Arab Academy For Science & Technology

[Basic and Applied Science Dept.]

Math 1 [EB127]
Analytic Geometry , Determinants , Matrices , System of Linear Equations, Curve Fitting , Linear Programming

Math I – EB127

Syllabus for Mathematics I , Course no. : EB 127

Week	Sheet	Topics	Book Ref.
1	1	**Analytic Geometry** - Coordinates - General equation of a straight line	P. 99 P. 127
2	1	Equation of a straight line - Point slope form - Two point form - Slope and intersection point	P. 129-132
3	1	Parallel and Perpendicular lines	P.133-134
4	1	Equation of the parabola	P. 141
5	1	Equation of the hyperbola	
6	2	**Determinants** - 2^{nd} order determinant - 3^{rd} order determinant	
7	2	**7^{th} week exam**	
8	3	Solving linear system using Cramer's method	
9	3	**Matrices** - Algebra of matrices (Addition, Subtraction and Multiplication) -Transpose of a matrix	P.240 – 258 P. 259
10	3	- Inverse of a matrix -Solving linear system using Matrix Inverse	
11	4	**Curve Fitting** Least square method(linear regression)	
12	5	**12^{th} week Exam**	
13	6	**Linear Programming** - Linear inequalities - System of linear inequalities	P. 295 P. 297
14	6	- Feasible region - Maximum & Minimum values	P. 299 - 305
15		**Final Revision**	

Sheet (1) : Analytic Geometry

1) Coordinates and Straight line

Some Basic rules :

Slope of a Straight line: $m = \frac{y_2 - y_1}{x_2 - x_1}$,

Equation of a straight line: $y = m\,x + c$,

Point-slope form: $(y - y_1) = m(x - x_1)$,

Two-points form: $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$,

Parallel lines: $m_1 = m_2$, Perpendicular lines: $m_1 m_2 = -1$.

Lecture problems :

1) Find and sketch the equation of the straight line passing through each pair of the following points and find its slope

   i. (2 , 3); (1 , 0) ii. (-1 , 1); (2 , 3)

   iii. (-1 , 2) ; (-1 ; -2) iv. (3 , 2) ; (5 ; 2)

2) Find and sketch the equation of the straight line passing through the point P and having the slope m:

   i) P (1 , 2) ; m = 2

   ii) P (2 , 4) ; m = -3/4

   iii) P (-1 , 3) ; m = 0

3) For the two points A(2,-1) , B(-4,1) find :

   a) The slope m of the straight line passing through A and B.

   b) The equation of the straight line passing through A and B.

   c) The equation of the straight line passing through the point A with slope m = - 4 .

4) Find and sketch the equation of the straight line passing through the point of intersection of the two lines x + 5y = 6 and 3x – 4y = - 1 and the point (-2,6) .

5) Find and sketch the equation of the straight line passing through the point of intersection of the two lines x + 4y = 6 and 2x – 3y = 1 and its slope 3 .

6) Find and sketch the equation of the straight line passing through the point of intersection of the two lines $2x + 3y = 6$ and $x - y = -2$ and perpendicular to the line $3x + 4y = 12$. Does the line pass through the point (0, 3)?

7) Find and sketch the equation of the straight line passing through the point of intersection of the two lines $x + 3y = 2$ and $2x - y = 4$ and parallel to the line $8x - 4y = 16$.

8) For the following simultaneous system of linear equations:

$$y - 2x = 1 \quad , \quad x + 2y = 2$$

i. Find slope of each line.

ii. State the type of this system i.e parallel or orthogonal?

iii. Find the intersection point, if possible.

Section problems:

1) Find and sketch the equation of the straight line passing through each pair of the following points and find its slope

i. (1 , 3); (2 , 4)

ii. (-1 , 3); (-2 , 4)

iii. (0 , 0) ; (1 ; 2)

iv. (0 , -2) ; (2 ; -2)

2) For the two points A(-3,-1) , B(2,1) find :

a) The slope m the straight line passing through A and B .

b) Equation of the straight line passing through A and B .

c) Equation of the straight line passing through point B with slope m = -1/2 .

3) Find and sketch the equation of the straight line passing through the point of intersection of the two lines $2x + 5y = 12$ and $3x - 4y = -5$ and the point (-2,6) .

3) Find and sketch the equation of the straight line passing through the point (2 ,-3) and parallel to the line passing through the points (4,1) and (-2,2) .

4) Find and sketch the equation of the straight line passing through the point of intersection of the two lines $3x - 2y = 9$ and $4x + 7y = -17$ and perpendicular to the line $2y + 3x = 7$.

5) Find and sketch the equation of the straight line passing through the point of intersection of the two lines $3x - 2y = -8$ and $-2x + 5y = 9$ and parallel to the straight line $5x - 2y = 3$.

