CHAPTER (7)

MAGNETIC FORCE MAGNETIC POLARIZATION MAGNETIC MATERIALS

Force Between Conductors Carrying Current:

Force of Translation:

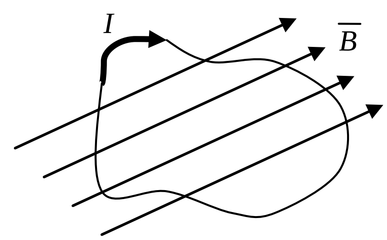
Since the force on charge element dQ due to $\vec{E}$ is given by:

$$\overrightarrow{dF}_e = dQ\,\vec{E}$$

So, the force on current element $Id\vec{l}$ due to $\vec{B}$ is given by:

$$\overrightarrow{dF}_m = Id\vec{l}\ x\ \vec{B} \qquad (N)$$

$$\vec{F}_m = \int Id\vec{l}\ x\ \vec{B}$$

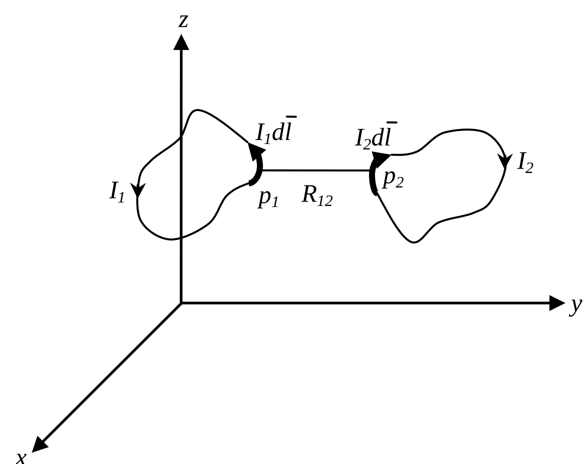


$$\overrightarrow{dH}_{12} = \frac{I_1\,d\vec{l}_1\ x\ \hat{a}_{R_{12}}}{4\pi R_{12}^2}$$

$$\overrightarrow{dB}_{12} = \mu_o \frac{I_1\,d\vec{l}_1\ x\ \hat{a}_{R_{12}}}{4\pi R_{12}^2}$$

$$d\left(\overrightarrow{dF}_{12}\right) = I_2\,d\vec{l}_2\ x \mu_o \frac{I_1\,d\vec{l}_1\ x\ \hat{a}_{R_{12}}}{4\pi R_{12}^2}$$

$$\overrightarrow{dF}_{12} = I_2\,d\vec{l}_2 x \int_{l_1} \mu_o \frac{I_1\,d\vec{l}_1\ x\ \hat{a}_{R_{12}}}{4\pi R_{12}^2}$$



$$\vec{F}_{12} = \int_{l_2} I_2\, \overrightarrow{dl}_2 \ x \ {}_2 \int_{l_1} \mu_o \frac{I_1\, \overrightarrow{dl}_1 \ x\, \hat{a}_{R_{12}}}{4\pi R_{12}^2}$$

$$\vec{F}_{12} = \frac{\mu_o I_1 I_2}{4\pi} \oint \overrightarrow{dl}_2 x \oint \frac{\overrightarrow{dl}_1 \ x\, \hat{a}_{R_{12}}}{4\pi R_{12}^2}$$

Example:

Find the force of translation per unit length between two infinite wires carrying currents I_1 and I_2 as shown in figure.

Solution:

$$\overrightarrow{dF}_{12} = I_2 \overrightarrow{dl}_2 \ x\, \vec{B}_{12}$$

$$\vec{B}_{12} = \mu_o \vec{H}_{12} = \mu_o \frac{I_1}{2\pi r_c} \hat{a}_\varphi$$

$$\hat{a}_\varphi = \hat{a}_l \ x\, \hat{a}_{r_c} = \hat{a}_z \ x\, \hat{a}_y = -\hat{a}_x$$

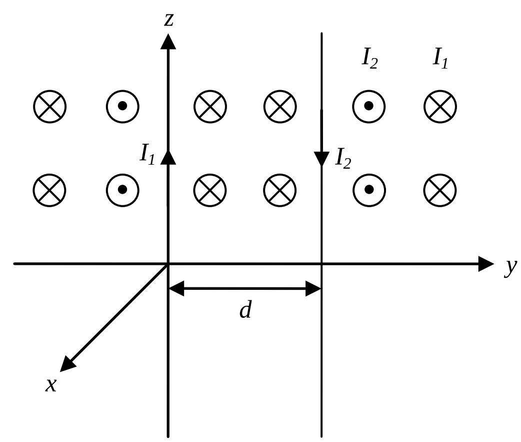
$$r_c = d$$

$$\vec{B}_{12} = -\mu_o \frac{I_1}{2\pi d} \hat{a}_x$$

$$\overrightarrow{dF}_{12} = I_2 \overrightarrow{dl}_2 \ x\, (-\hat{a}_x)\, \mu_o \frac{I_1}{2\pi d}$$

$$\overrightarrow{dF}_{12} = I_2 dz_2 \hat{a}_z \ x\, (-\hat{a}_x)\, \mu_o \frac{I_1}{2\pi d}$$

$$\overrightarrow{dF}_{12} = -\hat{a}_y\, \mu_o \frac{I_1 I_2}{2\pi d} dz_2$$



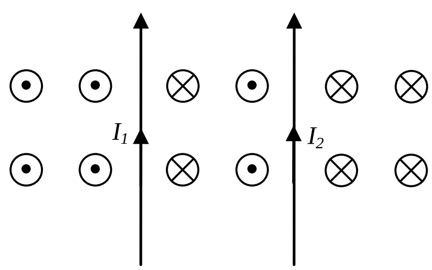
$$\vec{F}_{12} = -\hat{a}_y \, \mu_o \, \frac{I_1 I_2}{2\pi d} \int_{l/2}^{-l/2} dz_2 = \hat{a}_y \, \mu_o \, \frac{I_1 I_2 l}{2\pi d} \quad (N)$$

$$\frac{\vec{F}_{12}}{l} = \hat{a}_y \, \mu_o \, \frac{I_1 I_2}{2\pi d} \quad \left({}^{N}/{}_{m}\right)$$

The force of repulsion

Note:

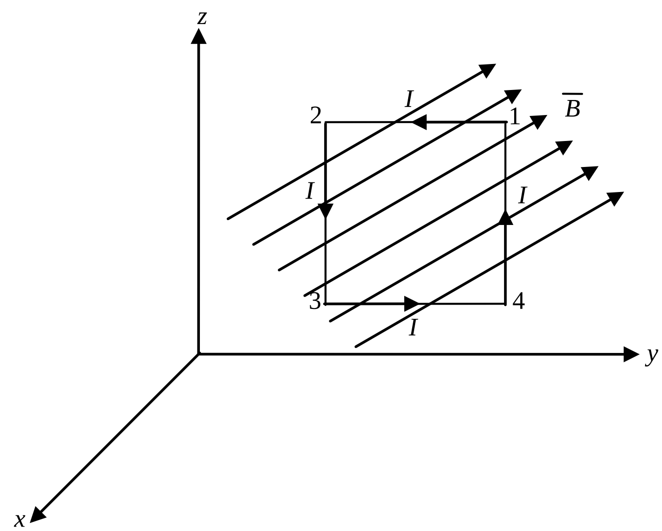
If the current in the two wires is in the same direction, then the force is attraction force.



