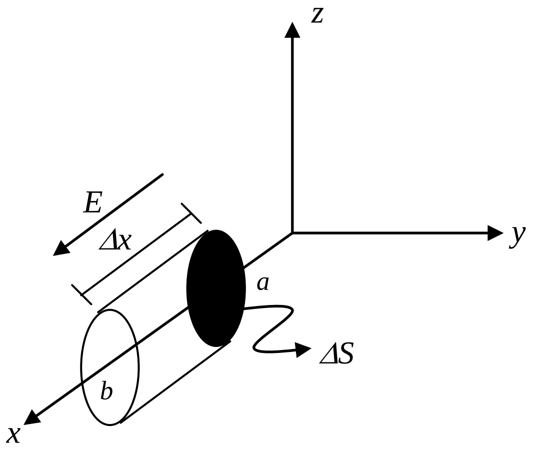
# CHAPTER (5)

# Moving charges
# Conductors – Semiconductors
# Dielectrics – Capacitors

# Convection Current Density

**Assume a cylinder of $\rho_V$ in space, as shown in figure. Also assume electric field $\vec{E} = \hat{a}_x \vec{E}_x$. The charge will move with velocity $\vec{u} = \hat{a}_x \vec{u}_x$.**




$$\Delta Q = \rho_V \Delta v = \rho_V \Delta S \Delta x$$

$$\therefore \frac{\Delta Q}{\Delta t} = \rho_V \Delta S \frac{\Delta x}{\Delta t}$$

$$\therefore \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \rho_V \Delta S \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

$$\therefore \Delta I = \rho_V \Delta S u_x$$

$$\text{Let:} \quad J_x = \lim_{\Delta t \to 0} \frac{\Delta I}{\Delta S} \quad \Rightarrow \quad \text{Current density}$$

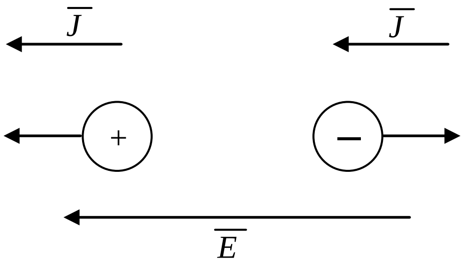
$$\therefore J_x = \rho_V u_x$$

$$\text{Similarly:} \quad J_y = \rho_V u_y \quad , \quad J_z = \rho_V u_z$$

$$\text{Therefore:} \quad \overline{J} = \rho_V \overline{u} \quad A/m^2 \quad \text{(generally)}$$

**If $\rho_v$ is – ve, the direction of current is still in the same direction of the electric field but the velocity will be in – ve direction of *x*, and the electric current will be in the direction of + ve charges.**

$$\therefore \overline{J} = \rho_{v_+} \overline{u_+} + \rho_{v_-} \overline{u_-}$$

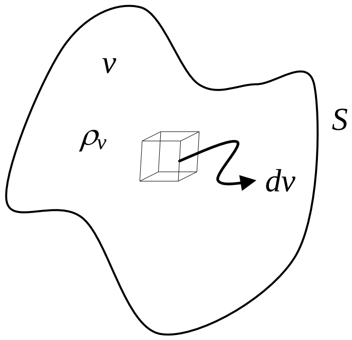



**This type of current is called convectional current**

$$I = \oint_S \overline{J} . d\overline{S}$$

# Conservation of charge and continuity equation:

**Assume a volume *v* is charged by $\rho_v$**




$$Q_{en} = \int \rho_v \, dv$$

**The charge $Q_{en}$ can increase or decrease by flowing of charges in and out of the surface *S* .**

**The rate of decrease of $Q_{en}$ means a net current $I_{outward}$.**

$$I_S = -\frac{\partial Q_{en}}{\partial t}$$

**Applying the Divergence theorem,**

$$\therefore I_S = \oint \bar{J}.d\bar{S} = -\int \frac{\partial \rho_v}{\partial t} dv = \int \nabla.\bar{J} \, dv$$

$$\therefore \nabla.\bar{J} = -\frac{\partial \rho_v}{\partial t} \qquad \text{(continuity equation)}$$

**This equation represents point form of continuity equation. It states that the current per unit volume emitting from a point equals the time rate decrease of volume charge density at the same point.**

# Conductors and conductivity:

The conductor has many free electrons in the conduction band. These electrons move in different directions (random motion) when there is no external electric field, i.e. net current = zero.

Now if we apply an electric field, the electrons will drift slowly in $E$-direction with drift velocity $\vec{u}_d$.

The drift velocity is related to the electric field by:

$\vec{u}_d = -\mu_e \vec{E}$ where $\mu_e$ is the electron mobility.

$$\overline{J} = \rho_{ve}\overline{u}$$

$$\therefore \overline{J} = -\mu_e \rho_{ve}\overline{E} = \sigma \overline{E} \quad , \quad \sigma \Rightarrow \text{conductivity (S/m) or (Mho/m)}$$

$$\therefore \sigma = -\mu_e \rho_{ve}$$

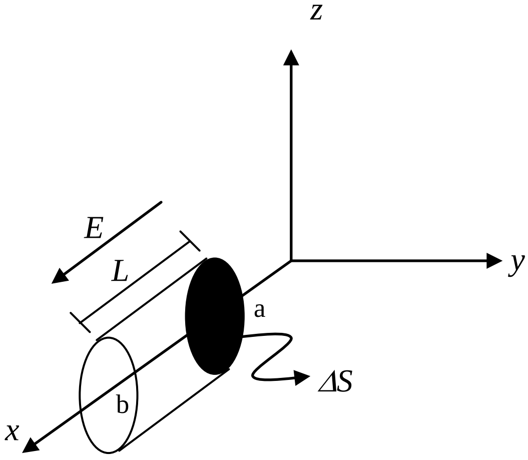
## Note:

- σ is a measure of the free electrons in the conductor.
- In good conductor σ decreases as temperature increases.

