Arab Academy for Science and Technology and Maritime Transport

College of Engineering and Technology

Computer Engineering Department

CC216 Digital Logic Design

Lecture 3

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Binary Logic and Gates

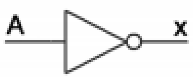
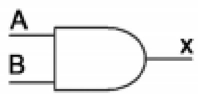
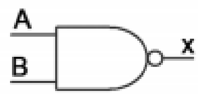
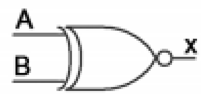
- Binary **logic** deals with binary variables (i.e. can have two values, “0” and “1”)
- Binary variables can undergo three basic logical operators **AND, OR and NOT**:

  - AND is denoted by a dot (·)
  - OR is denoted by a plus (+).
  - NOT is denoted by an overbar (¯), a single quote mark (') after the variable.

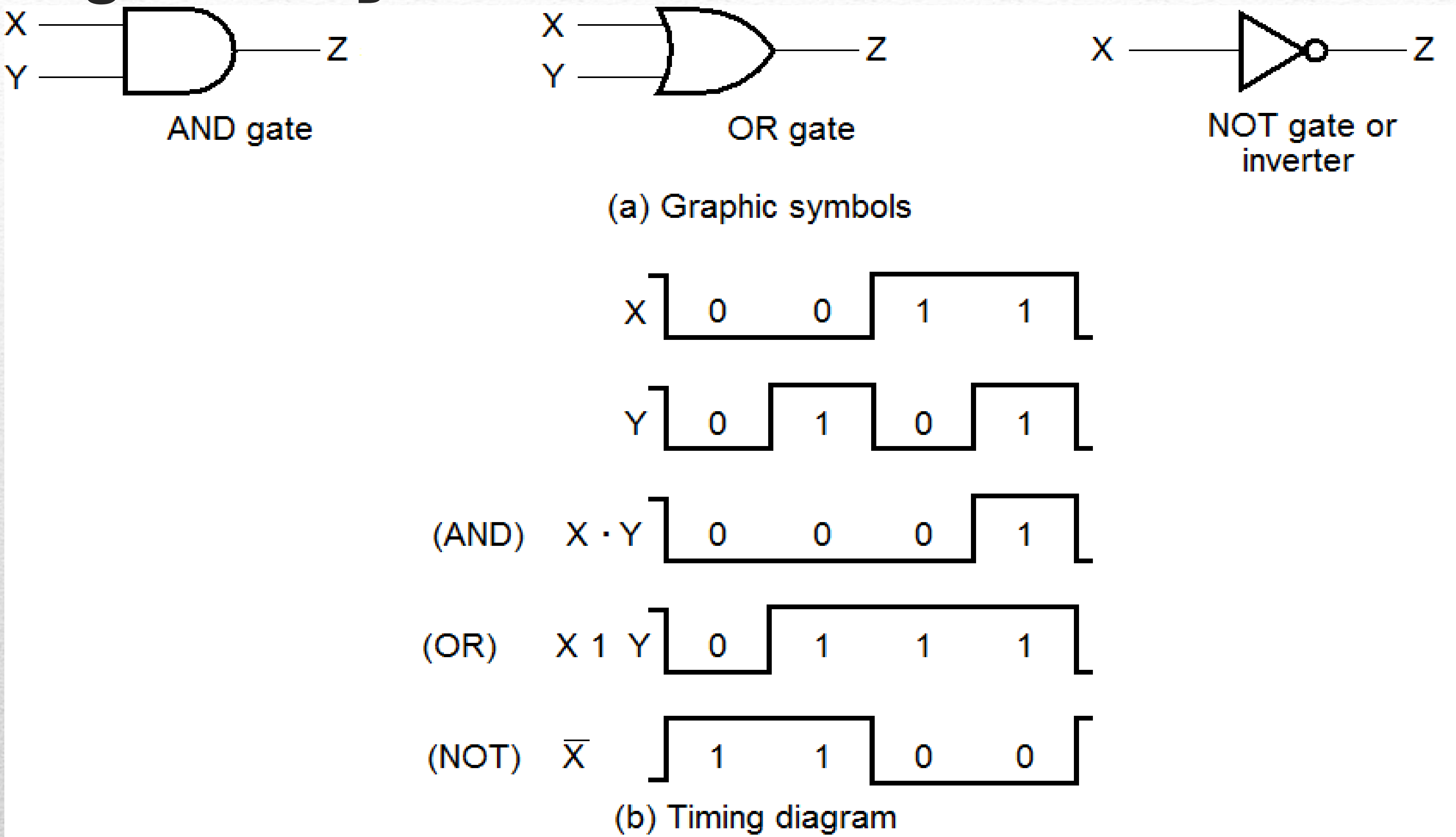
Operator Definitions and Truth Tables

- ***Truth table*** **- a tabular listing of the values of a function for *all possible combinations of values on its arguments***
- **Example: Truth tables for the basic logic operations:**