Example:

Find the force of translation on a current loop as shown in figure when the applied magnetic field is uniform.

Solution:



$$\overrightarrow{dF_1} = Idy\hat{a}_y \, x \, \vec{B}$$

$$\vec{F}_1 = -Il\left(\hat{a}_y \, x \, \vec{B}\right)$$

$$\overrightarrow{dF_2} = Idz\hat{a}_z \, x \, \vec{B}$$

$$\vec{F}_2 = -Il\left(\hat{a}_z \, x \, \vec{B}\right)$$

$$\overrightarrow{dF_3} = Idy\hat{a}_y \, x \, \vec{B}$$

$$\vec{F}_3 = Il\left(\hat{a}_y \, x \, \vec{B}\right)$$

$$\overrightarrow{dF_4} = Idz\hat{a}_z \, x \, \vec{B}$$

$$\vec{F}_2 = Il\left(\hat{a}_z \, x \, \vec{B}\right)$$

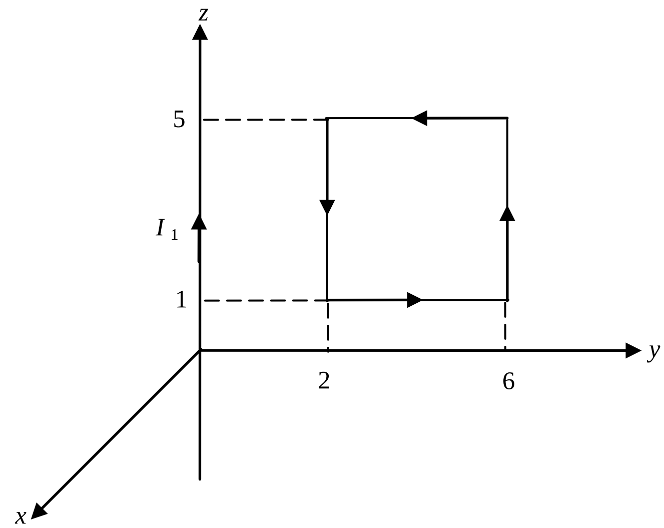
So, $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 0$

Note: **The force of translation = 0, if the applied magnetic field is uniform.**

Example:

Find the force of translation on the square loop shown in figure.

Solution:



$$\vec{B}_1 = \mu_o \vec{H}_1 = \mu_o \frac{I_1}{2\pi r_c} \hat{a}_\varphi$$

$$\hat{a}_\varphi = \hat{a}_l \, x \, \hat{a}_{r_c} = \hat{a}_z \, x \, \hat{a}_y = -\hat{a}_x$$

$$\vec{B}_1 = \mu_o \vec{H}_1 = -\mu_o \frac{I_1}{2\pi y} \hat{a}_x$$

$$\overrightarrow{dF}_{1ab} = I_{ab} \overrightarrow{dl}_{ab} \, x \, (-\hat{a}_x) \, \mu_o \frac{I_1}{2\pi y}$$

$$\overrightarrow{dF}_{1ab} = I_{ab} dy \, \hat{a}_y \, x \, (-\hat{a}_x) \, \mu_o \frac{I_1}{2\pi y}$$

$$\overrightarrow{dF}_{1ab} = \hat{a}_z \;\; \mu_o \frac{I_1 I_2}{2\pi} \int_2^6 \frac{dy}{y} \, dy$$

$$\vec{F}_{1ab} = \hat{a}_z \;\; \mu_o \frac{I_1 I_2}{2\pi} \, lin\left(\frac{6}{2}\right)$$

$$\vec{F}_{1ab} = \hat{a}_z \;\; \mu_o \frac{I_1 I_2}{2\pi} \, lin(3)$$

$$\overrightarrow{dF}_{1bc} = I_{bc} \overrightarrow{dl}_{bc} \, x \, (-\hat{a}_x) \, \mu_o \frac{I_1}{2\pi 6}$$

$$\overrightarrow{dF}_{1bc} = I_{bc}dz\,\hat{a}_z \;x\,(-\hat{a}_x)\,\mu_o\,\frac{I_1}{2\pi 6}$$

$$\overrightarrow{dF}_{1bc} = \hat{a}_y \;\; \mu_o\,\frac{I_1 I_2}{12\pi}\,dz$$

$$\vec{F}_{1bc} = \hat{a}_y \;\; \mu_o\,\frac{I_1 I_2}{12\pi}\int_1^5 dz$$

$$\vec{F}_{1bc} = \hat{a}_y \;\; \mu_o\,\frac{I_1 I_2}{3\pi}$$

$$\overrightarrow{dF}_{1cd} = I_{cd}dy\,\hat{a}_y\,x\,(-\hat{a}_x)\,\mu_o\,\frac{I_1}{2\pi y}$$

$$\overrightarrow{dF}_{1cd} = \hat{a}_z \;\; \mu_o\,\frac{I_1 I_2}{2\pi}\int_6^2 \frac{dy}{y}\,dy$$

$$\vec{F}_{1cd} = \; -\hat{a}_z \;\; \mu_o\,\frac{I_1 I_2}{2\pi}\,lin(3)$$

$$\overrightarrow{dF}_{1da} = I_{da}\overrightarrow{dl}_{da}\,x\,(-\hat{a}_x)\,\mu_o\,\frac{I_1}{2\pi 2}$$

$$\overrightarrow{dF}_{1da} = I_{da}dz\,\hat{a}_z \;x\,(-\hat{a}_x)\,\mu_o\,\frac{I_1}{2\pi 2}$$

$$\vec{F}_{1da} = \hat{a}_y \;\; \mu_o\,\frac{I_1 I_2}{4\pi}\int_5^1 dz$$

$$\vec{F}_{1da} = -\,\hat{a}_y \;\; \mu_o\,\frac{I_1 I_2}{\pi}$$

$$\vec{F}_{1total} = \;\vec{F}_{1ab} + \vec{F}_{1bc} + \vec{F}_{1cd} + \vec{F}_{1da}$$

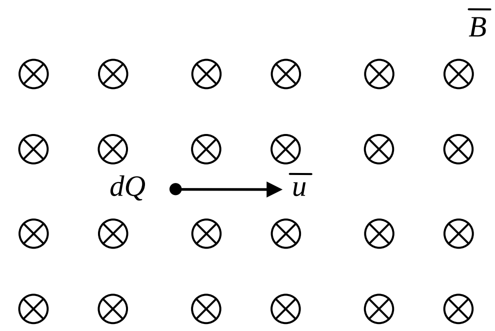
$$\vec{F}_{1total} = \hat{a}_y \;\; \mu_o\,\frac{I_1 I_2}{3\pi} - \;\hat{a}_y \;\; \mu_o\,\frac{I_1 I_2}{\pi}$$

$$\vec{F}_{1total} = -\hat{a}_y\,\mu_o\,\frac{2 I_1 I_2}{3\pi} \quad (N)$$

Note:

If the current loop is placed in a non uniform magnetic field a force of translation occurs.