## Examples:

| Material | σ | Material | σ |
|---|---|---|---|
| Silver<br>Copper | $6.1\text{x}10^{7}$<br>$5.8\text{x}10^{7}$ | Sea water<br>S.C. | 4<br>$0.44\text{x}10^{-3}$ |

## Ohm's law and resistance:




$$I_C = \int \overline{J}.d\overline{S} = \int \sigma \overline{E}.d\overline{S}$$

$$\therefore I_C = \sigma E S$$

$$V_{ab} = -\int_b^a \overline{E_x}.d\overline{l} = -E_x \int_b^a dx = E_x L$$

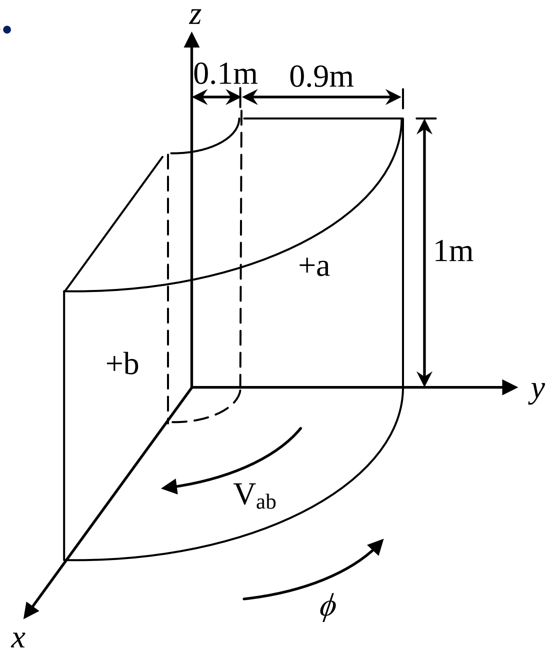
$$\text{But:} \quad V_{ab} = \frac{I_C}{\sigma S} L = \frac{L}{\sigma S} I_C = R I_C$$

$$R = \frac{L}{\sigma S} = \rho \frac{L}{S}$$

$$\text{In general:} \quad R = \frac{V_{ab}}{I_C} = \frac{\left| -\int_b^a \overline{E}.d\overline{l} \right|}{\left| \int_S \sigma \overline{E}.d\overline{S} \right|}$$

## Example:

**Find the resistance between the $\phi = 0$ and $\phi = \pi/2$ surface for the section shown in figure where $\sigma = 4 \times 10^7$ S/m and $\vec{E} = -\frac{1}{r_c}\hat{a}_\varphi \ V/m$.**




$$R = \left|\frac{V_{ab}}{I}\right| \quad , \quad V_{ab} = -\int_b^a \bar{E}.d\bar{l} \quad , \quad d\bar{l} = dr_c\,\hat{r}_c + r_c d\phi\,\hat{\phi} + dz\,\hat{z}$$

$$\therefore V_{ab} = -\int_b^a -\frac{1}{r_c}\hat{\phi}.r_c d\phi\,\hat{\phi} = \int_0^{\frac{\pi}{2}} d\phi = \frac{\pi}{2} \text{ volts}$$

$$, I = \int_S \sigma \bar{E}.d\bar{S} = \int_S \sigma\left(-\frac{1}{r_c}\hat{\phi}\right).dr_c dz\,(-\hat{\phi})$$

$$= \int_0^1 \int_{0.1}^1 \sigma\left(-\frac{1}{r_c}\hat{\phi}\right) dr_c dz\,\hat{\phi} = \sigma(\ln r_c)_{0.1}^1 . (z)_0^1$$

$$= 4\times 10^7 [\ln(1) - \ln(0.1)] = 4\times 10^7 \ln(10) = 0.017\Omega$$

# Conductor properties under static conditions:

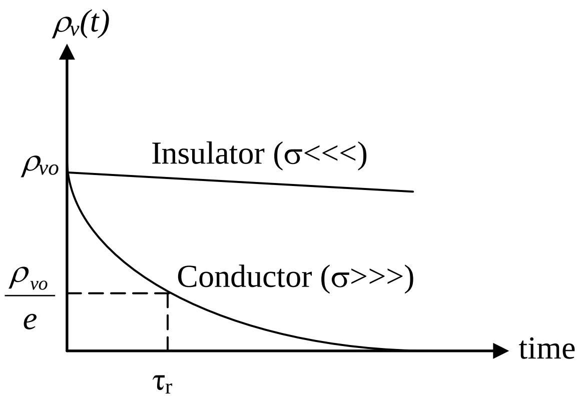
$$\because \nabla . \overline{J} = -\frac{\partial \rho_{ve}}{\partial t} \quad , \quad \because \overline{J} = \sigma \overline{E}$$

$$\therefore \nabla . \left(\sigma \overline{E}\right) = -\frac{\partial \rho_{ve}}{\partial t}$$

$$\therefore \nabla . \overline{E} = -\frac{1}{\sigma} \frac{\partial \rho_{ve}}{\partial t} \quad \rightarrow \quad (1)$$

$$\because \nabla . \overline{D} = \rho_{ve} \quad , \quad \overline{D} = \varepsilon \overline{E}$$

$$\therefore \nabla . \overline{E} = \frac{\rho_{ve}}{\varepsilon}$$




from equations (1) , (2) :

$$\frac{\rho_{ve}}{\varepsilon} = -\frac{1}{\sigma} \frac{\partial \rho_{ve}}{\partial t}$$

$$\therefore \frac{\partial \rho_{ve}}{\partial t} + \frac{\sigma}{\varepsilon} \rho_{ve} = 0$$

Diffrential equation solution :

$$\rho_{ve}(t) = \rho_{ve_0} e^{-\frac{t}{\tau_r}} \quad , \quad \text{where} : \tau_r = \frac{\varepsilon}{\sigma} = \text{relaxation time}$$

**for** $\boldsymbol{\sigma >>> \ \Rightarrow \ \tau_r <<<}$

**for coductor (copper)** $\boldsymbol{\sigma = 4 \times 10^7}$

$$\tau_r = \frac{\varepsilon_o}{\sigma} = \frac{10^{-9}}{36\pi \times 4 \times 10^7} = 1.5 \times 10^{-19} \ \text{sec}$$

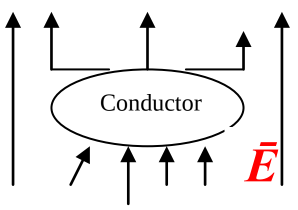
for good insulator :

$$\tau_r = \frac{10^{-9}}{36\pi \times 4 \times 10^{-12}} = 1.5 \ \text{sec}$$

## Thus conductor properties under static conditions are:

1- The net $\rho_{ve} = 0$ inside the conductor.
2- The electric field inside the conductor is zero.
3- The conductor is an equi-potential surface.
4- The charge density $\rho_s$ is present (it's found only on the surface).
5- The tangent component of the electric field = 0, i.e. $E_t = 0$. Therefore $J_t = \sigma E_t = 0$.
6- The normal component of $\bar{E}$ on conductor isn't zero if $\rho_s$ exists.

