STEADY STATE BEHAVIOR OF SERIES CONNECTED SYNCHRONOUS MOTOR

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Abstract - Series Connected Synchronous Motor (SCSM) is a slip-ring induction motor with stator and rotor windings connected in series. This motor runs synchronously at double the rated speed when runs as a normal induction motor. This paper presents theoretical and experimental investigation for steady state performance based on the d-q model as derived for a synchronously rotating frame. The model is extended to include saturation.

1. Introduction

The principle of operation of the series connected synchronous machines for generator mode of operation was studied and analyzed using d-q model [1], Floquet theory [2] and phasor diagram [3]. SCSM is basically a three phase slip ring induction machine whose stator and rotor windings are connected in series with sequence of two phases reversed as shown in Fig. 1. With reference to Fig. 2, synchronous mode of operation is possible when stator and rotor MMFs rotate synchronously opposite to each other at an absolute speed equal to half rotor speed. This machine is capable of operating at higher speeds than conventional induction or synchronous motors fed by same supply frequency. Since rotor and stator windings are connected in series, this machine can operate at higher voltage levels without affecting conductor insulation requirements. Theory and analysis of SCSM was presented in [4]. Saturation was included by expressing inductance of quadrature-axis mathematically as a function of quadrature-axis current while inductance of direct-axis was constant. It was reported that after pulling out of synchronism, the motor still operates at less than synchronous speed. In a later paper [5], transient performance of series connected induction motors at less than synchronous speed was explained. In the present paper, operation of SCSM is studied. A d-q model is suggested and a steady state analysis is introduced. The model is extended to include saturation. Theoretical and experimental results are compared.

2. D-Q Model Representation

To analyze this machine, a reference frame rotates synchronously with stator and rotor MMFs is chosen. Since the rotor actually rotates at double reference frame speed and with reference to Fig. 3, the following relation holds for every rotor position:

$$\theta = \beta \tag{1}$$

Reversing stator and rotor phase sequence windings is expressed mathematically by:

$$\begin{bmatrix} i_{ar} \\ i_{br} \\ i_{cr} \end{bmatrix} = \begin{bmatrix} i_{as} \\ i_{cs} \\ i_{bs} \end{bmatrix} \tag{2}$$

Equivalent stator and rotor d-q currents in terms of actual phase currents and mutual displacements are given by the well known transformations [6]:

$$\begin{bmatrix} i_{dr} \\ i_{qr} \end{bmatrix} = \frac{2}{3}\begin{bmatrix} cos(\beta) & cos\left(\beta+\frac{2\pi}{3}\right) & cos\left(\beta-\frac{2\pi}{3}\right) \\ -sin(\beta) & -sin\left(\beta+\frac{2\pi}{3}\right) & -sin\left(\beta-\frac{2\pi}{3}\right) \end{bmatrix}\begin{bmatrix} i_{ar} \\ i_{br} \\ i_{cr} \end{bmatrix} \tag{3}$$

and

$$\begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} = \frac{2}{3}\begin{bmatrix} cos(\theta) & cos\left(\theta-\frac{2\pi}{3}\right) & cos\left(\theta+\frac{2\pi}{3}\right) \\ sin(\theta) & sin\left(\theta-\frac{2\pi}{3}\right) & sin\left(\theta+\frac{2\pi}{3}\right) \end{bmatrix}\begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} \tag{4}$$

Substituting by (1) and (2) into (3) and comparing the result with (4) yield:

$$\begin{Bmatrix} i_{ds} = \;\; i_{dr} = i_d \\ i_{qs} = -i_{qr} = i_q \end{Bmatrix} \tag{5}$$

Implementing the conditions derived in (5) to the d-q model results in the interconnections between axes equivalent coils shown in Fig. 4. It is seen that:

$$\begin{Bmatrix} v_d = v_{ds} + v_{dr} \\ v_q = v_{qs} - v_{qr} \end{Bmatrix} \tag{6}$$

The relation between d-q axes voltages and currents can be expressed as:

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} R_a + L_d P & L_q P\theta \\ -L_d P\theta & R_a + L_q P \end{bmatrix}\begin{bmatrix} i_d \\ i_q \end{bmatrix} \tag{7}$$

where

$$\begin{Bmatrix} R_a = R_s + R_r \\ L_d = L_s + L_r + 2M \\ L_q = L_s + L_r - 2M \end{Bmatrix} \tag{8}$$

The series connection between stator and rotor phase windings resulted in effective saliency as indicated by the difference in axes inductances which shows synchronous operation of this machine.