6) Find and sketch the equation of the straight line passing through the point of intersection of the two lines $x - y = 4$ and $2x + 5y = 15$ and the point of intersection of the two lines $x + 3y = 2$ and $2x - y = 4$.

Homework problems:

1) Find and sketch the equation of the straight line passing through each pair of the following points and find its slope

   i. (-4 , 5); (3 , 0)
   ii. (-2 , 3); (0 , -4)
   iii. (-1 , 3) ; (-2 ; -4)
   iv. (0 , 0) ; (5 ; 2)
   v. (0 , 0) ; (-1 ; 0)
   vi. (-3 , 2) ; (-1 ; 2)

2) Find and sketch the equation of the straight line passing through the points A(2,-1) and B(4,-3) .

3) Find the equation of the straight line with slope m = 0.5 and passes through the point (-1, 3).

4) Find and sketch the equation of the straight line passing through the point of intersection of the two lines $x - y = 0$ and $2x - y = 3$ and the point (4,-1).

5) Find and sketch the equation of the straight line passing through the point of intersection of the two lines $5x + 4y = 10$ and $6x + 5y = 13$ and parallel to the line $4x + 2y = 9$.

6) Find and sketch the equation of the straight line passing through the point (3,-3) and perpendicular to the line $2x - 3y = 11$.

7) Find and sketch the equation of the straight line through the intersection point of the two lines: $x + 4y = -3$, $x + y = 0$ and parallel to the straight line passing through the points $(2,-1)$ and $(4,6)$.

8) Find and sketch the equation of the straight line passing through the point (2,-4) and perpendicular to the line passing through the points (4,1) and (-2,2) .

9) Find and sketch the equation of the straight line passing through the point of intersection of the two lines $x + 2y = 3$ and $3x + y = 4$ and perpendicular to the straight line passing through the points (1, 1) and (4, 6).

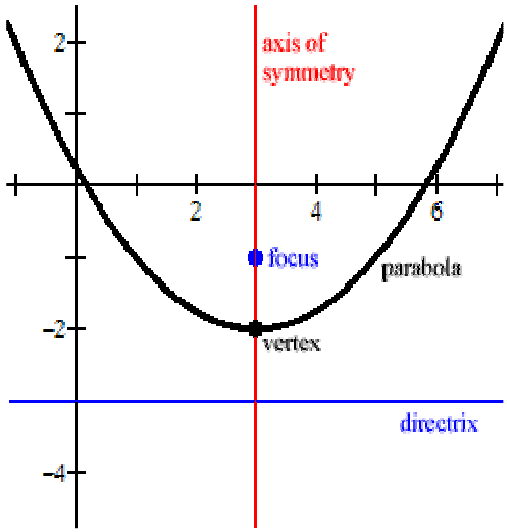
10) Find and sketch the equation of the straight line passing through the point of intersection of the two lines $x + 2y = 3$ and $3x + y = 4$ and the point of intersection of the two lines $x + y = 1$ and $3x - 4y = 10$.

2) Equation of a parabola:

a. The general form of a vertical parabola :

$$(x-h)^2 = 4a(y-k)$$

vertex : (h,k) ,
axis of symmetry : $x=h$,
focus : $(h,k+a)$,
equation of directrix : $y=k-a$.



Another form:

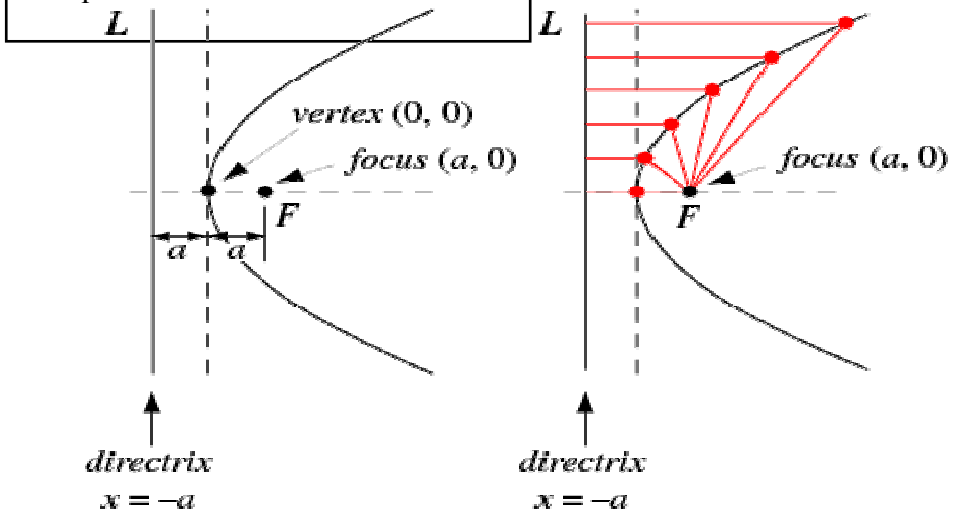
$y = ax^2 + bx + c$; **where** $h = \dfrac{-b}{2a}$; $k = y(h)$

vertex : (h,k) ,
axis of symmetry : $x=h$

b. The general form of a horizontal parabola:

$$(y-k)^2 = 4a(x-h)$$

vertex : (h,k) ,
axis of symmetry : $y=k$,
focus : $(h+a,k)$,
equation of directrix : $x=h-a$.