## Note:




If conductor is imposed in an $\bar{E}_a$ the flux adapt it self to be normal to the surface. If a slab of conductor is immersed in an electric field $\bar{E}_a$ (applied electric field), the free charges will be forced to the surface, Produce $\bar{E}_a$ = - $\bar{E}_i$, thus the net electric field inside the conductor = $\bar{E}_a - \bar{E}_i = 0$.

If an elongated conducting object where its axis // to $\bar{E}a$, the net field inside = 0 and the charges are accumulated to thin edges.

# Semiconductors:

**The free electrons in conduction band for a semiconductor material is very small compared with the free electrons in a conductor. The charge carriers in intrinsic semi-conductor are free electrons (-ve) and holes (+ ve). The density of the free electrons equal to the density of holes. The conduction current in intrinsic S.C is:**

$$\overline{J_c} = \left(-\mu_e \rho_{ve} + \mu_h \rho_{vh}\right)\overline{E} = \sigma \overline{E}$$

$$\therefore \sigma = -\mu_e \rho_{ve} + \mu_h \rho_{vh}$$

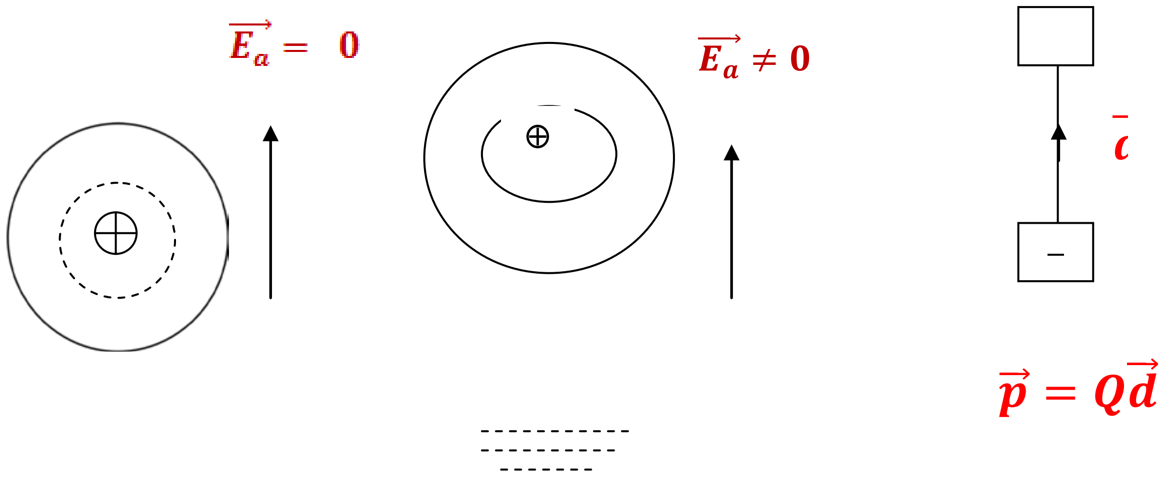
# Dielectrics (insulators):

**Dielectrics are poor conductors where $\sigma \ll 1$, the main characteristic of dielectrics is polarization in the form of electric dipoles. When a dielectric material is placed in an electric field $\vec{E}$, a displacement takes place between the average equilibrium positions of + ve and - ve bound charges of atoms and molecules. This displacement gives rise to an electric dipole moment $(\vec{p} = Q\vec{d})$. These electric dipoles have an effect on $\vec{E}$ in which the dielectric materials is placed.**

## There are three basic polarization mechanisms:

### 1- Electronic polarization:

**It exists in an atom where $\bar{E}_a$ is applied, where the center of the cloud electrons is displace relative to the center of nucleus.**



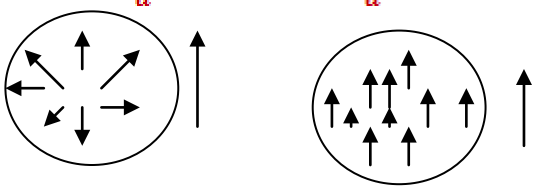

### 2- Ionic polarization:

**Exists in molecules having ionic bonds such as $Na^+Cl^-$. When $\bar{E}_a$ is applied it will displace the + ve ions relative to the –ve ions making dipoles.**

## 3- Orientation  polarization (polar polarization):

It exists in material whose molecules have permanent dipoles due to its structure but in random directions. The net dipole moments equal zero, but under the influence of external electric field these dipoles take direction of the electric field thus the net dipole moments ≠ 0. This kind of polarization exists in $H_2O$.

$\overrightarrow{E_a} = 0$ $\overrightarrow{E_a} \neq 0$



$\vec{p} = 0$ $\vec{p} \neq 0$

## Polarization Vector $\overrightarrow{P}$

The polarization of a material is simply the total dipole moment for a unit volume.

$$\overrightarrow{P} = lim_{\Delta v \to 0} \frac{\sum_{i=1}^{N} \vec{p}_i}{\Delta v}$$

where $\Delta v$ is the overall volume of the sample, and . $N$ is the dipole density.

- Since $\sum_{i=1}^{N} \vec{p}_i$ is a vector sum, a material may contain dipoles without having any net polarization, since dipole moments can cancel out.