Force on Moving Point Charge:



$$\overrightarrow{dF} = \vec{J}\, dv \; x\, \vec{B}$$

Since the convection current density is given by:

$$\vec{J}_{conv} = \rho_v\, \vec{u}$$

$$\overrightarrow{dF} = \vec{J}\, dv \; x\, \vec{B} = \rho_v\, \vec{u}\, dv \; x\, \vec{B}$$

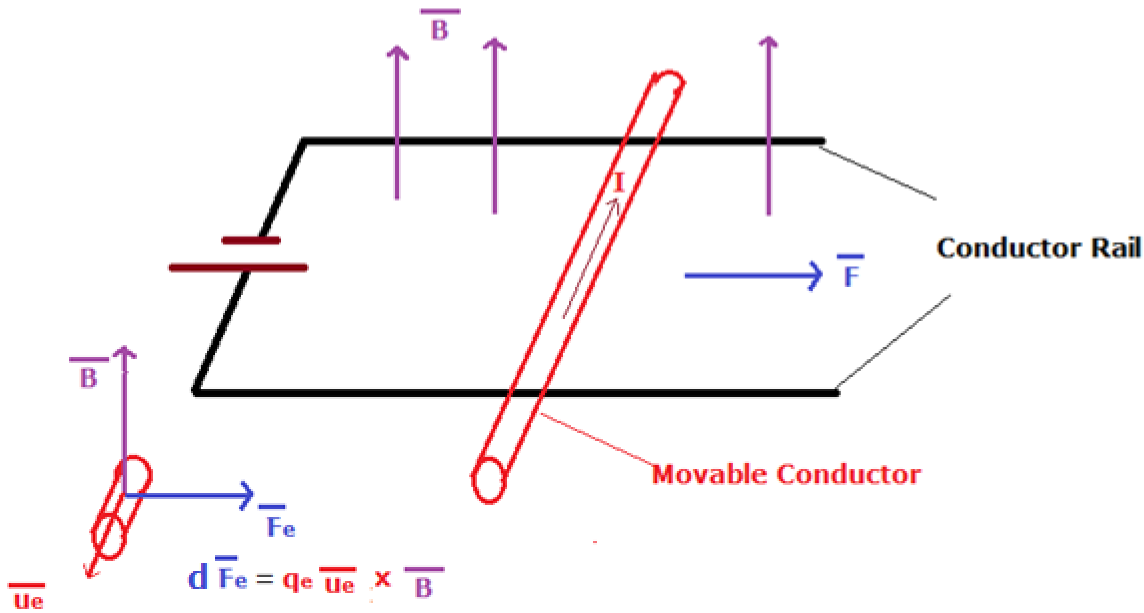
$$\overrightarrow{dF} = \rho_v\, dv\, \vec{u} \; x\, \vec{B}$$

$$\overrightarrow{dF} = dQ\, \vec{u} \; x\, \vec{B} \qquad (N)$$

- **The electron beam in a cathode – ray tube is a good example of convection current. It can be deflected by placing a $\vec{B}$ field perpendicular to the direction of the beam. In this case, the force $\overrightarrow{dF}$ can be viewed as either on each electron or on a volume dv of electrons.**

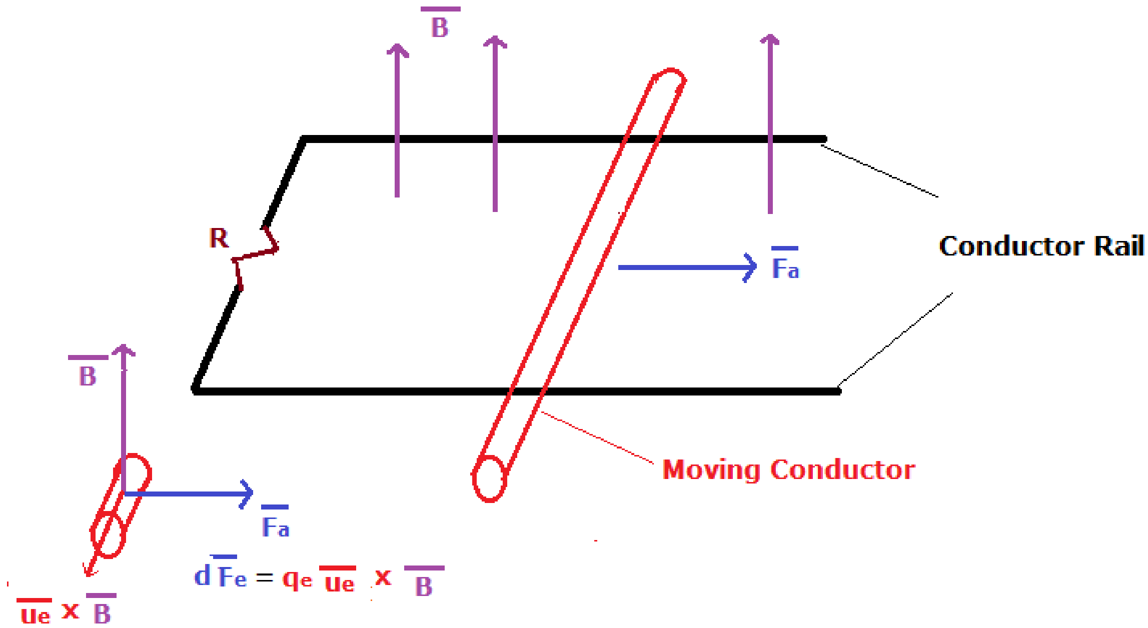
Applications on the Force on Moving Point Charge

1- Idea of Simple Electric Motor



- **A simple electric motor, is shown in the figure, where the force $d\vec{F}_e$ on the moving electron and thus on the conductor is shown.**

2- Idea of Simple Electric Generator



When the conductor is moved by some applied force $\vec{F}_a$ in a magnetic field as shown in the figure, the velocity due to $\vec{F}_a$ is imparted to both the free electrons and the positive lattice ions. The force $\overrightarrow{dF}_e$ will exist on both the electrons and positive lattice ions, but only the free electrons will experience translation. The term $(\vec{U}_e\ x\ \vec{B})$ is called the apparent electric field intensity and is obtained as

$$\frac{\overrightarrow{dF}_e}{dQ} = \vec{U}\ x\ \vec{B} \qquad N/c$$

The movement of the electrons in the conductor will give rise to a current and a build – up of potential across the resistance R