3. Steady State Analysis

The model given in (7) represents the dynamic behavior of the SCSM. At steady state, if the applied voltage is sinusoidal and balanced, the operator P and the term $P\theta$ become zero and 0.5ω respectively. The transformed voltage equation (7) tends to:

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} R_a & \frac{\omega}{2} L_q \\ -\frac{\omega}{2} L_d & R_a \end{bmatrix} \begin{bmatrix} I_d \\ I_q \end{bmatrix} \quad (9)$$

The volatge balance equation in (9) suggests a phasor diagram as shown in Fig. 5. It is seen that:

$$\begin{Bmatrix} V_d = \sqrt{2}\ V_a \sin(\delta) \\ V_q = \sqrt{2}\ V_a \cos(\delta) \end{Bmatrix} \quad (10)$$

Rearragning (9) to determine d-q currents yields:

$$\begin{bmatrix} I_d \\ I_q \end{bmatrix} = \frac{1}{\left(R_a^2 + \frac{\omega^2}{4} L_d L_q \right)} \begin{bmatrix} R_a & -\frac{\omega}{2} L_q \\ \frac{\omega}{2} L_d & R_a \end{bmatrix} \begin{bmatrix} V_d \\ V_q \end{bmatrix} = \bar{I}_{dq} \quad (11)$$

The motor phase current is given by:

$$I_a = \sqrt{\frac{\left(I_d^2 + I_q^2 \right)}{2}} \quad (12)$$

The power factor can be expressed by:

$$PF = \cos(\delta - \psi) \quad (13)$$

4. Saturation Representation

The iron and eddy current losses are neglected. The model can possibly be extended to include saturation in the iron parts. From conventional open circuit test, the relationship between M and I_m shown in Fig. 6 is evaluated which can be expressed mathematically using curve fitting techniques. Self inductances are related to leakage and mutual inductances by:

$$\begin{Bmatrix} L_r = l_r + KM \\ L_s = l_s + \frac{M}{K} \end{Bmatrix} \quad (14)$$

Substituting by(14) into (8) results in:

$$\begin{Bmatrix} L_d = l_s + l_r + \left(K + \frac{1}{K} + 2 \right) M \\ L_q = l_s + l_r + \left(K + \frac{1}{K} - 2 \right) M \end{Bmatrix} \quad (15)$$

From the MMFs relationship, the magnetization current is related to d-q currents by:

$$I_m = \sqrt{\left(I_{ds} + KI_{dr} \right)^2 + \left(I_{qs} + KI_{qr} \right)^2} \quad (16)$$

Substituting by (5) into (16) results in:

$$I_m = \sqrt{(1+K)^2 I_d^2 + (1-K)^2 I_q^2} \quad (17)$$

For each δ, d-q voltages are obtained from (10). By assuming an initial value for I_m, M is determined from Fig. 6. Hence d-q inductances and currents are calculated as given in (15) and (11) respectively. I_m can now be evaluated as in (17). I_m is changed until the initial assumed value equals to that evaluated by (17).

5. Torque and Power Developed by SCSM

Speed voltage coefficient matrix G can be concluded from (7) as follows:

$$G = \begin{bmatrix} 0 & L_q \\ -L_d & 0 \end{bmatrix} \quad (18)$$

The air-gap torque developed by SCSM is given by:

$$T_g = \frac{3}{2} \frac{p}{4}\ \bar{I}_{dq}^T\ G\ \bar{I}_{dq} \quad (19)$$

The developed torque T_o is the air gap torque T_g minus losses due to friction and windage. It is important to note that for p pole SCSM, the relation between the electrical angles (θ_e) and mechanical angles (θ_m) is given by:

$$\theta_e = \frac{p}{4}\ \theta_m \quad (20)$$

Also, shaft speed N_r is related to frequency of MMFs by:

$$N_r = 2 * \frac{120f}{p} \quad (21)$$

Input and output power are given by:

$$\begin{Bmatrix} P_{in} = 3 \,.\, I_a \,.\, V.\ PF \\ P_{out} = T_o \,.\, \frac{\omega}{2} \end{Bmatrix} \quad (22)$$

