) + (6x^5 - 1)^3$

4) $y = \frac{\sqrt{x}}{(1+x^2)}$

5) $y = \tan x + \sin x$

6) $y = x^2 \sin x + (1+x^2)^4$

7) $y = \sec x + \cos x$

Section

Find $\frac{dy}{dx}$ for :

1) $y = x^3(1+x^2) + (x^2+1)^2$

2) $y = x^3 \cos x + (1-x^2)^3$

3) $y = \frac{x^3}{(1-x^2)^2} + \tan x$

$$4) y = \frac{x^2}{(x^2 + \cos x)}$$

$$5) y = \cos x + \sin x$$

$$6) y = x\sqrt{5-x^2} + (3-5x^2)^4$$

$$7) y = \frac{\sqrt{x}}{(3-\cos x)^2} + x^2 \sin x$$

Home

Find $\frac{dy}{dx}$ for :

$$1. y = x^2 + \sqrt[3]{x} + \sqrt[4]{x^3}$$

$$2. y = \sqrt{x}(1+3x^2)^3 - \sin x$$

$$3. y = \frac{1-\cos x}{1+\sqrt{x}}$$

$$4. y = \frac{x^2}{(1+\sqrt[3]{x})^3}$$

$$5. y = x^2 - \frac{1}{\sqrt{1+3x}}$$

$$6. y = \tan x + \cos x$$

$$7. y = \cos x + \frac{\sqrt{x}}{(1+\cos x)}$$

Sheet (9)

Application for differentiation

Lecture

1. Find the equation of the tangent and normal lines to the curve

$$y = x^2 - \frac{1}{\sqrt{1+3x}} \text{ at } (1, 0.5)$$

2. The distance of a particle is given by $S = 2t^3 + 3t + 6$ in m where t is time in sec. Find the velocity and acceleration at $t = 10$ sec.
3. The distance of a train is given by $S = t^3 - 4t^2 + 3t + 15$ in km. And t time in hr. Find the train velocity while moving by decreasing acceleration 2 km/hr^2 .

Section

1. Find the equation of the tangent and normal lines to the curve

$$y = x^2(1+x^2) - \frac{x}{1+\sqrt{x}} \text{ at point } (1, 1.5)$$

2. The distance of a train is given by $S = 3t^2 + 14t + 5$ in km. where t is time in hours. Find the velocity and acceleration when the train moves a distance of 10 km.
3. Find the acceleration of a car arriving to rest if the distance moved by the car is given by $S = 3t^3 + 12t^2 - 9t + 17$ in km and t is the time in hours.

Home

1. Find the equation of the normal and tangent lines to the curve

$$y = \frac{x}{1 + \sqrt{x}} - \sqrt{x^2 + 3} \quad \text{at } (1, -1.5)$$

2. If the distance of a particle moving is given by $S = 3t^2 + 6t + 5$ in km, where t is the time in hours, find acceleration and velocity after the particle moves a distance 14 km.
3. Find the acceleration and distance moved by a car moving by constant velocity 30 km/hr if the car distance is given by $S = 3t^3 + 15t + 3$ where t in hrs. and S in km.

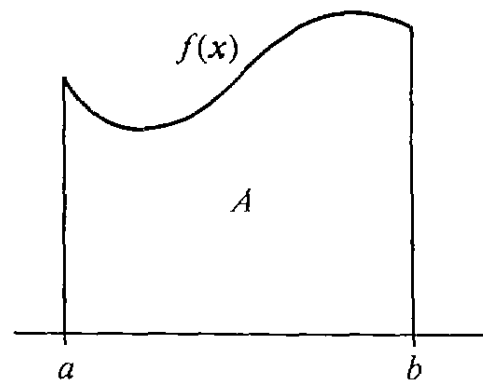
Integration

1. Basic Rules

- $\int (x^n) dx = \frac{x^{n+1}}{n+1} + c \quad n \neq -1$
- $\int k dx = kx + c$, k is constant
- $\int (x^n) dx = \frac{x^{n+1}}{n+1} + c \quad n \neq -1$
- $\int_a^b f(x) dx = [F(x)] = F(b) - F(a)$, where $\frac{d}{dx} F(x) = f(x)$
- $\int \sin(ax) dx = \frac{-\cos(ax)}{a} + c$
- $\int \cos(ax) dx = \frac{\sin(ax)}{a} + c$

2. Area under the Curve

Area $A = \int_a^b f(x) dx$



Sheet (10)

Integration

Lecture

Evaluate

1. $\int (3x^2 + 6x + 3) dx$

2. $\int \left[\sqrt[5]{x^2} + x(1+x^2) \right] dx$

3. $\int (3x^2 + 3)^2 dx$

4. $\int_0^1 \left[(1+3x)^3 + \cos x \right] dx$

5. $\int_0^1 \frac{1}{\sqrt[4]{3x+1}} dx$

Section

Evaluate

1. $\int (x^3 + 2x)^2 dx$

2. $\int \sqrt{x}(1+x^2) dx$

$$3. \int \left(\frac{\sqrt{x} + 5}{x^2} \right) dx$$

$$4. \int \frac{1}{\sqrt[3]{2 + 3x}} dx$$

$$5. \int_1^4 \left(x^3 + \frac{1}{\sqrt{x}} + 3x^2 \right) dx$$

$$6. \int_0^1 \frac{dx}{\sqrt[3]{1 - x}}$$

$$7. \int \cos(5x) dx$$

Home

Evaluate

$$1. \int (x^2 + 1) dx$$

$$2. \int (x^2 + 1)^2 dx$$

$$3. \int (x + 1)^2 dx$$

$$4. \int \left(\frac{1 + x^3}{x^2} \right) dx$$

$$5. \int \frac{dx}{\sqrt[4]{1 - 3x}}$$

$$6. \int_2^3 (x^3 + 2x + 1) dx$$

Sheet (11)

Application for Integration

Evaluate the area under the following curves

Lecture

1. $y = 2x^2 + 3x - 1, \quad x \in [0, 2]$
2. $y = x^3, \quad x \in [0, 3]$
3. $y = 2 \sin(x), \quad x \in [0, \frac{\pi}{2}]$
4. $y = \cos\left(\frac{x}{2}\right), \quad x \in [0, \pi]$

Section

1. $y = 2x^2 + x - 1, \quad x \in [0, 1]$
2. $y = x^2 - 1, \quad x \in [0, 2]$
3. $y = 2 \sin(x) + x, \quad x \in [0, \frac{\pi}{2}]$
4. $y = \cos(x), \quad x \in [0, \frac{\pi}{2}]$

Home

1. $y = 2x^2 - 1, \quad x \in [0, 4]$
2. $y = x^3 - 5, \quad x \in [0, 3]$
3. $y = 4 \sin(x), \quad x \in [0, \frac{\pi}{2}]$
4. $y = x + \cos\left(\frac{x}{2}\right), \quad x \in [0, \pi]$