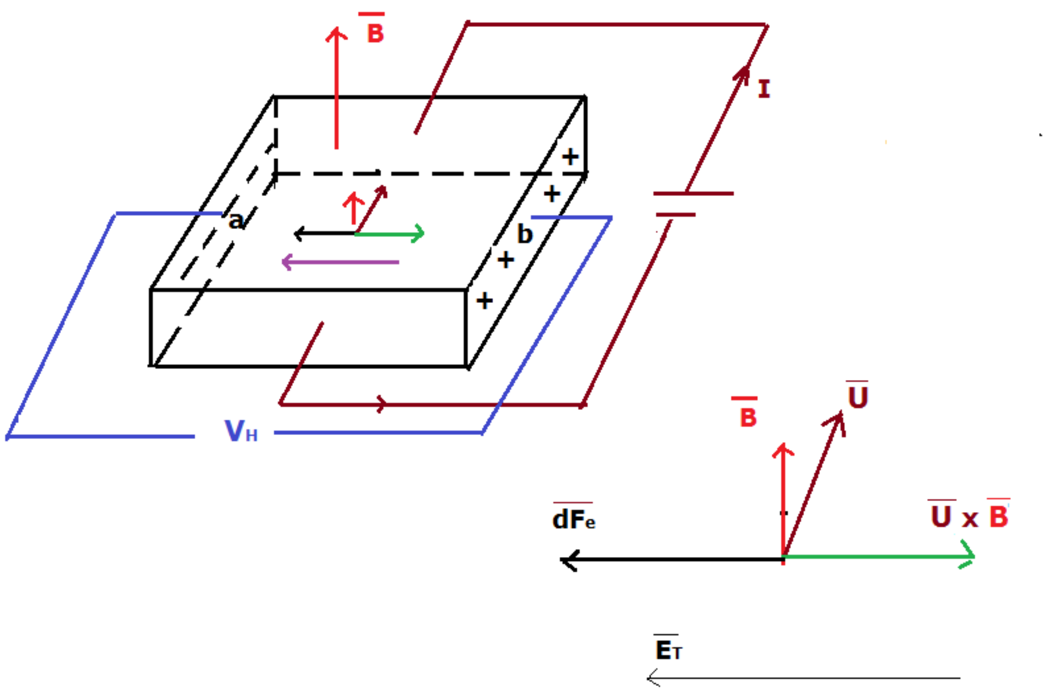
Note: Lorentz force equation

When dQ is immersed in combined $\vec{E}$ and $\vec{B}$ fields, the combined force becomes

$$\overrightarrow{dF} = \left(dQ\,\vec{E} + dQ\,\vec{U}\,x\,\vec{B}\right)$$

The previous equation is known as the Lorentz force equation, whose solution is required in determining the motion of a point charge in combined $\vec{E}$ and $\vec{B}$ fields. The first and second terms of Lorentz force equation are the electric and magnetic forces, respectively, on the charge dQ.

3 – Hall Effect



As mentioned, the magnetic force on moving electrons in a conductor will produce a slight displacement between the electrons and the positive ions of the stationary lattice structure.
This displacement of electrons will give rise to surface charges, as shown in the figure.
The build up of the surface charge will stop when the transversal electric field $\vec{E}_T$, due to surface charges, is just cancelled by the apparent electric field $\vec{U}\ x\ \vec{B}$, due to the magnetic force on the electron, or

$$\vec{E}_T + \vec{U}\ x\ \vec{B} = 0$$

A potential V_H,the Hall potential, will be found between directly opposite points on the side walls.

The Hall potential, due to the surface charges, is equal to

$$V_H = V_{ba} = -\int_a^b \vec{E}_T \,.\, \vec{dl} = -\int_a^b \left(-\vec{U}\; x\; \vec{B}\right).\, \vec{dl}$$

$$= U_x\, B_z\, W$$

Where, W is the width of the slab.

Through the use of $\vec{J} = \rho_{ve}\,\vec{U}$, the Hall potential will become

$$V_H = \frac{J_x}{|\rho_{ve}|}\; B_z\, W = \frac{{}^{I}/_{Wd}}{|\rho_{ve}|}\; B_z\, W = \frac{I\; B_z}{n_e q_e d}$$

Where, n_e, is the free electron density.

The Hall voltage polarity can be used as an indicator of the polarity of the carrier of I in a semiconductor material.

Thus, we can distinguish between n-type and p-type semiconductor.

The magnitude of V_H can be also used as an indicator of the magnitude of $\vec{B}$ field.

Magnetic Materials:

The prominent characteristic of magnetic materials is the magnetization (Formation of Magnetic dipoles).

MAGNETIC DIPOLE MOMENT

- **Magnetic properties of materials involve concepts based on the magnetic dipole moment. Consider a current loop, as shown in Figure 1, where the circulating current is I. This may, for example, be a coil carrying a current. For simplicity we will assume that the current loop lies within a single plane. The area enclosed by the current I is A. Suppose that $\hat{\mu}_n$ is a unit vector coming out from the area A. The direction of $\hat{\mu}_n$ is such that looking along it, the current circulates clockwise.**
- **Then the magnetic dipole moment, or simply the magnetic moment $\vec{\mu}_m$, is given by**

$$\vec{\mu}_m = IA\hat{\mu}_n$$

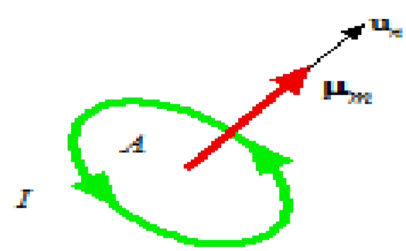


Fig. 1: Definition of a magnetic dipole moment.

- **When a magnetic moment is placed in a magnetic field , it experiences a torque that tries to rotate the magnetic moment to align its axis with the magnetic field, as depicted in Figure 2.**

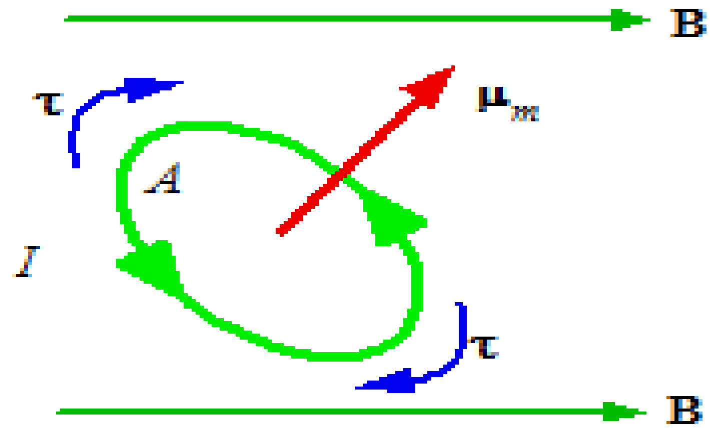


Fig. 2: A magnetic dipole moment in an external field experiences a torque.

