# Solving simultaneous linear equations ( two variables ) :

Consider the following linear system of the two unknowns x and y

$$a_{11}x + a_{12}y = b_1 \;,$$
$$a_{21}x + a_{22}y = b_2$$

Solving this system , is to find the values of x and y which satisfy that system. We apply one of the two following methods:

## *i) Elimination Method*

We eliminate one of the two variables from the two equations to get the value of the other , then by direct substitution of the obtained value in any equation we get the value of the other unknown variable .

Example :

If we have

$8x - 2y = 5$ & multiplying by 3

$10x - 3y = 3$ multiplying by -2

then we have

$24x - 6y = 15$ &

$-20x + 6y = -6$

by adding the two equations , implies

$4x = 9$ so $x = 9/4$

by substitution of x in any of the two equations , we have

$y = 13/2$

## *ii) Cramer's Method*

The solution of the system is given by

$$x = \frac{\Delta_x}{\Delta} \quad , \quad y = \frac{\Delta_y}{\Delta}$$

Where

$$\Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0 \;, \; \Delta_x = \begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix} \text{ and } \Delta_y = \begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}$$

Note : $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a \times d - c \times b$

From the previous example, we have

$$\Delta = \begin{vmatrix} 8 & -2 \\ 10 & -3 \end{vmatrix} = -24 - (-20) = -4 \ ,$$

$$\Delta_x = \begin{vmatrix} 5 & -2 \\ 3 & -3 \end{vmatrix} = -15 - (-6) = -9$$

$$\Delta_y = \begin{vmatrix} 8 & 5 \\ 10 & 3 \end{vmatrix} = 24 - 50 = -26$$

This implies $x = 9/4$ *and* $y = 13/2$

## **Solving quadratic equations :**

1. by factorization :
   Examples :

1. $x^2 - 7x + 12 = 0$
   so we have $x^2 - 7x + 12 = (x - 3) . (x - 4) = 0$
   then we must have $(x - 3) = 0$ or $(x - 4) = 0$
   so $x = 3$ or $x = 4$

( not here that the last term ( the constant term ) has a positive sine so the tow factors must have the same sine of the middle term ( term in x ))

2. $x^2 - 17x - 84 = 0$
   so we have $x^2 - 17x - 84 = (x - 21) . (x + 4) = 0$
   then we must have $(x - 21) = 0$ or $(x + 4) = 0$
   so $x = 21$ or $x = -4$

(not here that the last term ( the constant term ) has a negative sine so the tow factors must have an opposite sine such that the sum of the tow terms in x resulting from the product of the tow factors gives the middle term ( term in x ))

2. by the formula :

| | |
|---|---|
| if we have | $a x^2 + b x + c = 0$ |
| then we have | $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ |

Example :

if we have $3x^2 + 2x - 1 = 0$

then a = 3 & b = 2 & c = -1

so $x = \dfrac{-2 \pm \sqrt{(-2)^2 - 4(3)(-1)}}{2(3)} = \dfrac{-2 \pm \sqrt{16}}{6} = \dfrac{-2 \pm 4}{6}$

then $x = \dfrac{1}{3}$ *or* $x = -1$

**Sheet (1)**

## <u>Solution of linear equations :</u>

1) Solve the following simultaneous linear equations for $x$ and $y$ :

Lecture

1. $\begin{aligned} 3x + 4y &= 11 \\ 5x - 2y &= 1 \end{aligned}$

2. $\begin{aligned} 6x + y &= 2 \\ 4x + 5y &= -3 \end{aligned}$

3. $\begin{aligned} 3x + 2y - 6 &= 0 \\ 4x + 4y - 9 &= 0 \end{aligned}$

Section

1. $\begin{aligned} 5x + 3y &= 1 \\ 2x + y &= 1 \end{aligned}$

2. $\begin{aligned} 2x + 6y &= 5 \\ 2y - 5x &= 13 \end{aligned}$

3. $\begin{aligned} 5x + 6y &= 72 \\ 7x + 8y &= 98 \end{aligned}$

Home work

1. $\begin{aligned} 8x - 2y &= 5 \\ 10x - 3y &= 3 \end{aligned}$

2. $\begin{aligned} 6x - 5y &= 75 \\ 5x - 3y &= 59 \end{aligned}$

3. $\begin{aligned} 6x + 7y &= 2 \\ 4x + 4y &= 1 \end{aligned}$

4. $\begin{aligned} 5x + 7y &= 63 \\ 3x - 6y &= -54 \end{aligned}$

# Solving quadratic equations :

1) Solve the following quadratic equations by factorization :

Lecture

1. $x^2+7x+12=0$
2. $p^2-17p+60=0$
3. $x^2+3xy-18y^2=0$
4. $6x^2-17x+12=0$
5. $4-3y-27y^2=0$

Section

1. $x^2+16x+63=0$
2. $z^2+2z-120=0$
3. $a^2b^2+15ab-54=0$
4. $3x^2-11x-42=0$
5. $8x^2+37a^2x-15a^4=0$

Home work

1. $x^2-7x+12=0$
2. $y^2-23y+132=0$
3. $z^2+28z-128=0$
4. $a^2x^2+12ax-189=0$
5. $x^2-3x-10=0$
6. $x^2+4x-32=0$
7. $x^2-12ax-64a^2=0$
8. $x^2+5x-84=0$
9. $x^2-17x-84=0$
10. $a^2b^2+15ab+54=0$

2) Solve the following quadratic equations by using the formula ( $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ ) :

Lecture

1. $y^2-5y-12=0$
2. $x^2+x-300=0$
3. $12x^2-11x+2=0$
4. $3.2x^2-12x+10.35=0$
5. $x^2-18x+87=0$

Section

1. $3y^2-9y-11=0$
2. $8z^2+5z-3=0$
3. $15x^2-8x-12=0$
4. $42x^2+167x+165=0$
5. $x^2-36x-323.7=0$

Home work

1. $x^2+x-1=0$
2. $-x^2-18x+87=0$
3. $57x^2-88x-105=0$
4. $x^2=40-3x$
5. $21x^2-29x+10=0$
6. $x^2-6x+7=0$
7. $3x^2+2x-1=0$
8. $7x^2+3x=5$
9. $3y^2-7y-3=0$
10. $4z^2-5z-5=0$