## surface and volume bounded charges $\rho_{sb}, \rho_{vb}$

**If the applied electric field $\overrightarrow{E_a}$ on a dielectric slab is uniform thus the distribution of dipoles is also uniform. The upper surface will have +ve bound charges $\rho_{sb}^{+}$ , but the lower surface will have -ve bound charge $\rho_{sb}^{-}$.**

### The surface bound charge density $\rho_{sb}$:

$$\rho_{sb} = \overrightarrow{P} \cdot \hat{n}$$

**Where,**

$\overrightarrow{P}$ **is the electric polarization vector**

$\hat{n}$ **is the unit vector normal to the surface**

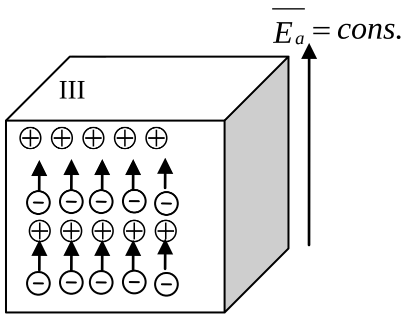
**While, volume bound charge density**

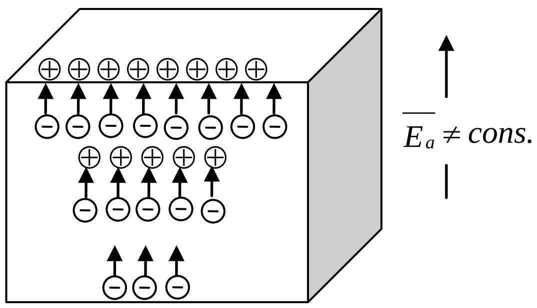
$$\rho_{vb} = -\nabla \cdot \overrightarrow{P} = 0$$

**If the electric field $\overrightarrow{E_a}$ is non-uniform:**

### Volume bound charge density $\rho_{vb}$:

$$\rho_{vb} = -\nabla \cdot \overrightarrow{P}$$



## Effect of polarization on electric fields:

- Since, the electric flux density in free space $\vec{D}_f$ is related to the electric field intensity $\vec{E}_f$ due to free volume charge density $\rho_v$ by

$$\nabla . \vec{D}_f = \rho_v$$

$$\vec{D}_f = \varepsilon_o \vec{E}_f$$

- But in dielectric materials, the electric flux density will not be changed since it depends upon the free volume charge density $\rho_v$ and equal to $\vec{D}_f$, while the electric field intensity $\vec{E}_f$ will be changed into $\vec{E}$.

- The dielectric can be considered as a free space in addition to the Volume bounded charge density $\rho_{vb}$, so

$$\nabla . \varepsilon_o \vec{E} = \rho_v + \rho_{vb}$$

Since,

$$\rho_{vb} = -\nabla . \vec{P}$$

So,

$$\nabla . (\varepsilon_o \vec{E}) = \rho_v - \nabla . \vec{P}$$

$$\nabla . (\varepsilon_o \vec{E} + \vec{P}) = \rho_v$$

Since $\vec{D}_f$ depends only on the free volume charge density $\rho_v$, so

$$\vec{D} = \vec{D}_f = \varepsilon_o \vec{E} + \vec{P}$$

**For linear isotropic dielectric material, the relation between $\vec{P}$ and $\vec{E}$ is**

$$\vec{P} = \varepsilon_o \chi_e \vec{E}$$

**Where, $\chi_e$ is the electric susceptibility**

**Then,**

$$\vec{D} = \varepsilon_o \vec{E} + \varepsilon_o \chi_e \vec{E}$$

$$\vec{D} = \varepsilon_o (1 + \chi_e) \vec{E}$$

$$\vec{D} = \varepsilon_o \varepsilon_r \vec{E} = \varepsilon \vec{E}$$

- **Where, $\varepsilon_r$ is the relative permittivity**

$$\varepsilon_r = (1 + \chi_e)$$

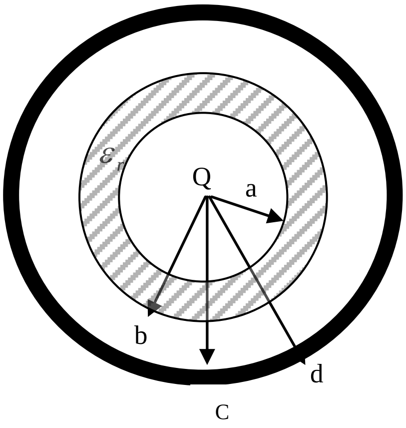
**Note:**

**The electric flux density $\bar{D}$ is constant elsewhere but the electric field is only varying from a medium to another**

## Example:

**For the following concentric spheres shown in figure a point charge Q is placed at the center. regions 1, 3, 5 are free spaces, while region 2 is of a dielectric constant $\varepsilon_r$ and region 4 is a conductor find: $\overline{D}, \overline{E}, \overline{p}, \rho_{Sb}, \rho_{vp}, \rho_S$ in all regions.**

## Solution:




For region (1) : $0 < r_s < a$

$$D.4\pi r_S^2 = Q$$

$$\overline{D} = \frac{Q}{4\pi r_S^2}\hat{r}_s = \varepsilon_o \overline{E}$$

$$\overline{E} = \frac{Q}{4\pi\varepsilon_o r_S^2}\hat{r}_s$$

$$\overline{p} = \overline{D} - \varepsilon_O \overline{E} = \frac{Q}{4\pi r_S^2}\hat{r}_s - \frac{\varepsilon_O Q}{4\pi \varepsilon_O r_S^2}\hat{r}_s = 0$$

$$\overline{p} = \varepsilon_O \chi_e \overline{E} = \varepsilon_O(\varepsilon_r - 1)\overline{E} = \varepsilon_O(1-1)\overline{E} = 0$$

$$\rho_{Sb} = bound - charge = \overline{p}.\hat{n} = 0$$

$$\rho_{vb} = -\nabla.\overline{p} = 0$$

$$\rho_S = zero$$

**Note:** **in applying Gauss's law we take only free charge not bound charge".**

For region (2) : $a < r_s < b$

$$D.4\pi r_S^2 = Q$$

$$\overline{D} = \frac{Q}{4\pi r_S^2}\hat{r}_S = \varepsilon_O \varepsilon_r \overline{E}$$

