

Solving simultaneous linear equations (two variables) :

Consider the following linear system of the two unknowns x and y

$$\begin{aligned}a_{11}x + a_{12}y &= b_1, \\ a_{21}x + a_{22}y &= b_2\end{aligned}$$

Solving this system, is to find the values of x and y which satisfy that system. We apply one of the two following methods:

i) Elimination Method

We eliminate one of the two variables from the two equations to get the value of the other, then by direct substitution of the obtained value in any equation we get the value of the other unknown variable.

Example :

If we have

$$\begin{aligned}8x - 2y &= 5 && \text{\&} && \text{multiplying by 3} \\ 10x - 3y &= 3 && && \text{multiplying by -2}\end{aligned}$$

then we have

$$\begin{aligned}24x - 6y &= 15 && \text{\&} \\ -20x + 6y &= -6\end{aligned}$$

by adding the two equations, implies

$$4x = 9 \quad \text{so} \quad x = 9/4$$

by substitution of x in any of the two equations, we have

$$y = 13/2$$

ii) Cramer's Method

The solution of the system is given by

$$x = \frac{\Delta_x}{\Delta}, \quad y = \frac{\Delta_y}{\Delta}$$

Where

$$\Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0, \quad \Delta_x = \begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix} \quad \text{and} \quad \Delta_y = \begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}$$

Note : $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a \times d - c \times b$

From the previous example, we have

$$\Delta = \begin{vmatrix} 8 & -2 \\ 10 & -3 \end{vmatrix} = -24 - (-20) = -4 ,$$

$$\Delta_x = \begin{vmatrix} 5 & -2 \\ 3 & -3 \end{vmatrix} = -15 - (-6) = -9$$

$$\Delta_y = \begin{vmatrix} 8 & 5 \\ 10 & 3 \end{vmatrix} = 24 - 50 = -26$$

This implies $x = 9/4$ and $y = 13/2$

Solving quadratic equations :

1. by factorization :

Examples :

1. $x^2 - 7x + 12 = 0$

so we have $x^2 - 7x + 12 = (x - 3) \cdot (x - 4) = 0$

then we must have $(x - 3) = 0$ or $(x - 4) = 0$

so $x = 3$ or $x = 4$

(not here that the last term (the constant term) has a positive sine so the tow factors must have the same sine of the middle term (term in x))

2. $x^2 - 17x - 84 = 0$

so we have $x^2 - 17x - 84 = (x - 21) \cdot (x + 4) = 0$

then we must have $(x - 21) = 0$ or $(x + 4) = 0$

so $x = 21$ or $x = -4$

(not here that the last term (the constant term) has a negative sine so the tow factors must have an opposite sine such that the sum of the tow terms in x resulting from the product of the tow factors gives the middle term (term in x))

2. by the formula :

if we have	$ax^2 + bx + c = 0$
then we have	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example :

if we have $3x^2 + 2x - 1 = 0$
then $a = 3$ & $b = 2$ & $c = -1$
so $x = \frac{-2 \pm \sqrt{(-2)^2 - 4(3)(-1)}}{2(3)} = \frac{-2 \pm \sqrt{16}}{6} = \frac{-2 \pm 4}{6}$
then $x = \frac{1}{3}$ or $x = -1$

Sheet (1)

Solution of linear equations :

1) Solve the following simultaneous linear equations for x and y :

Lecture

1. $3x + 4y = 11$ $5x - 2y = 1$	2. $6x + y = 2$ $4x + 5y = -3$
3. $3x + 2y - 6 = 0$ $4x + 4y - 9 = 0$	

Section

1. $5x + 3y = 1$ $2x + y = 1$	2. $2x + 6y = 5$ $2y - 5x = 13$
3. $5x + 6y = 72$ $7x + 8y = 98$	

Home work

1. $8x - 2y = 5$ $10x - 3y = 3$	2. $6x - 5y = 75$ $5x - 3y = 59$
3. $6x + 7y = 2$ $4x + 4y = 1$	4. $5x + 7y = 63$ $3x - 6y = -54$

Solving quadratic equations :

1) Solve the following quadratic equations by factorization :

Lecture

1. $x^2 + 7x + 12 = 0$
2. $p^2 - 17p + 60 = 0$
3. $x^2 + 3xy - 18y^2 = 0$
4. $6x^2 - 17x + 12 = 0$
5. $4 - 3y - 27y^2 = 0$

Section

1. $x^2 + 16x + 63 = 0$
2. $z^2 + 2z - 120 = 0$
3. $a^2b^2 + 15ab - 54 = 0$
4. $3x^2 - 11x - 42 = 0$
5. $8x^2 + 37a^2x - 15a^4 = 0$

Home work

1. $x^2 - 7x + 12 = 0$
2. $y^2 - 23y + 132 = 0$
3. $z^2 + 28z - 128 = 0$
4. $a^2x^2 + 12ax - 189 = 0$
5. $x^2 - 3x - 10 = 0$
6. $x^2 + 4x - 32 = 0$
7. $x^2 - 12ax - 64a^2 = 0$
8. $x^2 + 5x - 84 = 0$
9. $x^2 - 17x - 84 = 0$
10. $a^2b^2 + 15ab + 54 = 0$

2) Solve the following quadratic equations by using the formula

$$\left(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right) :$$

Lecture

1. $y^2 - 5y - 12 = 0$
2. $x^2 + x - 300 = 0$
3. $12x^2 - 11x + 2 = 0$
4. $3.2x^2 - 12x + 10.35 = 0$
5. $x^2 - 18x + 87 = 0$

Section

1. $3y^2 - 9y - 11 = 0$
2. $8z^2 + 5z - 3 = 0$
3. $15x^2 - 8x - 12 = 0$
4. $42x^2 + 167x + 165 = 0$
5. $x^2 - 36x - 323.7 = 0$

Home work

1. $x^2 + x - 1 = 0$
2. $-x^2 - 18x + 87 = 0$
3. $57x^2 - 88x - 105 = 0$
4. $x^2 = 40 - 3x$
5. $21x^2 - 29x + 10 = 0$
6. $x^2 - 6x + 7 = 0$
7. $3x^2 + 2x - 1 = 0$
8. $7x^2 + 3x = 5$
9. $3y^2 - 7y - 3 = 0$
10. $4z^2 - 5z - 5 = 0$