Lecture problems:

1) Discuss and sketch the parabola $x^2 = -8y$.

2) Discuss and sketch the parabola $x^2 - 4y = 0$.

3) Discuss and sketch the parabola $x^2 + 8x - 8y + 40 = 0$.

4) Discuss and sketch the parabola $x^2 + 4x + y + 5 = 0$.

5) Discuss and sketch the parabola $x^2 + 6x - 4y + 29 = 0$.

Section problems:

1) Discuss and sketch the parabola $x^2 = -16y$.

2) Discuss and sketch the parabola $x^2 + 2x + 2y + 3 = 0$.

3) Discuss and sketch the parabola $x^2 + 4x - 4y - 8 = 0$.

4) Discuss and sketch the parabola $x^2 - 2x - 4y + 1 = 0$.

Homework problems:

1) Discuss and sketch the parabola $x^2 = -20y$.

2) Discuss and sketch the parabola $(x+5)^2 = -12(y-2)$.

3) Discuss and sketch the parabola $x^2 + 2x - 8y - 16 = 0$.

4) Discuss and sketch the parabola $x^2 - 4x + y = 0$.

5) Discuss and sketch the parabola $x^2 + 4x - 3y - 8 = 0$.

6) Discuss and sketch the parabola $x^2 - 2x + 2y - 5 = 0$.

7) Discuss and sketch the parabola $x^2 + 6x + 2y + 13 = 0$.

8) Discuss and sketch the parabola $x^2 - 4x - 3y + 1 = 0$.

9) Discuss and sketch the parabola $x^2 + 2x + 4 - 7y = 0$.

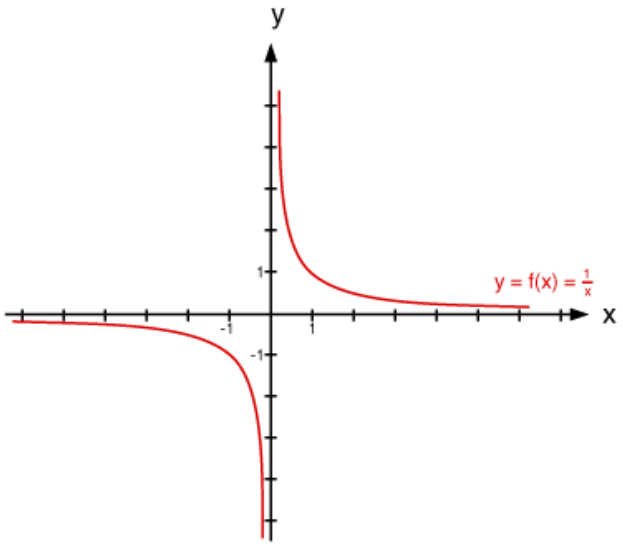
10) Discuss and sketch the parabola $x^2 - 6x - y + 7 = 0$.

3) Equation of a Hyperbola:

$$y = \frac{ax+b}{cx+d} \quad , \quad ad \prec bc$$

$$y - k = \frac{l}{x - h}$$

Center (h , k)
vertical asymptote $x = h = -d/c$
horizontal asymptote $y = k = a/c$



Lecture problems:

1) Determine the center and asymptotes of the following hyperbolas, and then sketch the graph:

a. $y = \frac{x+1}{2x-4}$.

b. $y = \frac{4x+1}{x-1}$

c. $y = \frac{2x+3}{x+1}$

d. $y = \frac{2x+5}{3x+2}$

Section problems:

2) Determine the center and asymptotes of the following hyperbolas, and then sketch the graph:

a. $y = \frac{x+1}{3x+2}$.

b. $y = \frac{-2x+5}{x+2}$

c. $y = \frac{4x+3}{2x+1}$

d. $y = \frac{3x+2}{3x-1}$

Homework problems:

3) Determine the center and asymptotes of the following hyperbolas, and then sketch the graph:

a. $y = \dfrac{x+5}{x+2}$.

b. $y = \dfrac{-4x+1}{2x+1}$

c. $y = \dfrac{2x+5}{x+2}$

d. $y = \dfrac{2x+3}{x-2}$

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Sheet (2) : Determinants

[I] a) Evaluating 2nd order determinant :