$$\overline{E} = \frac{Q}{4\pi \varepsilon_O \varepsilon_r r_S^2}\hat{r}_S$$

$$\overline{p} = \overline{D}_2 - \varepsilon_0 \overline{E} = \frac{Q}{4\pi r_S^2}\hat{r}_S - \frac{\varepsilon_0 Q}{4\pi \varepsilon_0 \varepsilon_r r_S^2}\hat{r}_S$$

$$= \frac{Q}{4\pi r_S^2}\left(1-\frac{1}{\varepsilon_r}\right)\hat{r}_S$$

$$\rho_{sb}(r_s = b) = \overline{p}.\hat{n} = \frac{Q}{4\pi r_S^2}\left(1-\frac{1}{\varepsilon_r}\right)\hat{r}_S . \hat{r}_S = \frac{Q}{4\pi b^2}\left(1-\frac{1}{\varepsilon_r}\right) C/m^2$$

$$\rho_{sb}(r_s = a) = \frac{Q}{4\pi r_S^2}\left(1-\frac{1}{\varepsilon_r}\right)\hat{r}_S .(-\hat{r}_s) \quad [at\, \hat{r}_S = a]$$

$$= -\frac{Q}{4\pi a^2}\left(1-\frac{1}{\varepsilon_r}\right) C/m^2$$

$$\rho_S = zero$$

$$\rho_{Vb} = -\nabla.\overline{p} = \frac{1}{r_S^2 \sin\theta}\left[\frac{\partial}{\partial r_S}\left(r_S^2 \sin \frac{Q}{4\pi r_S^2}\left(1-\frac{1}{\varepsilon_r}\right)\right)\right] = 0$$

For region (4) : $c < r_s < d$

$$\overline{D} = 0$$

$$\overline{E} = 0$$

$$\overline{p} = 0$$

$$\rho_{Sb} = 0$$

$$\rho_{vb} = 0$$

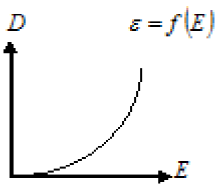
$$\rho_s = -\frac{Q}{4\pi c^2} C/m^2 \quad at\, r_s = c$$

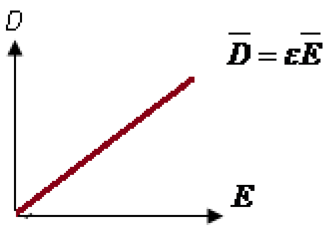
$$\rho_s = d = \frac{Q}{4\pi d^2} C/m^2 \quad at\ r_s = d$$

**If the conducting region at $r_s = d$ is connected to earth the electric field in the region 5 is equal to zero and the surface free charge at $r_s = d$ is equal to zero.**

## DEFINITIONS:

**• Linearity:** A dielectric material is considered linear if the relation between the magnitude of electric field intensity and the magnitude of electric flux density is a linear function. So the relative permittivity is not function of the magnitude of electric field intensity. But if the permittivity at any point is a function of magnitude of electric field intensity the material is non-linear.




**•Homogeneity:** A dielectric material is homogeneous when $\varepsilon$ does not change from a point to another, i.e. $\varepsilon$ is not a function of position. If $\varepsilon$ varies with position $\varepsilon$ = f (x,y,z ) the material is non homogeneous

**•Isotropy:** If $\vec{P}$ *and* $\vec{E}$ are in the same direction the material is isotropic, which means that $\chi_e$ does not change with the direction of $\vec{E}$

$$\vec{P} = \varepsilon_o \chi_e \vec{E}$$

## ● For non-isotropic material:

$$P_x = \varepsilon_o \chi_{e11} E_x + \varepsilon_o \chi_{e12} E_y + \varepsilon_o \chi_{e13} E_z$$
$$P_y = \varepsilon_o \chi_{e21} E_x + \varepsilon_o \chi_{e22} E_y + \varepsilon_o \chi_{e23} E_z$$
$$P_z = \varepsilon_o \chi_{e31} E_x + \varepsilon_o \chi_{e32} E_y + \varepsilon_o \chi_{e33} E_z$$

**and**

$$\vec{D} = \varepsilon_o \vec{E} + \varepsilon_o \chi_e \vec{E}$$

$$D_x = \varepsilon_o E_x + P_x$$

$$D_x = \varepsilon_o (1 + \chi_{e11}) E_x + \varepsilon_o \chi_{e12} E_y + \varepsilon_o \chi_{e13} E_z$$

$$D_x = \varepsilon_{11} E_x + \varepsilon_{12} E_y + \varepsilon_{13} E_z$$

$$D_y = \varepsilon_{21} E_x + \varepsilon_{22} E_y + \varepsilon_{23} E_z$$

$$D_z = \varepsilon_{31} E_x + \varepsilon_{32} E_y + \varepsilon_{33} E_z$$

$$\vec{D} = \begin{vmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{vmatrix} \begin{vmatrix} \vec{E}_x \\ \vec{E}_y \\ \vec{E}_z \end{vmatrix}$$

$$\vec{D} = |\varepsilon| |\vec{E}|$$

## ●Dispersion:

**The dielectric material is considered non dispersive, if the dielectric constant $\varepsilon_r$ is not function of frequency, and so as an example the velocity of waves will be constant and do not change with frequency ($v = \frac{1}{\sqrt{\mu_o \varepsilon_o \varepsilon_r}}$).**