Operator Definitions and Truth Tables

Name	NOT	AND	NAND	OR	NOR	XOR	XNOR
Alg. Expr.	$\overline{A}$	AB	$\overline{AB}$	$A+B$	$\overline{A+B}$	$A \oplus B$	$\overline{A \oplus B}$
Symbol							
Truth Table	**A** **X** 0 **1** 1 **0**	**B** **A** **X** 0 0 **0** 0 1 **0** 1 0 **0** 1 1 **1**	**B** **A** **X** 0 0 **1** 0 1 **1** 1 0 **1** 1 1 **0**	**B** **A** **X** 0 0 **0** 0 1 **1** 1 0 **1** 1 1 **1**	**B** **A** **X** 0 0 **1** 0 1 **0** 1 0 **0** 1 1 **0**	**B** **A** **X** 0 0 **0** 0 1 **1** 1 0 **1** 1 1 **0**	**B** **A** **X** 0 0 **1** 0 1 **0** 1 0 **0** 1 1 **1**

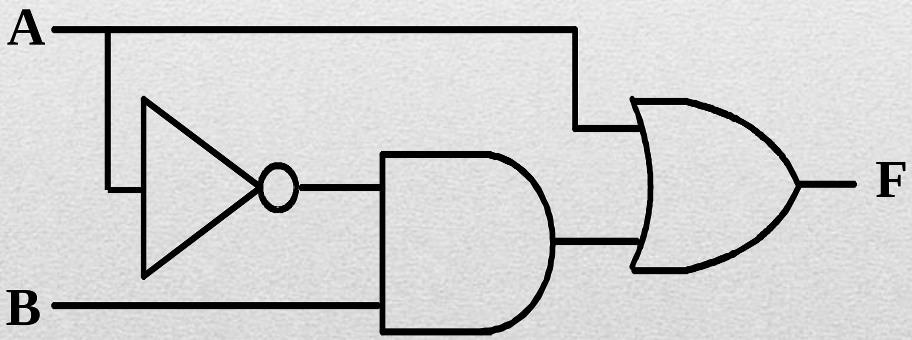
Logic Gate Symbols and Behavior



Logic Diagrams and Expressions

$$F = A + A.\overline{B}$$

Logic Diagram



Truth Table

A	B	F
0	0	
0	1	
1	0	
1	1	

Boolean Algebra

- **Boolean algebra deals with binary variables and a set of three basic logic operations: AND (.), OR (+) and NOT (—) that satisfy basic identities**

Basic identities

1. $X + 0 = X$
2. $X \cdot 1 = X$
3. $X + 1 = 1$
4. $X \cdot 0 = 0$
5. $X + X = X$
6. $X \cdot X = X$
7. $X + \overline{X} = 1$
8. $X \cdot \overline{X} = 0$
9. $\overline{\overline{X}} = X$

Boolean Algebra

Boolean Theorems of multiple variables

10.	$X + Y = Y + X$	Commutative	11.	$XY = YX$
12.	$(X + Y) + Z = X + (Y + Z)$	Associative	13.	$(XY)Z = X(YZ)$
14.	$X(Y + Z) = XY + XZ$	Distributive	15.	$X + YZ = (X + Y)(X + Z)$
16.	$\overline{X + Y} = \overline{X} \cdot \overline{Y}$	DeMorgan's	17.	$\overline{X \cdot Y} = \overline{X} + \overline{Y}$

Example: Boolean Algebraic Proof

- **$A + A \cdot B = A$** **(Absorption Theorem)**

Proof Steps	**Justification**
$A + A \cdot B$	
$= A \cdot 1 + A \cdot B$	(Operation with 1)
$= A \cdot (1 + B)$	(Distributive Law)
$= A \cdot 1$	(Operation with 1)
$= A$	

Example: Boolean Algebraic Proof

$$A + A'B = A + B$$

$$A+AB+A'B$$

$$A+B(A+A')$$

$$A+B(1)$$

$$A+B$$

DeMorgan

Use DeMorgan's Theorem to complement a function:

- Interchange AND and OR operators
- Complement each constant value and literal

Example: DeMorgan's theorem

Exercise: find $\overline{G}$

$G = UX(Y+VZ)$

$F = AB + C\,(E+D)$

Find $\overline{F}$