

Recursion

Week #6

Recursive Functions

- **What is recursion?**

When one function calls ITSELF directly or indirectly.

- A function is a *recursive* if a statement in the body of the function calls the function that contains it.

Example 1: Multiplication

- Suppose we wish to write a recursive function to multiply an integer m by another integer n using addition. [We can add, but we only know how to multiply by 1].
- The best way to go about this is to formulate the solution by identifying the base case and the recursive case.
- The base case is if n is 1. The answer is m .
- The recursive case is: $m * n = m + m (n-1)$.

$$m * n \begin{cases} m, & n = 1 \\ m + m (n-1), & n > 1 \end{cases}$$

Recursive Functions

- *Recursive Function Multiply*

```
int multiply ( int m, int n)
{
int ans;
If (n==1) ans=m;
else
ans= m + multiply (m, n-1);
return (ans);
}
```

Example 1: Multiplication ...

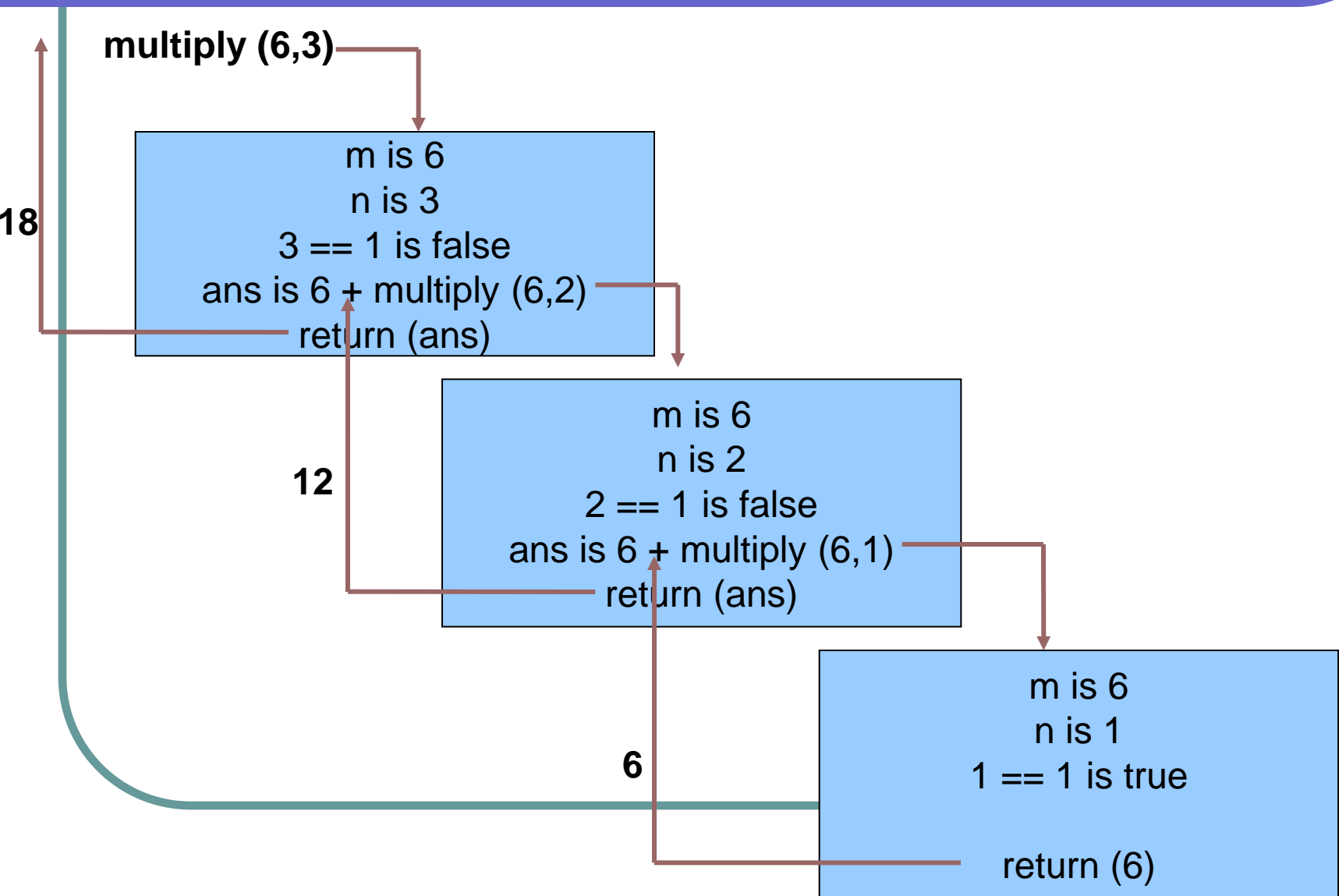
$$\begin{aligned}\text{multiply}(6,3) &= 6 + \text{multiply}(6, 2) \\ &= 6 + (6 + \text{multiply}(6, 1)) \\ &= 6 + (6 + (6))\end{aligned}$$

Expansion
phase

$$\begin{aligned}&= 6 + (6 + (6)) \\ &= 6 + (12) \\ &= 18\end{aligned}$$

Substitution
phase

Recursive Functions



Factorial

$$\begin{aligned}\text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * (3 * \text{factorial}(2)) \\ &= 4 * (3 * (2 * \text{factorial}(1))) \\ &= 4 * (3 * (2 * (1 * \text{factorial}(0))))\end{aligned}$$

Expansion
phase

$$\begin{aligned}&= 4 * (3 * (2 * (1 * 1))) \\ &= 4 * (3 * (2 * 1)) \\ &= 4 * (3 * 2) \\ &= 4 * 6 \\ &= 24\end{aligned}$$

Substitution
phase

Recursive Functions

- *Factorial (Recursive)*

```
Int Factr (int n)
{
int answer;
if (n==1) return(1);
answer=factr(n-1)*n;
return (answer)
}
```

- *Factorial (non-recursive)*

```
Int Fact (int n)
{ int t, answer;
answer = 1;
for (t=1; t<n; t++)
answer= answer * (t);
return (answer);
}
```


Example 3: Power function

- Suppose we wish to define our own power function that raise a double number to the power of a non-negative integer exponent. x^n , $n \geq 0$.
- The base case is if n is 0. The answer is 1.
- The recursive case is: $x^n = x * x^{n-1}$.

$$x^n \begin{cases} 1, & n = 0 \\ x * x^{n-1}, & n > 0 \end{cases}$$

Example 3: Power function ...

```
double pow(double x, int n)
{
    if (n == 0)
        return 1;  /* simple case */
    else
        return x * pow(x, n - 1); /* recursive step */
}
```