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12}$$

b) Evaluating 3rd order determinant :

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

c) Cramer's method :

If

$$a_{11}x + a_{12}y + a_{13}z = b_1 \ ,$$
$$a_{21}x + a_{22}y + a_{23}z = b_2 \ ,$$
$$a_{31}x + a_{32}y + a_{33}z = b_3 \ .$$

then $x = \frac{\Delta_x}{\Delta}$, $y = \frac{\Delta_y}{\Delta}$, $z = \frac{\Delta_z}{\Delta}$. where

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \neq 0 \ , \ \Delta_x = \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix} \ , \ \Delta_y = \begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix} \ , \ \Delta_z = \begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}$$

Lecture problems :

1) Evaluate the following 2^{nd} order determinants

a) $\begin{vmatrix} 2 & -3 \\ 4 & 1 \end{vmatrix}$ b) $\begin{vmatrix} -4 & -2 \\ -3 & 3 \end{vmatrix}$ c) $\begin{vmatrix} 5 & 3 \\ 4 & -2 \end{vmatrix}$.

2) Evaluate the following 3^{rd} order determinants

a) $\begin{vmatrix} 2 & -1 & 2 \\ 1 & 4 & -3 \\ -2 & 3 & 1 \end{vmatrix}$ b) $\begin{vmatrix} -6 & 21 & -30 \\ 1 & -3 & 5 \\ 2 & 7 & -4 \end{vmatrix}$

c) $\begin{vmatrix} -3 & 2 & 5 \\ 3 & 1 & -6 \\ -2 & 4 & 3 \end{vmatrix}$ d) $\begin{vmatrix} -2 & 4 & -3 \\ 1 & -3 & 5 \\ 0 & 5 & -4 \end{vmatrix}$

3) Solve the following systems

a) $\begin{aligned} x + y - z &= 3 , \\ 2y + 2z &= 4 , \\ 4x + y - 3z &= -2 . \end{aligned}$ $(sol: x=-3, y=4, z=-2)$

b) $\begin{aligned} 2x + 2y - z &= -6 , \\ x - 2z &= -3 , \\ -3x + y + 4z &= 2 . \end{aligned}$ $(sol: x=1, y=-3, z=2)$

Section problems :

1) Evaluate the following 2^{nd} order determinants

a) $\begin{vmatrix} 4 & 2 \\ 5 & 3 \end{vmatrix}$ b) $\begin{vmatrix} 7 & -4 \\ 6 & -3 \end{vmatrix}$ c) $\begin{vmatrix} -5 & 1 \\ -4 & -3 \end{vmatrix}$ d) $\begin{vmatrix} 2 & -4 \\ 4 & -2 \end{vmatrix}$.

2) Evaluate the following 3^{rd} order determinants

a) $\begin{vmatrix} 2 & 1 & 2 \\ 1 & 0 & 1 \\ -1 & 3 & -2 \end{vmatrix}$ b) $\begin{vmatrix} 6 & -13 & -2 \\ 5 & -30 & -5 \\ 1 & 8 & 4 \end{vmatrix}$ c) $\begin{vmatrix} -6 & -2 & -15 \\ 3 & -3 & 2 \\ -2 & 5 & 4 \end{vmatrix}$.

3) Solve the following systems

a) $\begin{aligned} x - 2y &= 3 , \\ 2x + y + 2z &= 5 , \\ 3x - y - z &= 2 . \end{aligned}$ $(sol: x=1, y=-1, z=2)$

b) $\begin{aligned} x - 2y &= 7 , \\ 2x + 4y + z &= -3 , \\ -2x + y + 2z &= 9 . \end{aligned}$ $(sol: x=1, y=-3, z=7)$

c) $\begin{aligned} -y + 3z &= 5 , \\ 2x + 5z &= 10 , \\ 3x + 3y - 4z &= -5 . \end{aligned}$ $(sol: x=0, y=1, z=2)$

Homework problems :

1) Evaluate the following 2^{nd} order determinants

a) $\begin{vmatrix} 6 & -2 \\ -5 & 5 \end{vmatrix}$ b) $\begin{vmatrix} 4 & -5 \\ 7 & -6 \end{vmatrix}$ c) $\begin{vmatrix} -3 & 2 \\ -8 & -1 \end{vmatrix}$ d) $\begin{vmatrix} 8 & -7 \\ -2 & 2 \end{vmatrix}$.