- **The magnetic dipoles in magnetic material are due to the following:**
  1- Orbiting of the electrons around the nucleus
  2- Electron spin: orbiting of electrons around it self
  3- Nucleus spin: orbiting of the nucleus around it self

- **In the absence of the magnetic field, the dipole moments have random directions, and therefore it's net magnetic dipole moments will equal to zero**

$$\sum_i \vec{\mu}_{mi} = 0$$

- **In the presence of the magnetic field, these dipole moments will take the direction of the applied magnetic field, and therefore it's net magnetic dipole moments equal not to zero**

$$\sum_i \vec{\mu}_{mi} \neq 0$$

Definition:

Magnetic polarization $\vec{M}$:

The magnetic polarization $\overline{\mathrm{M}}$ **is defined by the total magnetic dipole moments per unit volume**

$$\vec{M} = lim_{\Delta v \to 0} \frac{\sum_{i=1} \vec{\mu}_{mi}}{\Delta v}$$

If the distribution of magnetic dipoles is uniform and *n* is the number of magnetic dipoles in unit volume:

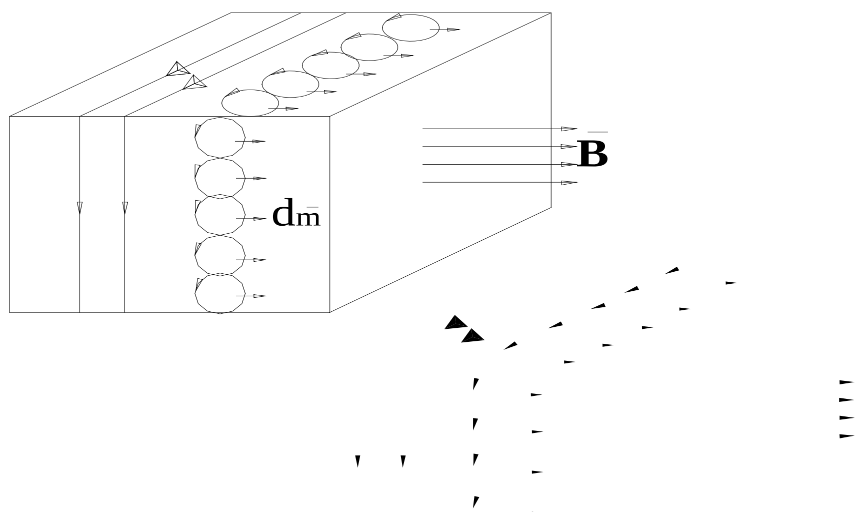
$$\vec{M} = n\,\vec{\mu}_m$$

Bound Magnetization Current:

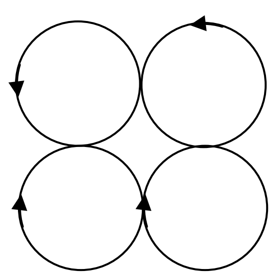
When the magnetic material is instead in uniform magnetic field, all the magnetic dipoles moments will be aligned in the direction of the magnetic field and a bound magnetic surface current will be produced, which is given by:

$$\vec{J}_{sm} = \overrightarrow{M}\, x\, \hat{a}_n$$

Where, $\hat{a}_n = unit\ vector\ \perp to\ the\ surface$



But inside the material all the current of the loops will be opposite therefore, the bound magnetic volume current = zero.



$$\vec{J}_m = 0$$

If the magnetic material is inserted in a non-uniform magnetic field, the adjacent current loops will not cancel each other totally, which induces bound magnetic volume current $\vec{J}_m$ which is given by:

$$\vec{J}_m = \nabla \times \vec{M} \quad \left({}^{A}/_{m^2} \right)$$

Example:

Along a cylinder of magnetic material of radius (a) along the z-axis the magnetic polarization $\vec{M} = 10\,\hat{a}_z \quad \left({}^{A}/_{m} \right)$ within the cylinder. Find $\vec{J}_m, \vec{J}_{sm}$.

Solution:

$$\vec{J}_m = \nabla \times \vec{M} = \frac{1}{r_c}\begin{bmatrix} \hat{a}_{r_c} & \hat{a}_\varphi & \hat{a}_z \\ \frac{\partial}{\partial r_c} & \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial z} \\ 0 & 0 & 10 \end{bmatrix} = 0$$

$\therefore$ The magnetic field is uniform $\rightarrow \vec{J}_m = 0$

$$\vec{J}_{sm} = \vec{M} \times \hat{a}_n = 10\,\hat{a}_z \times \hat{a}_{r_c} = 10\,\hat{a}_\varphi$$

Effect of Magnetization on Magnetic Field:

- Since, the magnetic flux density $\vec{B}_f$ in free space is related to the magnetic field intensity $\vec{H}_f$ due to conduction current density $\vec{J}$ by

$$\vec{B}_f = \mu_o \vec{H}_f$$

$$\vec{H}_f = \vec{H} = \frac{\vec{B}_f}{\mu_o}$$

$$\nabla X \vec{H}_f = \vec{J}$$

But in magnetic materials, the magnetic flux density becomes $\vec{B}$, while the magnetic field intensity $\vec{H}_f$ will not change, since it depends upon the conduction current and equal to $\vec{H}$

- The magnetic materials can be considered as a free space in addition to bound magnetic volume current density $\vec{J}_m$, so

$$\nabla X \frac{\vec{B}}{\mu_o} = \vec{J} + \vec{J}_m$$

$$\nabla X \frac{\vec{B}}{\mu_o} = \vec{J} + \nabla X \vec{M}$$

$$\nabla X \left(\frac{\vec{B}}{\mu_o} - \vec{M}\right) = \vec{J} = \nabla X \vec{H}$$

$$\frac{\vec{B}}{\mu_o} - \vec{M} = \vec{H}$$

$$\vec{B} = \mu_o\left(\vec{H} + \vec{M}\right)$$

For linear isotropic magnetic materials

$$\vec{M} = \chi_e \vec{H}$$

Where, χ_e is the magnetic susceptibility

$$\vec{B} = \mu_o\left(\vec{H} + \chi_e \vec{H}\right)$$

$$\vec{B} = \mu_o(1 + \chi_e)\vec{H}$$

$$\vec{B} = \mu_o\mu_r \vec{H}$$

$$\vec{B} = \mu \vec{H}$$

Where, μ_r is the relative permeability

$$\mu_r = 1 + \chi_e$$

Magnetic Material Classification:

- **In general, magnetic materials are classified into five distinct groups: diamagnetic, paramagnetic, ferromagnetic, antiferromagnetic, and ferrimagnetic.**

DIAMAGNETISM

Typical diamagnetic materials have a magnetic susceptibility that is negative and small. For example, the silicon crystal is diamagnetic with $\chi_m = -5.2x10^{-6}$.

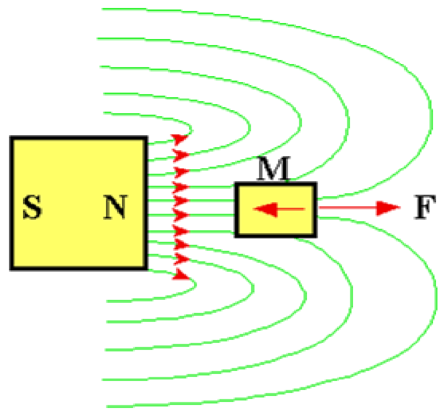
The relative permeability of diamagnetic materials is slightly less than unity.

When a diamagnetic substance such as a silicon crystal is placed in a magnetic field, the magnetization vector $\vec{M}$ in the material is in the opposite direction to the applied field intensity $\vec{H}$ and the resulting magnetic flux $\vec{B}$ within the material is less than $\mu_o \vec{H}$.

The negative susceptibility can be interpreted as the diamagnetic substance trying to expel the applied magnetic field from the material.

