Fundamental Parameters of Antennas

Ranga Rodrigo

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Lecture notes are fully based on books, Balanis [?] Kraus *et al.* [?], and Rao [?]. Some diagrams are directly from the books. These are acknowledged by inserting the citation.

Contents

Important Parameters

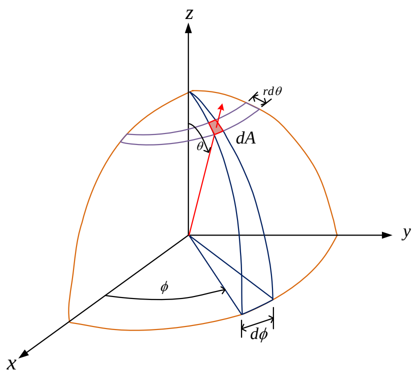
In order to describe the performance of an antenna, we use various, sometimes interrelated, parameters.

- Radiation pattern, beam width
- Power
- Directivity, gain, aperture
- Radiation resistance

1 Radiation Pattern

Definition 1 (Antenna Radiation Pattern)**.** An antenna radiation pattern or antenna pattern is defined as a mathematical function or a graphical representation of the radiation properties of the antenna as a function of space coordinates.

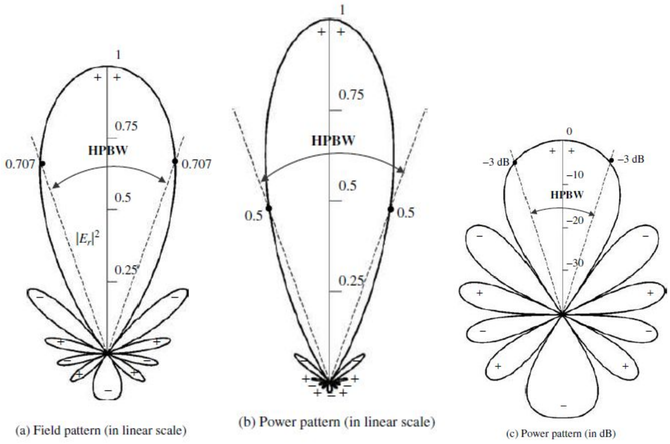
- Defined for the far-field.
- As a function of directional coordinates.
- There can be *field patterns* (magnitude of the electric or magnetic field) or *power patterns* (square of the magnitude of the electric or magnetic field).
- Often normalized with respect to their maximum value.
- The power pattern is usually plotted on a logarithmic scale or more commonly in decibels (dB).



Radiation patterns are conveniently represented in spherical coordinates. Pattern: $E(\theta,\phi)$.

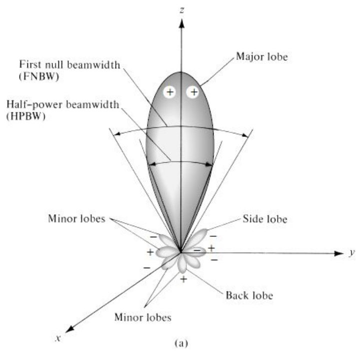
$$dA = r^2 \sin\theta \, d\theta \, d\phi.$$

Azimuth: ϕ Elevation: $\pi/2-\theta$.



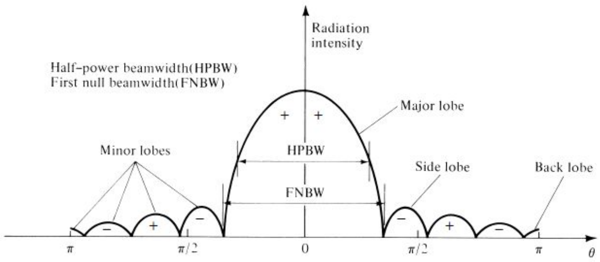
- All three patterns yield the same angular separation between the two half-power points, 38.64°, on their respective patterns, referred to as HPBW.