2) Evaluate the following 3^{rd} order determinants

a) $\begin{vmatrix} 2 & 6 & 4 \\ 1 & 3 & 0 \\ -1 & 0 & -2 \end{vmatrix}$ b) $\begin{vmatrix} -6 & -3 & -4 \\ 2 & -15 & 5 \\ -1 & 7 & 2 \end{vmatrix}$ c) $\begin{vmatrix} 3 & -2 & 10 \\ 3 & -3 & 4 \\ -5 & 2 & 3 \end{vmatrix}$.

3) Solve the following systems

a)
$$\begin{aligned} 2x + y + 2z &= -1 , \\ x + z &= -1 , \\ -x + 3y - 2z &= 7 . \end{aligned}$$
(sol : $x = 2$, $y = 1$, $z = -3$)

b)
$$\begin{aligned} 2x + 5y &= 3 , \\ 3x + 3y - z &= 4 , \\ -x - 7y + z &= 8 . \end{aligned}$$
(sol : $x = 4$, $y = -1$, $z = 5$)

c)
$$\begin{aligned} -x + 3y - z &= -9 , \\ 2x + 3z &= 8 , \\ 5x + 2y &= -5 . \end{aligned}$$
(sol : $x = 1$, $y = -2$, $z = 2$)

Sheet (3) : Matrices

1) Algebraic operation of matrices :

a) Addition

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} + \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{pmatrix}$$

b) Subtraction

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} - \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11}-b_{11} & a_{12}-b_{12} \\ a_{21}-b_{21} & a_{22}-b_{22} \end{pmatrix}$$

c) Multiplication

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22} \\ a_{21}b_{11}+a_{22}b_{21} & a_{21}b_{12}+a_{22}b_{22} \end{pmatrix}$$

Lecture problems :

1) If

$$A = \begin{pmatrix} 2 & -1 \\ 0 & -4 \\ -5 & 3 \end{pmatrix} \quad , \quad B = \begin{pmatrix} 5 & -2 & 6 \\ -1 & 4 & -2 \end{pmatrix} \quad , \quad C = \begin{pmatrix} 2 & 1 & -1 \\ 0 & -2 & 3 \\ -6 & 4 & -2 \end{pmatrix}$$

and $D = \begin{pmatrix} 1 & 3 \\ -2 & 5 \\ -4 & 7 \end{pmatrix}$

Find if possible:

a) A + D b) 2D – 3A c) AB d) BC e) (A + D) C f) (DB) C

Section problems :

1) If

$$A = \begin{pmatrix} 3 & -2 & 1 \\ -1 & 4 & -7 \\ 0 & 5 & 8 \end{pmatrix} \quad , \quad B = \begin{pmatrix} 7 & -4 \\ -5 & 9 \\ -1 & 2 \end{pmatrix} \quad , \quad C = \begin{pmatrix} 2 & 1 & -1 \\ 0 & -2 & 3 \\ -6 & 4 & -2 \\ -3 & 3 & 1 \end{pmatrix} \quad ,$$

$$D = \begin{pmatrix} 1 & 3 & 2 & -3 \\ -2 & 5 & 4 & 6 \end{pmatrix} \quad \text{and} \quad E = \begin{pmatrix} -6 & -1 \\ 5 & 3 \\ 4 & -5 \end{pmatrix} \quad .$$

Find if possible:

a) 2A + E b) E-C c) AE d) DC e) (A+ E)D f) (DC)B

Homework problems :

1) If

$$A = \begin{pmatrix} 5 & -6 & 4 \\ -3 & 0 & -2 \end{pmatrix} \quad , \quad B = \begin{pmatrix} -1 & 3 \\ -2 & 7 \\ 6 & -4 \end{pmatrix} \quad , \quad C = \begin{pmatrix} 0 & 5 & -4 \\ 1 & -3 & 3 \\ -2 & 1 & -7 \end{pmatrix} \quad ,$$

$$D = \begin{pmatrix} 1 & 3 & 2 \\ -2 & 5 & 4 \end{pmatrix} \quad \text{and} \quad E = \begin{pmatrix} -3 & -6 \\ 4 & 3 \\ 1 & -1 \\ 2 & 7 \end{pmatrix} \quad .$$

Find if possible:

a) A + D b) D – A c) EA d) 3BD e) B(A+D) f) (EA)C

2) Transpose of a matrix :

If

$$A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \quad , \quad \text{then} \quad \boxed{A^T=\begin{pmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{pmatrix}}$$

3) Inverse of a matrix :

If

$$A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \quad , \quad \text{then} \quad \boxed{A^{-1}=\frac{1}{|A|}\,adj(A)\,,\ |A|\neq 0}$$

Lecture problems :

1) If

$$A=\begin{pmatrix} 6 & 8 & -4 \\ -1 & -7 & 3 \\ -5 & 3 & 2 \end{pmatrix} \quad \text{and} \quad B=\begin{pmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{pmatrix} .$$