# Dielectric Boundary conditions:

## *Normal components:*

$$\oint \overline{D}.d\overline{S} = Q_{en}$$

$$D_{n1}\Delta S - D_{n2}\Delta S = \rho_S \Delta S$$

$$\boldsymbol{D_{n1} - D_{n2} = \rho_s}$$

## *Tangent component:*

$$\oint \overline{E}.d\overline{l} = 0$$

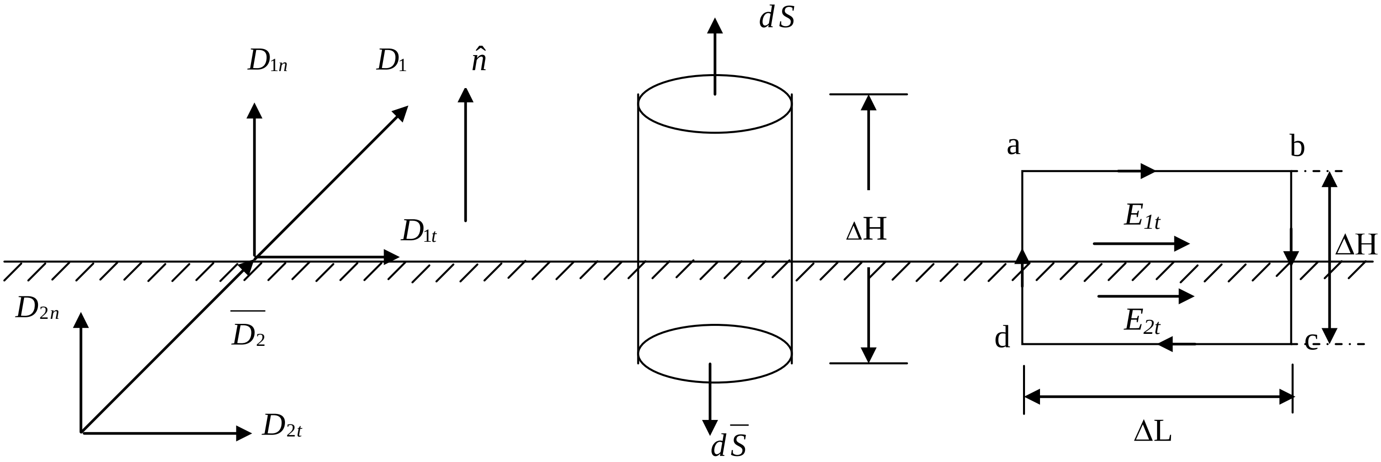
$$E_{t1}.\Delta l + \int_b^C \overline{E}.d\overline{l} - E_{t2}.\Delta l + \int_d^a \overline{E}.dl = 0$$

$$\int_b^C \overline{E}.d\overline{l} = zero \Rightarrow \Delta l = 0$$

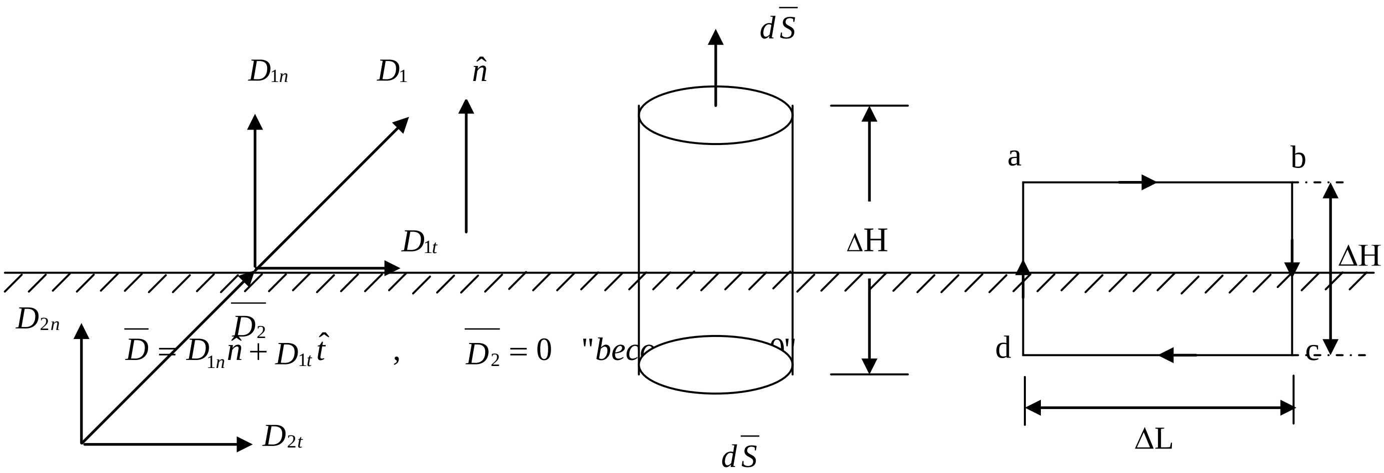
$$\int_d^a \overline{E}.d\overline{l} = zero \Rightarrow \Delta l = 0$$

$$\therefore E_{t1} = E_{t2}$$

$$\boldsymbol{E_{1t} = E_{2t}}$$



# Conductor-free space boundary conditions:




**The boundary condition on the normal field component can be found by applying Gauss's law to a small cylinder at the boundary as $\Delta H \rightarrow 0$.**

$$\oint \overline{D}.d\overline{S} = Q_{en} = \int_{top} \overline{D_1}.d\overline{S} + \int_{bottom} \overline{D_2}.d\overline{S} + \lim_{\Delta H \to 0} \int_{side} \overline{D}.d\overline{S} = Q_{en}$$

$$\therefore D_{1n}.\Delta S + 0 + 0 = \rho_S \,\Delta S$$

$$\therefore D_{1n} = \rho_s$$

$$,\ E_{1n} = \frac{D_{1n}}{\varepsilon_o} = \frac{\rho_s}{\varepsilon_o}$$

**The boundary conditions for tangent field components can be obtained by using the conservative property of the electric field:**

$$\oint_L \overline{E}.d\overline{l} = 0$$

$$\oint_l \overline{E}.d\overline{l} = \int_a^b \overline{E_1}.d\overline{l} + \int_b^C \overline{E}.dl + \int_c^d E_{.2}.dl \int_d^a \overline{E}.dl = 0$$

$$\therefore E_{1t}.\Delta l + 0 + 0 + 0 = 0$$

$$\therefore E_{1t} = 0$$

## Note:

$$\lim_{\Delta H \to 0} \int_b^c \overline{E}.d\overline{l} = 0 \quad and \quad \lim_{\Delta h - 0} \int_d^0 \overline{E}.d\overline{l} = 0$$