Radiation Pattern Lobes



A radiation lobe is a portion of the radiation pattern bounded by regions of relatively weak radiation intensity.

- Main lobe
- Minor lobes
- Side lobes
- Back lobes



- Minor lobes usually represent radiation in undesired directions, and they should be minimized. Side lobes are normally the largest of the minor lobes.
- The level of minor lobes is usually expressed as a ratio of the power density, often termed the *side lobe ratio* or *side lobe level.*
- In most radar systems, low side lobe ratios are very important to minimize false target indications through the side lobes (e.g., -30 dB).

Components in the Amplitude Pattern

- There would be, in general, three electric-field components (E_r, E_θ, E_ϕ) at each observation point on the surface of a sphere of constant radius.
- In the far field, the radial E_r component for all antennas is zero or vanishingly small.
- Some antennas, depending on their geometry and also observation distance, may have only one, two, or all three components.
- In general, the magnitude of the total electric field would be $|E| = \sqrt{|E_r|^2 + |E_\theta|^2 + |E_\phi|^2}$.

Isotropic, Directional, and Omnidirectional Patterns

Definition 2 (Isotropic Radiator)**.** A hypothetical lossless antenna having equal radiation in all directions.

Definition 3 (Omnidirectional Radiator)**.** An antenna having an essentially nondirectional pattern in a given plane (e.g., in azimuth) and a directional pattern in any orthogonal plane.

Definition 4 (Directional Radiator)**.** An antenna having the property of radiating or receiving more effectively in some directions than in others. Usually the maximum directivity is significantly greater than that of a half-wave dipole.

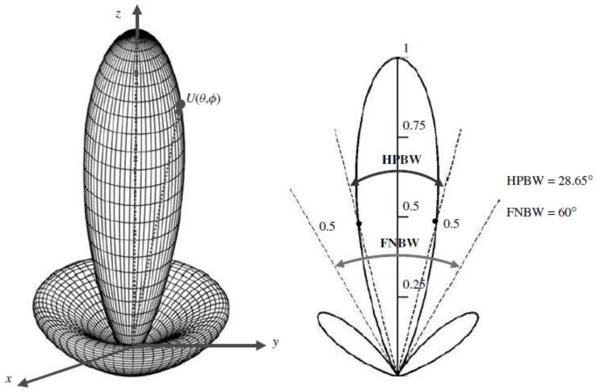
2 Beamwidth

- The beamwidth of an antenna is a very important figure of merit and often is used as a trade-off between it and the side lobe level; that is, as the beamwidth decreases, the side lobe increases and vice versa.

- The beamwidth of the antenna is also used to describe the resolution capabilities of the antenna to distinguish between two adjacent radiating sources or radar targets.

Definition 5 (Half-Power Beam Width (HPBW))**.** In a plane containing the direction of the maximum of a beam, the angle between the two directions in which the radiation intensity is one-half value of the beam.

Definition 6 (First-Null Beamwidth (FNBW))**.** Angular separation between the first nulls of the pattern.



Resolution

- The most common resolution criterion states that the resolution capability of an antenna to distinguish between two sources is equal to half the first-null beamwidth (FNBW/2), which is usually used to approximate the HPBW.

- That is, two sources separated by angular distances equal or greater than FNBW/2 ≈ HPBW of an antenna with a uniform distribution can be resolved.

- If the separation is smaller, then the antenna will tend to smooth the angular separation distance.

Example 7. An antenna has a field pattern given by

$$E(\theta) = \cos^2(\theta), \quad 0^\circ \le \theta \le 90^\circ.$$

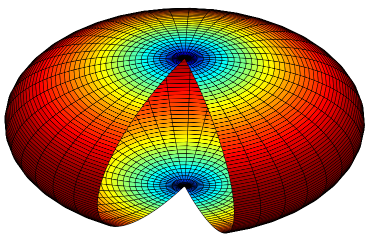
Find the half-power beamwidth HPBW (in radians and degrees).