Find:

a) A^T b) $(A+B)^T$ c) B^{-1} d) AB e) B^TA^T

2) Find x , y, and z such that:

$$\begin{pmatrix} 3x & -z \\ 3 & y \end{pmatrix}+2\begin{pmatrix} 1 & 4 \\ 2 & 1 \end{pmatrix}=\begin{pmatrix} 8 & 7 \\ 7 & 11 \end{pmatrix}^T$$

3) If

$$A=\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} \quad \text{and} \quad B=\begin{pmatrix} 4 & 7 \\ 2 & 6 \end{pmatrix}$$

Find:

a) A^{-1} b) B^T c) AB d) $A^{-1}B^{-1}$

Section problems :

1) If

$$A = \begin{pmatrix} 7 & -4 & -6 \\ -3 & 5 & 2 \\ 1 & 2 & -1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -4 & 8 & -1 \\ 2 & -1 & 3 \\ 6 & 0 & 4 \end{pmatrix} .$$

Find :

a) B^T b) $(A + B)^T$ c) A^{-1} d) AB e) $B^T A^T$

2) If

$$A = \begin{pmatrix} 5 & 4 \\ 6 & 3 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 3 & 4 \\ 8 & 24 \end{pmatrix}$$

Find:

b) A^{-1} b) B^T c) AB d) $A^{-1}B^{-1}$

Homework problems :

1) If

$$A = \begin{pmatrix} 10 & 8 & -5 \\ -2 & 3 & 4 \\ 7 & -2 & -1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 2 & 6 & 7 \\ -2 & -5 & 1 \\ 0 & 4 & -3 \end{pmatrix} .$$

Find :

a) A^T b) $(A+B)^T$ c) $(AB)^T$ d) A^{-1} e) B^{-1}

2) If

$$A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$$

Find:

c) A^T b) B^{-1} c) AB d) $A^{-1}B^{-1}$

Sheet (4): Solving System of Linear Equations

Inverse Matrix Method

If

$$a_{11}x + a_{12}y + a_{13}z = b_1 \ ,$$
$$a_{21}x + a_{22}y + a_{23}z = b_2 \ ,$$
$$a_{31}x + a_{32}y + a_{33}z = b_3 \ .$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \ ,$$

Let $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ then find the inverse matrix A^{-1}

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}. \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Lecture problems :

1) Solve the following systems

a)
$$x + y - z = 3 \ ,$$
$$2y + 2z = 4 \ ,$$
$$4x + y - 3z = -2 \ .$$
$(sol: \ x = -3 , \ y = 4 , \ z = -2)$

b)
$$2x + 2y - z = -6 \ ,$$
$$x - 2z = -3 \ ,$$
$$-3x + y + 4z = 2 \ .$$
$(sol: \ x = 1 , \ y = -3 , \ z = 2)$

c) $\begin{aligned} x + y - z &= 4, \\ x - 2y + 3z &= -6, \\ 2x + 3y + z &= 7. \end{aligned}$
$(sol: x=1, y=2, z=-1)$

d) $\begin{aligned} -x + 3y - z &= -9, \\ 2x + 3z &= 8, \\ 5x + 2y &= 1. \end{aligned}$
$(sol: x=1, y=-2, z=2)$

e) $\begin{aligned} x - 2y &= 3, \\ 2x + y + 2z &= 5, \\ 3x - y - z &= 2. \end{aligned}$
$(sol: x=1, y=-1, z=2)$

Section problems :

4) Solve the following systems

a) $\begin{aligned} x - 2y &= 3, \\ 2x + y + 2z &= 5, \\ 3x - y - z &= 2. \end{aligned}$
$(sol: x=1, y=-1, z=2)$

b) $\begin{aligned} x - 2y &= 7, \\ 2x + 4y + z &= -3, \\ -2x + y + 2z &= 9. \end{aligned}$
$(sol: x=1, y=-3, z=7)$

c) $\begin{aligned} -y + 3z &= 5, \\ 2x + 5z &= 10, \\ 3x + 3y - 4z &= -5. \end{aligned}$
$(sol: x=0, y=1, z=2)$

d) $\begin{aligned} x - z &= -5, \\ 3x - 4y &= -2, \\ -4x + 6y + z &= 5. \end{aligned}$
$(sol: x=-2, y=-1, z=3)$

e) $\begin{aligned} -x - 7y + z &= 8, \\ 2x + 5y &= 3, \\ 3x + 3y - z &= 4. \end{aligned}$
$(sol: x=4, y=-1, z=5)$

Homework problems :