## Very important note:

**The electric field cannot be found tangent to the conductor but may be found normal.**

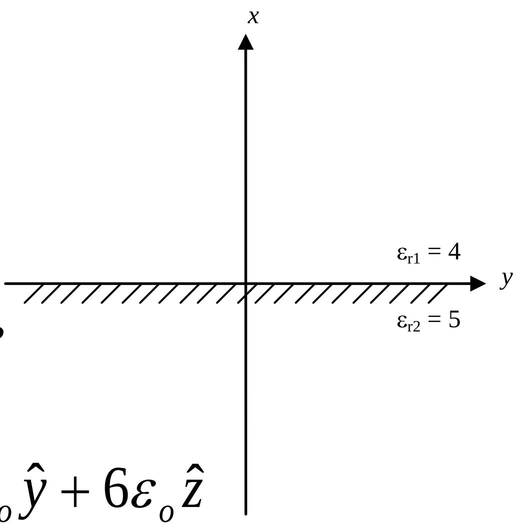
## Example:

**A boundary between two dielectrics is found at *x* = 0 plane. If material (1) exists for *x* > 0 with $\varepsilon_{r1}$ = *4*, and material (2) exists for *x* < 0 with $\varepsilon_{r2}$ = *5* and** $\overline{E}_1 = 2\hat{x} + 3\hat{y} - 6\hat{z}$ **at the boundary. Find:**

1- $\overline{D_1}$ 2- $\overline{p_1}$ 3- $\overline{E}_2$

4- $\overline{D_2}$ 5- $\overline{p_2}$ 6- $\rho_{sb}$ assume $\rho_s$=0 at the boundary

## Solution:




$$\overline{D_1} = \varepsilon_o \varepsilon_r \overline{E_1}$$

$$\overline{D_1} = 4\varepsilon_o [2\hat{x} + 3\hat{y} - 6\hat{z}]$$

$$\overline{D_1} = 8\varepsilon_o \hat{x} + 12\varepsilon_o \hat{y} - 24\varepsilon_o$$

$$\overline{p_1} = \overline{D_1} - \varepsilon_o \overline{E_1}$$

$$\overline{p_1} = 8\varepsilon_o \hat{x} + 12\varepsilon_o \hat{y} - 24\varepsilon_o \hat{z} - 2\varepsilon_o \hat{x} - 3\varepsilon_o \hat{y} + 6\varepsilon_o \hat{z}$$

$$\overline{p_1} = 6\varepsilon_o \hat{x} + 9\varepsilon_o \hat{y} - 18\varepsilon_o \hat{z}$$

$$\overline{E_{1_t}} = \overline{E_{2_t}} = 3\hat{y} - 6\hat{z}$$

$$\because D_{n_1} - D_{n_2} = \rho_s$$

$$\therefore 8\varepsilon_o - D_{n_2} = 0$$

$$\therefore D_{n_2} = 8\varepsilon_o$$

$$E_{n_2} = \frac{D_{n_2}}{\varepsilon_o \varepsilon_r} = \frac{8\varepsilon_o}{\varepsilon_o \varepsilon_{r2}} = \frac{8}{\varepsilon_{r2}} = \frac{8}{5}$$

$$\therefore \overline{E_{n_2}} = \frac{8}{5}\hat{x}$$

$$\therefore \overline{E_2} = \overline{E_{n_2}} + \overline{E_{t_2}}$$

$$\therefore \overline{E_2} = \frac{8}{5}\hat{x} + 3\hat{y} - 6\hat{z}$$

$$\overline{D_2} = \varepsilon_o \varepsilon_r \overline{E_2} = 5\varepsilon_o \left( \frac{8}{5}\hat{x} + 3\hat{y} - 6\hat{z} \right) = 8\,\varepsilon_o \hat{x} + 15\varepsilon_o \hat{y} - 30\varepsilon_o \hat{z}$$

$$\overline{p_2} = \varepsilon_o \chi_{t2} \overline{E_2} = \varepsilon_o (\varepsilon_{r2} - 1)\overline{E_2} = 4\varepsilon_o \overline{E_2} = 4\varepsilon_o \left( \frac{8}{5}\hat{x} + 3\hat{y} - 6\hat{z} \right)$$

$$= \frac{32}{5}\varepsilon_o \hat{x} + 12\varepsilon_o \hat{y} - 24\varepsilon_o \hat{z}$$

$$\rho_{sb} = \overline{p_1} . \hat{n}_1$$

$$\rho_{sb} = \overline{p_2} . \hat{n}_2$$

## Capacitance:

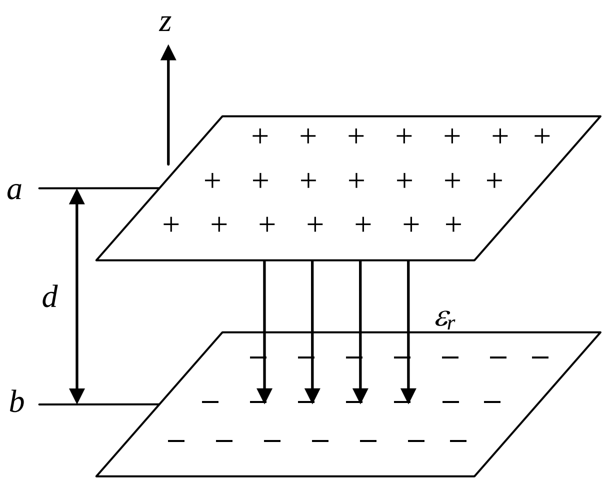
Any two conducting bodies separated by free space or dielectric material has a capacitance between them any voltage difference applied results a charge $Q$ on one conductor and – $Q$ on the other conductor. The ratio of the absolute value of the charge $Q$ to the voltage difference between the two conductors is defined as capacitance, i.e.:

$$C = \frac{|Q|}{|V_{ab}|}$$

$$or: \quad C = \frac{\left|\int \rho_s .ds\right|}{\left|-\int \overline{E}.d\overline{l}\right|} = \frac{\left|\int \varepsilon \overline{E}.d\overline{s}\right|}{\left|-\int \overline{E}.d\overline{l}\right|}$$

## Example:

Find the capacitance of the parallel plate capacitor. Assuming charge +$Q$ on the upper plate and – $Q$ on the lower one, the distance between the plates is d and the surface area of each plate is A. the dielectric between the plates $\varepsilon_r$.