6. Experimental Setup

The experimental setup is made from a slip ring three phase induction motor whose details are: 220/380 V, 6.3/3.6 A, 2.2 kW, 50 Hz, 4-pole and 1390 rpm. The parameters as have been obtained by standard tests at 50Hz are: R_s = 2.08 Ω, R_r = 1.96 Ω, X_s = 5.28 Ω, X_r = 3.92 Ω and K = 0.86. The relation between I_m and M is depicted by Fig. 6. The induction machine is connected in the SCSM mode as shown in Fig. 1 and coupled to a DC dynamometer which enables reading torque. The speed and load angle are measured by an ac tachometer and stroboscope respectively. Also phase voltage, current and input power are recorded. Starting methods of SCSM are similar to conventional synchronous motor. In this test, the machine is fed from a conventional synchronous generator to facilitate starting and speed control.

7. Results and Discussion

Experimental tests have been carried out at a line voltage of 220 V and a frequency of 40Hz. Thus, the running speed is 2400 rpm. The parts of Fig. 7 show the theoretical and experimental relation between the load angle δ versus output torque T_o, phase current I_a, input power P_{in}, power factor PF and efficiency η. It is seen that pull out occurs when δ exceeds 14 electrical degrees. The power factor is nearly constant while efficiency increases up to δ = 8 electrical degrees and then tends to decrease. From figure 7, it can be concluded that correlation between experimental and theoretical results shows satisfactory matching which proves the validity of the suggested model. It can also be concluded that the power factor is high. This can be attributed to the high L_d/L_q ratio (approximately 40). With reference to (15), in SCSM, the higher the rotor to stator turns ratio K, the higher the L_d/L_q ratio.

8. Conclusion

A d-q model for SCSM is derived. The model is extended to include saturation. The motor runs synchronously at double rated speed when it runs as an induction motor depending on number of motor poles and supply frequency. Like synchronous motors, this motor is not self starting and it starts with the same methods as the conventional synchronous motor. The L_d/L_q ratio depends on stator to rotor turns ratio. The steady state performance has been studied. Comparison between experimental and theoretical results showed satisfactory agreement which proves the validity of the suggest model. The results can be considered as useful guides to operate and design three phase SCSM.

References

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List of Symbols

f	frequency of MMFs
i_{ar} , i_{br} , i_{cr}	instantaneous rotor currents in phases a, b and c respectively, A
i_{as} , i_{bs} , i_{cs}	instantaneous stator currents in phases a, b and c respectively, A
i_{dr}, i_{qr}	instantaneous rotor currents in q-d axes respectively, A
i_{ds}, i_{qs}	instantaneous stator currents in q-d axes respectively, A
i_d , i_q	instantaneous currents in the q-d axes respectively, A
I_d , I_q	steady state currents in the q-d axes respectively, A
I_m	peak magnetization current, A
K	effective rotor to stator turns ration
L_d, L_q	self inductance of d & axes receptively, H
L_s, L_r	stator & rotor self inductance receptively, H
l_s , l_r	stator and rotor leakage inductances receptively, H
M	maximum mutual inductance between rotor & stator, H
N_r	Shaft speed, rpm
p	number of machine poles
P	differential operator
PF	motor power factor
P_{in} , P_{out}	input and output power respectively, W
R_s, R_r, R_a	stator, rotor and equivalent armature resistances respectively, Ω
T_g , T_o	air gap and net torque respectively, Nm
v_{ds} , v_{qs}	instantaneous stator voltages in the q-d axes
v_{dr} , v_{qr}	instantaneous rotor voltages in the q-d axes
v_d , v_q	instantaneous voltages in the q-d axes
V_d , V_q	steady state voltages in the q-d axes
V_a , I_a	RMS phase voltage (V) and current (A)
η	motor efficiency
δ	load angle, electrical degrees
ω	angular speed, rad/s