When a diamagnetic specimen is placed in a nonuniform magnetic field, the magnetization $\vec{M}$ of the material is in the opposite direction to $\vec{B}$ and the specimen experiences a net force toward smaller fields, as shown in following figure .

A substance exhibits diamagnetism whenever the constituent atom in the material have closed subshells and shells. This means that each constituent atom has no permenant magnetic moment in the absence of an applied field.



A diamagnetic material placed in a non-uniform magnetic field experiences a force towards smaller fields. This repels the diamagentic material away from a permanent magnet.

❑ PARAMAGNETISM

Paramagnetic materials have a small positive magnetic susceptibility. For example, oxygen gas is paramagnetic with $\chi_m = 2.1x10^{-6}$ at atmospheric pressure and room temperature. Each oxygen molecule has a net magnetic dipole moment $\vec{\mu}_{mol}$.

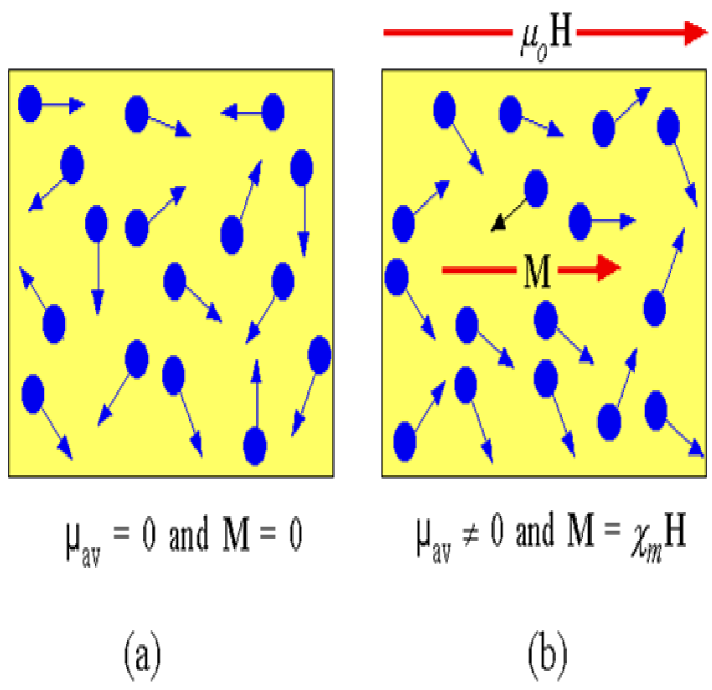
In the absence of an applied field, these molecular moments are randomly oriented due to the random collisions of the molecules, as illustrated in following figure. The magnetization of the gas is zero.

In the presence of an applied field, the molecular magnetic moments take various alignments with the field. The degree of alignment of $\vec{\mu}_{mol}$ with the applied field and hence magnetization $\vec{M}$ increases with the strength of the applied magnetic field $\vec{H}$.

Magnetization $\vec{M}$typically decreases with increasing temperature because at higher temperatures there are more

molecular collisions, which destroy the alignments of molecular magnetic moments with the applied field.

When a paramagnetic substance is placed in a nonuniform magnetic field, the induced magnetization $\vec{M}$ is along $\vec{B}$ and there is a net force toward greater fields. For example , when liquid oxygen is poured close to a magnet as depicted in the following figure, the liquid becomes attracted to the magnet.



: (a) In a paramagnetic material each individual atom possesses a permanent magnetic moment but due to thermal agitation there is no average moment per atom and M = 0. (b) In the presence of an applied field, individual magnetic moments take alignments along the applied field and M is finite and along B.

❑ FERROMAGNETISM

It is the basic mechanism by which certain materials (such as iron) form permanent magnets, or are attracted o magnets. In physics, several different types of magnetism are distinguished.

Ferromagnetic materials such as iron can possess large permanent magnetization even in the absence of an applied magnetic field.

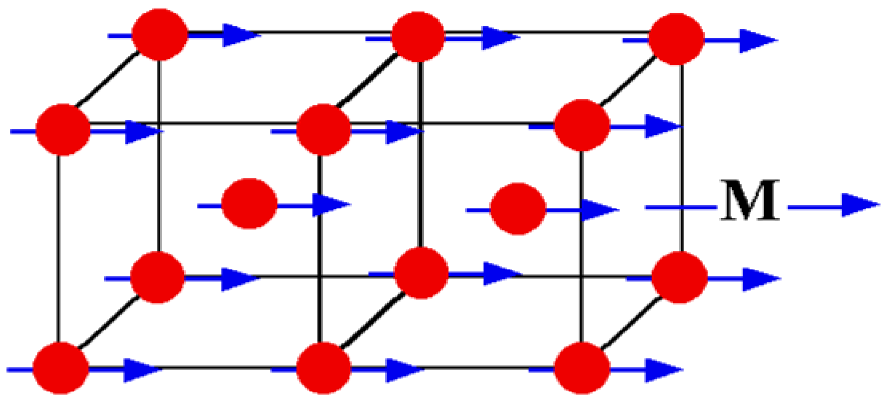
The magnetic susceptibility χ_m is typically positive and very large (even infinite) and, further, depends on the applied magnetic field intensity $\vec{H}$. The relationship between the magnetization $\vec{M}$ and the applied magnetic field intensity $\vec{H}$ is highly nonlinear.

At sufficiently high fields, the magnetization $\vec{M}$of the ferromagnet saturates.

The origin of the ferromagnetism is the quantum mechanical exchange interaction between the constituent atoms that results in regions of the material possessing permanent magnetization.

Following figure depicts a region of the Fe crystal, called a magnetic domain, that has a net magnetization vector $\vec{M}$ due to the alignment of the magnetic moments of all Fe atoms in this region. This crystal domain has magnetic ordering as all the atomic magnetic moments have been aligned parallel to each other.

Ferromagnetism occurs below a critical temperature called the Curie temperature T_c. At temperature above T_c, ferromagnetism is lost and the material becomes paramagnetic.



In a magnetized region of a ferromagnetic material such as iron all the magnetic moments are spontaneously aligned in the same direction There is a strong magnetization vector **M** even in the absence of an applied field.