4) Solve the following systems

a) $\begin{aligned} 2x + y + 2z &= -1, \\ x + z &= -1, \\ -x + 3y - 2z &= 7. \end{aligned}$
$(sol: x=2, y=1, z=-3)$

b) $\begin{aligned} 2x + 5y &= 3, \\ 3x + 3y - z &= 4, \\ -x - 7y + z &= 8. \end{aligned}$
$(sol: x=4, y=-1, z=5)$

c) $\begin{aligned} -x + 3y - z &= -9, \\ 2x + 3z &= 8, \\ 5x + 2y &= -5. \end{aligned}$
$(sol: x=1, y=-2, z=2)$

d) $\begin{aligned} x - 2y &= 7, \\ 2x + 4y + z &= -3, \\ -2x + y + 2z &= 9. \end{aligned}$
$(sol: x=1, y=-3, z=7)$

e) $\begin{aligned} -x + 2y + 5z &= 8, \\ + 3y + 7z &= 10, \\ 4x + y + 3z &= 0. \end{aligned}$
$(sol: x=-1, y=1, z=1)$

Sheet (5) : Curve Fitting

Linear Least Squares

The linear least squares solution for a given collection (x_i, y_i) , for $i = 1, 2, ..., n$ has the form $y = mx + c$, where you can find the two constants *m* and *c* by solving the following equations simultaneously:

$$\sum y = m \sum x + n c$$

$$\sum xy = m \sum x^2 + n \sum x$$

Lecture problems :

1) For the following data , fit a straight line to the x and y values

x	2	3	4	5	6	7	8	9
y	94	95	96	101	98	103	104	106

Find the value of y when x = 10 .

2) The data in the following table were obtained in a study a student's score on a test and his or her grade-point average. The study included eight randomly selected students. Use the equation of the regression line to predict the GPA for a student who scored 107 on the test. Please round to the nearest tenth of a grade-point.

score *x*	98	105	100	100	106	95	116	112
GPA *y*	1.9	2.2	2.8	2.9	2.2	2.3	3.2	3.4

3) The data in the following table were obtained in a study of age and exercise of six randomly selected persons. Use the equation of the regression line to predict the hours of exercise for a person who is 47 years old

Age *x*	18	26	32	38	52	59
Hours *y*	7	4	1	4	1.8	1.5

Section problems :

1) For the following data , fit a straight line to the x and y values

x	1	2	3	4	5	6	7
y	0.5	2.5	2	4	3.5	6	5.5

Find the value of y when x = 4.6

2) The data in the following table were obtained in a study of the amount of an alumnus's contribution in dollars and the years the alumnus has been out of school. The study included six randomly selected alumni. Use the equation of the regression line to predict the amount of contribution for an alumnus who has been out of school for 4 years

Years *x*	1	5	3	10	7	6
Contribution *y*, $	500	100	450	60	75	70

3) The data in the following table were obtained in a study of age and systolic blood pressure of six randomly selected persons. Use the equation of the regression line to predict the blood pressure for a person who is 57 years old. Please round to the nearest whole number.

Age *x*	43	48	56	61	67	70
Pressure *y*	123	127	129	144	144	147

Homework problems :

1) The data in the following table were obtained in a study of age and systolic blood pressure of six randomly selected subjects. Find the equation of the regression line. Please round to three decimal places.

Age *x*	43	48	56	61	67	70
Pressure *y*	123	129	126	143	139	152

2) The data in the following table were obtained in a study on the number of absences and the final grades of seven randomly selected students from a statistics class. Use the equation of the regression line to predict the grade of a student that his number of absences is 7. Please round to three decimal places.

Number of absences *x*	6	2	15	9	12	5	8
Final grade *y* (%)	84	87	45	68	55	89	74

3) The following table lists the college grade – point averages of 20 mathematics and computer science majors , together with the scores that these students received on the mathematics portion of the ACT (American College Testing Program) test while in high school . Find the equation of the least square line for these data

ACT Score	Grade – Point Average	ACT Score	Grade – Point Average
28	3.8	29	3.7
25	3.2	28	3.6
28	3.6	27	3.8
27	3.7	29	3.7
28	3.2	21	1.6
33	3.4	28	3.1
28	3.3	28	2.9
29	3.5	26	2.9
23	2.4	30	3.1
27	2	24	2.8

Sheet (6) : Linear Programming

1) Graphing linear inequalities :

Lecture problems :

1) Graph the following linear inequalities :

a) $2x + y \le 4$ b) $x + 3y \ge 6$ c) $3x - y \le 3$

2) Graph the following system of linear inequalities. Tell whether the graph is bounded or unbounded and list each corner point of the graph :

a) $\begin{cases} x + y \ge 2 \\ 2x + y \ge 3 \\ x \ge 0 \ , \ y \ge 0 \end{cases}$ b) $\begin{cases} x + y \le 6 \\ 2x + y \le 3 \\ x \ge 0 \ , \ y \ge 0 \end{cases}$ c) $\begin{cases} x + y \ge 2 \\ 2x + 3y \le 12 \\ 3x + 2y \le 12 \\ x \ge 0 \ , \ y \ge 0 \end{cases}$