## Solution:

Since,

$$C = \frac{|Q|}{|V_{ab}|}$$

Applying the boundary conditions between the lower face of the upper conductor (a) and the dielectric

$$D_{n1} - D_{n2} = \rho_s$$

$$0 - D_{n2} = \rho_s$$

So, the electric flux density in the dielectric $\vec{D}_2$ will be equal to

$$\vec{D}_2 = -\hat{a}_z \rho_s$$

and so, the electric field in the dielectric will be

$$\vec{E}_2 = -\hat{a}_z \frac{\rho_s}{\varepsilon_o \varepsilon_r}$$

and, the potential difference $V_{ab}$ will be,

$$V_{ab} = -\int_b^a \vec{E}_2 \, . \, \vec{dl}$$

$$V_{ab} = -\int_0^d \left(\vec{E}_2 = -\hat{a}_z \frac{\rho_s}{\varepsilon_o \varepsilon_r}\right) . \vec{dz}$$

$$V_{ab} = \frac{\rho_s}{\varepsilon_o \varepsilon_r} d$$

**The capacitance C is given as**

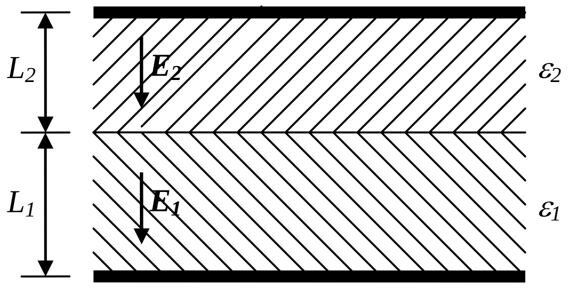
$$C = \frac{|Q|}{|V_{ab}|}$$

So,

$$C = \frac{\rho_s A}{\frac{\rho_s}{\varepsilon_o \varepsilon_r} d} = \frac{\varepsilon_o \varepsilon_r A}{d} \quad [F]$$

## Example:

**Find the capacitance of the parallel plate capacitor. Assuming charge +*Q* on the upper plate and – *Q* on the lower one, the distance between the plates is d and the surface area of each plate is A. there are two slabs of dielectric material between the plates $\varepsilon_{r1}$ and $\varepsilon_{r2}$.**




## Solution:

$$C = \frac{Q}{V_{ab}}$$

$$V_{ab} = -\int_0^{l_1} \overline{E}_1 . d\overline{l} - \int_{l1}^{l_1+l_2} \overline{E}_2 . d\overline{l}$$

$$\overline{E}_1 = \frac{\overline{D}_1}{\varepsilon_o \varepsilon_{r_1}} = \frac{\rho_s(-\hat{z})}{\varepsilon_o \varepsilon_{r_1}}$$

$$\overline{E}_2 = \frac{\overline{D}_2}{\varepsilon_o \varepsilon_{r_2}} = \frac{\rho_s(-\hat{z})}{\varepsilon_o \varepsilon_{r_2}}$$

$$\therefore V_{ab} = -\int_0^{l_1} \frac{\rho_s(-\hat{z})}{\varepsilon_o \varepsilon_{r_1}} .dz\ \hat{z} - \int_{l1}^{l_1+l_2} \frac{\rho_s(-\hat{z})}{\varepsilon_o \varepsilon_{r_2}} .dz\ \hat{z}$$

$$\therefore V_{ab} = \frac{\rho_s l_1}{\varepsilon_o \varepsilon_{r_1}} + \frac{\rho_s l_2}{\varepsilon_o \varepsilon_{r_2}}$$

$$\because C = \frac{Q}{V_{ab}}$$

$$\therefore C = \frac{\rho_s A}{\frac{\rho_s l_1}{\varepsilon_o \varepsilon_{r_1}} + \frac{\rho_s l_2}{\varepsilon_o \varepsilon_{r_2}}} = \frac{1}{\frac{l_1}{\varepsilon_o \varepsilon_{r_1} A} + \frac{l_2}{\varepsilon_o \varepsilon_{r_2} A}}$$

$$= \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2}$$

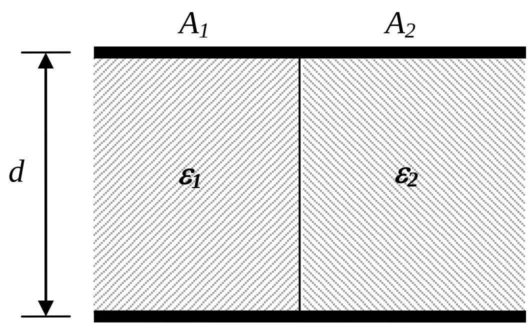
$\therefore$ Capacitors in series

## Notes:

1- **If the electric field is perpendicular to the interface between the two dielectrics, then:** ***capacitors are in series*.**

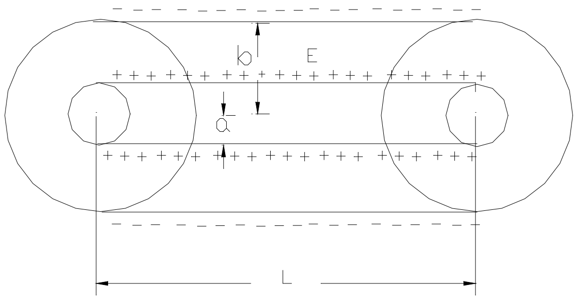
2- $C_1 = \frac{\varepsilon_o \varepsilon_{r1} A_1}{d}$ **,** $C_2 = \frac{\varepsilon_o \varepsilon_{r2} A_2}{d}$

$$\therefore C_t = C_1 + C_2$$



## Example:

**Find the capacitance of coaxial capacitor, shown in figure.**




## Solution:

$$C = \frac{Q}{V_{ab}}$$

$$Q = \rho_l . l$$

$$V_{ab} = -\int_b^a \overline{E}.d\overline{l}$$

$$= -\int_b^a \frac{\rho_l}{2\pi \varepsilon_o \varepsilon_r r_c} dr = \frac{\rho_l}{2\pi \varepsilon_o \varepsilon_r} \ln\left(\frac{b}{a}\right)$$

$$\therefore C = \frac{\rho_l L}{\frac{\rho_l}{2\pi \varepsilon_o \varepsilon_r} \ln\left(\frac{b}{a}\right)} = \frac{2\pi \varepsilon_o \varepsilon_r L}{\ln\left(\frac{b}{a}\right)} \quad \text{Farad}$$