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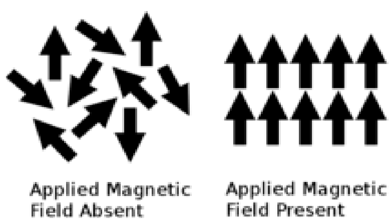
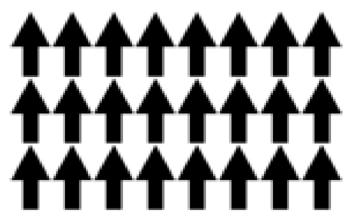
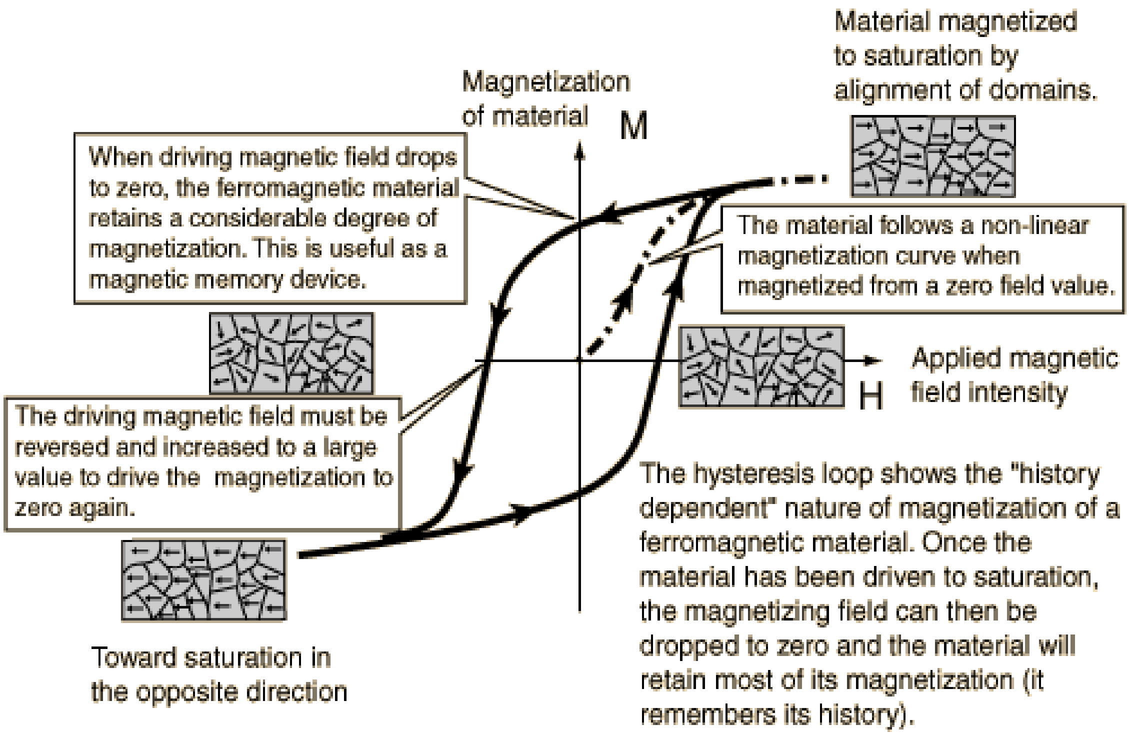




- **Figure 1 Below the Curie temperature neighbouring magnetic spins align in a ferromagnet in the absence of an applied [magnetic field]**

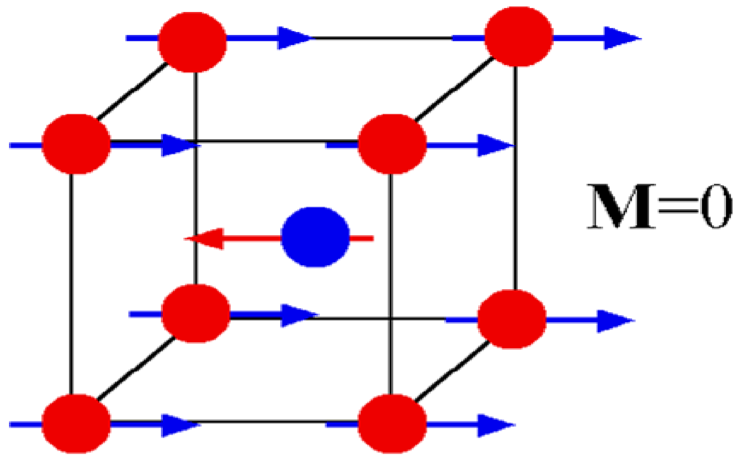
Figure 2 Above the Curie temperature, the magnetic spins are randomly aligned in a paramagnet unless a magnetic field is applied.



ANTIFERROMAGNETIC

- **Antiferromagnetic materials such as chromium have a small but positive susceptibility. They cannot possess any magnetization in the absence of an applied magnetic field, in contrast to ferromagnets.**
- **Antiferromagnetic materials possess a magnetic ordering in which the magnetic moments of alternating atoms in the crystals align in opposite directions, as schematically depicted in the following figure. The opposite alignments of atomic magnetic moments are due to quantum mechanical exchange forces. The net result is that in the absence of an applied magnetic field, there is no net magnetization.**

- **Antiferromagnetism occurs below a critical temperature called the Ne'el temperature T_N. Above T_N, antiferromagnetic materials becomes paramagnetic.**



In this antiferromagnetic BCC crystal (Cr) the magnetic moment of the center atom is cancelled by the magnetic moments of the corner atoms (an eighth of the corner atom belongs to the unit cell).

FERRIMAGNETISM

- **Ferrimagnetic materials such as ferrites (e.g.,Fe_3O_4) exhibit magnetic behavior similar to ferromagnetism below a critical temperature called the Curie temperature T_c. Above T_c they become paramagnetic.**
- **The origin of ferrimagnetism is based on magnetic ordering as schematically illustrated in the following figure. All A atoms have their spins aligned in one direction and all B atoms have their spins aligned in the opposite direction. As the magnetic moment of an A atoms**

is greater than that of **B** atoms, there is net magnetization $\vec{M}$ in the crystal.

- **Unlike the antiferromagnetic case, the oppositely directed magnetic moments have different magnitudes and do not cancel.**
- **The net effect is that the crystal can possess magnetization even in the absence of an applied magnetic field.**
- **Since ferrimagnetic materials are typically non-conducting and therefore do not suffer from eddy current losses, they are widely used in high –frequency electronic applications.**
- **All useful magnetic materials in electrical engineering are invariably ferromagnetic or ferrimagnetic.**

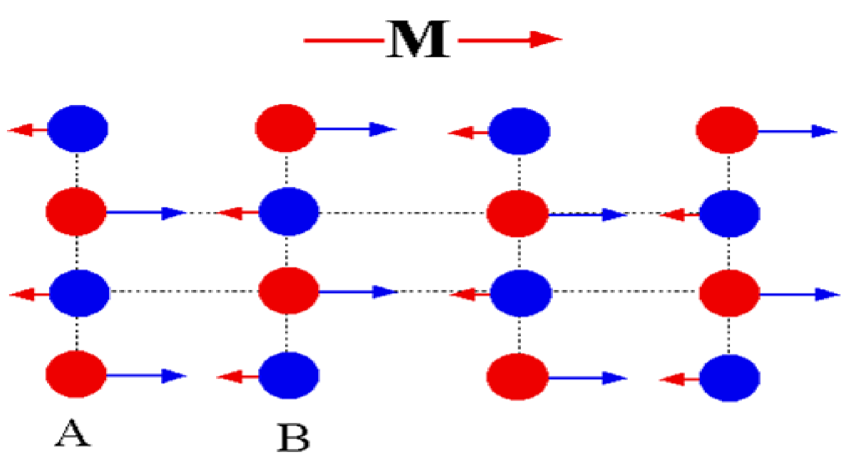


Illustration of magnetic ordering in a ferrimagnetic crystal. All A–atoms have their spins aligned in one direction and all B–atoms have their spins aligned in the opposite direction. As the magnetic moment of an A–atom is greater than that of a B–atom, there is net magnetization, M, in the crystal.