Section problems :

1) Graph the following linear inequalities :

a) $3x + 5y \le 15$ b) $3x - 4y \ge 12$ c) $x - 2y \le 4$

2) Graph the following system of linear inequalities. Tell whether the graph is bounded or unbounded and list each corner point of the graph :

a) $\begin{cases} x + 4y \le 8 \\ x - y \ge 3 \\ x \ge 0 \ , \ y \ge 0 \end{cases}$ b) $\begin{cases} 2x + 3y \ge 12 \\ x + y \ge 4 \\ x \ge 0 \ , \ y \ge 0 \end{cases}$ c) $\begin{cases} 2x + 3y \le 12 \\ 3x + y \le 6 \\ x \ge 0 \ , \ y \ge 0 \end{cases}$

Homework problems :

1) Graph the following linear inequalities :

a) $4x + 7y \le 28$ b) $2x + 4y \ge 8$ c) $x - y \le 6$

2) Graph the following system of linear inequalities. Tell whether the graph is bounded or unbounded and list each corner point of the graph :

a) $\begin{cases} 3x + 2y \le 18 \\ 3x + 4y \ge 12 \\ x \ge 0 \ , \ y \ge 0 \end{cases}$ b) $\begin{cases} 2x + y \le 6 \\ x + y \le 4 \\ x \ge 0 \ , \ y \ge 0 \end{cases}$ c) $\begin{cases} 3x - y \ge 6 \\ 2x + y \le 6 \\ x \ge 0 \ , \ y \ge 0 \end{cases}$

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2) Maximizing and minimizing a certain function :

Lecture problems :

1) Find the maximum and minimum values of the objective function $z = 2x + 5y$, subject to the following constrains :

$$\begin{aligned} 3x + 2y &\le 6 \\ -2x + 4y &\ge 8 \\ x \ge 0 \ , \ y &\ge 0 \end{aligned}$$

2) Maximize the function $z = 40x + 55y$ in the region determined by the following constrains :

$$\begin{aligned} 2x + y &\le 10 \\ x + y &\le 7 \\ 2x + 3y &\le 18 \\ x \ge 0 \ , \ y &\ge 0 \end{aligned}$$

3) Mike's Famous toy Trucks manufactures two kinds of toy trucks – a standard model and a deluxe model. In the manufacturing process each standard model requires 2 Hrs of grinding and 2 Hrs of finishing, and each deluxe model needs 2 Hrs of grinding and 4 Hrs of finishing . The company has two grinders and 3 finishers, each of whom works at most 40 Hrs per week . Each standard model toy truck bring a profit of $3 and each deluxe model a profit of $4 . Assuming that every truck will be sold , how many of each should be made to maximize profits ?

Section problems :

1) Find the maximum and minimum values of the objective function $z = 0.2x + 0.5y$, subject to the following constrains :

$$\begin{aligned} 2x + y &\ge 12 \\ 2x + 5y &\ge 20 \\ x \ge 0 \ , \ y &\ge 0 \end{aligned}$$

2) Minimize the function $z = 9x + 2y$ in the region determined by the following constrains :

$$\begin{aligned} -4x + 5y &\le 20 \\ 4x + 5y &\le 60 \\ x &\le 10 \\ x \ge 0 \ , \ y &\ge 0 \end{aligned}$$

3) A diet is to contain at least 400 units of vitamins, 500 units of minerals, and 1400 calories. Two foods are available: F1, which costs \$0.05 per unit, and F2, which costs \$0.03 per unit. A unit of food F1 contains 2 units of vitamins , 1 unit of minerals, and 4 calories ; a unit of food F2 contains 1 units of vitamins , 2 unit of minerals, and 4 calories. Find the minimum cost for a diet that consist of a mixture of these two foods and also meets the minimal nutrition requirements.

Homework problems :

1) Find the maximum and minimum values of the objective function $z = 1.5x + 2y$, subject to the following constrains :

$$3x + 2y \leq 36$$
$$3x + 10y \leq 60$$
$$x \geq 0 \ , \ y \geq 0$$

2) Minimize the function $z = 3x + 7y$ in the region determined by the following constrains :

$$x + y \geq 9$$
$$6x + 11y \geq 84$$
$$x \geq 0 \ , \ y \geq 0$$

3) Draw the following linear constrains

$2x + 5y \leq 20$, $2x + y \leq 12$, $x \geq 0$, $y \geq 0$

And then Find:

1. feasible Solution of that linear constrains .
2. The maximum value of the objective function z = 2x + 5y subjected to the given group.