Magnetic Boundary Condition:

Normal component:

$$\oint \overline{B} \cdot d\overline{S} = 0$$

$$B_{1n}\Delta S - B_{2n}\Delta S + \int_{side} \overline{B} \cdot d\overline{S} = 0$$

$$\because \oint \overline{B} \cdot d\overline{S} = 0$$

$$\boxed{\therefore B_{1n} = B_{2n}}$$

Tangent component:

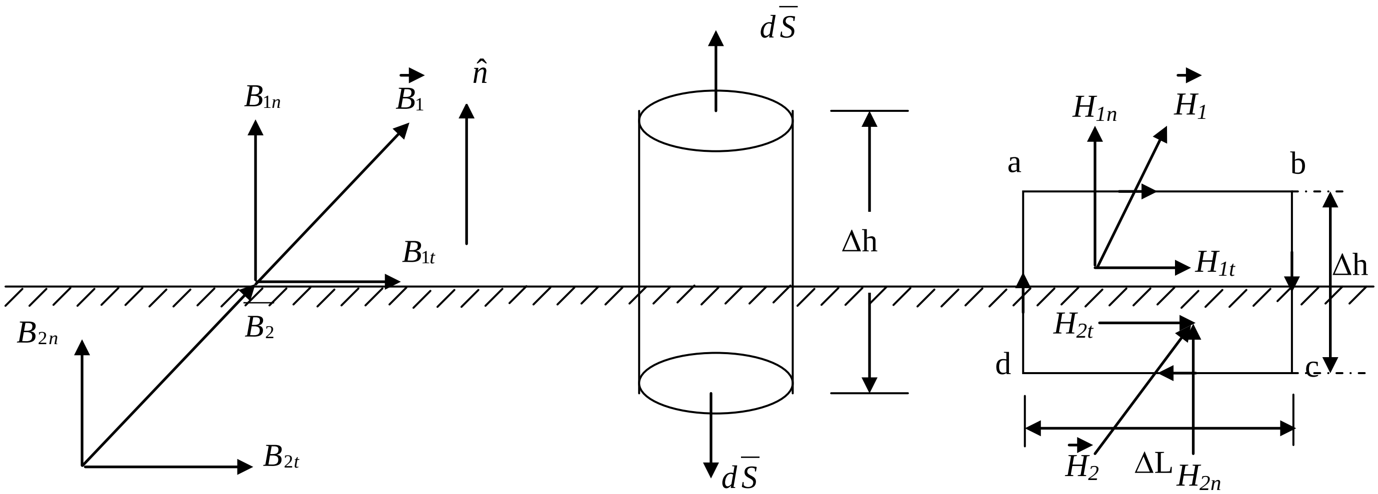
$$\oint \overline{H} \cdot d\overline{l} = I_{en}$$

$$\int_a^b \overline{H}_{t_1} \cdot d\overline{l} + \int_b^c \overline{H} \cdot d\overline{l} + \int_c^d \overline{H}_{t_2} \cdot d\overline{l} + \int_d^a \overline{H} \cdot d\overline{l} = J_s \cdot \Delta l$$

$$= H_{t_1}\Delta l + 0 + (-H_{t_2} \cdot \Delta l) + 0 = J_s \Delta l$$

$$\mathrm{H}_{1t} - \mathrm{H}_{2t} = J_s$$

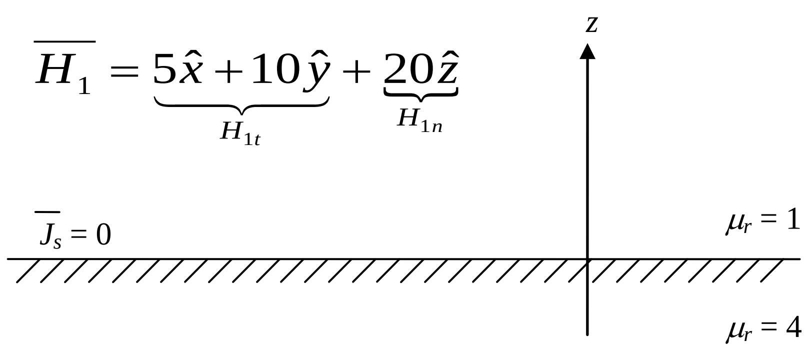
$$\boxed{\hat{n}_{21}\, x \left(\vec{H}_1 - \vec{H}_2 \right) = \vec{J}_s}$$



Example:

For the given figure, find: $\bar{B}_1$**,** $\bar{H}_2$ **and** $\bar{B}_2$**.**

$$\overline{H_1} = \underbrace{5\hat{x} + 10\hat{y}}_{H_{1t}} + \underbrace{20\hat{z}}_{H_{1n}}$$



Solution:

$$\overline{H_1} = \underbrace{5\hat{x} + 10\hat{y}}_{H_{1t}} + \underbrace{20\hat{z}}_{H_{1n}} \rightarrow \vec{B}_1 = \mu_1 \vec{H}_1 = 5\mu_o\hat{a}_x + 10\mu_o\hat{a}_y + 20\mu_o\hat{a}_z$$

$$\vec{H}_{1t} = 5\hat{a}_x + 10\hat{a}_y \qquad \vec{B}_{1t} = 5\mu_o\hat{a}_x + 10\mu_o\hat{a}_y$$

$$\vec{H}_{1n} = 20\hat{a}_z \qquad \vec{B}_{1n} = 20\mu_o\hat{a}_z$$

Applying the Boundary conditions

$$\vec{H}_{2n} = {}^{\vec{B}_{2n}}\!/_{\mu_2} = 5\hat{a}_z \quad \leftarrow \quad \vec{B}_{2n} = \vec{B}_{1n} = 20\mu_o\hat{a}_z$$

$$\hat{n}_{21}\, x\left(\vec{H}_1 - \vec{H}_2\right) = \vec{J}_s$$

$$\hat{a}_z x\left[\left(5\hat{a}_x + 10\hat{a}_y + 20\hat{a}_z\right) - \left(H_{2x}\hat{a}_x + H_{2y}\hat{a}_y + H_{2z}\hat{a}_z\right)\right] = 0$$

$$\hat{a}_z x\left[(5 - H_{2x})\hat{a}_x + \left(10 - H_{2y}\right)\hat{a}_y + (20 - H_{2z})\hat{a}_z\right] = 0$$

$$(5 - H_{2x})\hat{a}_y - \left(10 - H_{2y}\right)\hat{a}_x + 0 = 0$$

$$(5 - H_{2x}) = 0 \quad \rightarrow \quad H_{2x} = 5$$

$$\left(10 - H_{2y}\right) = 0 \rightarrow H_{2y} = 10$$

$$\vec{H}_2 = 5\hat{a}_x + 10\hat{a}_y + 5\hat{a}_z \quad \rightarrow \quad \vec{B}_2 = 20\,\mu_o\hat{a}_x + 40\mu_o\hat{a}_y + 20\mu_o\hat{a}_z$$

$=$