Example 8. The normalized radiation intensity of an antenna is represented by

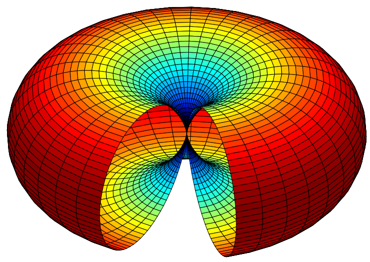
$$U(\theta) = \cos^2(\theta)\cos^2(3\theta), \quad 0^\circ \le \theta \le 90^\circ, \quad 0^\circ \le \phi \le 360^\circ.$$

Find the

1. half-power beamwidth HPBW (in radians and degrees).
2. first-null beamwidth FNBW (in radians and degrees).

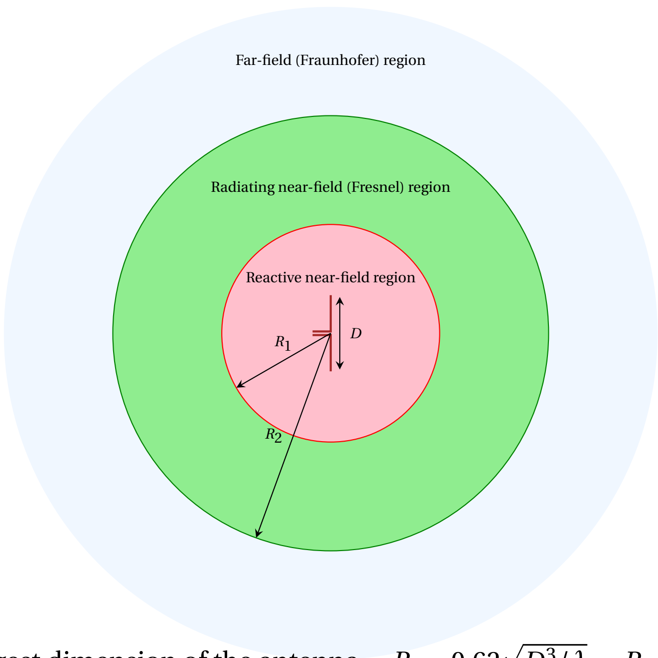


Isotropic



Omni-directional

Near- and Far-Fields



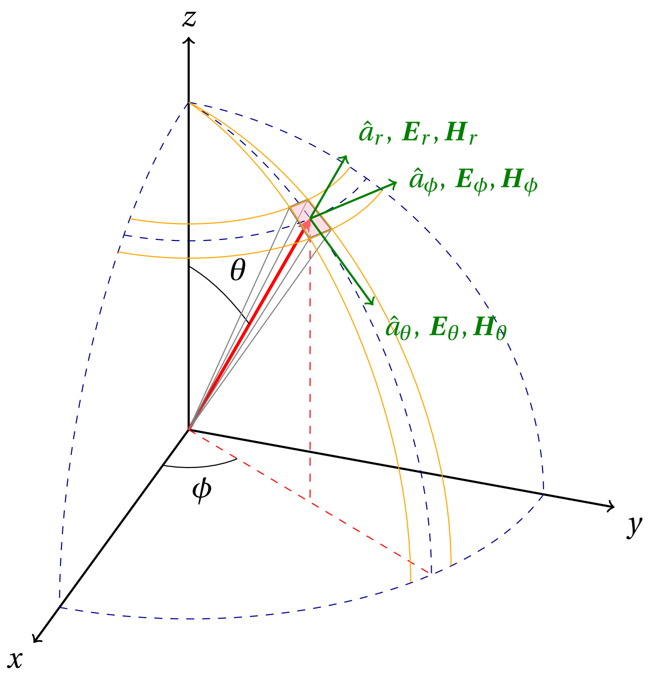
D = Largest dimension of the antenna. $\quad R_1 = 0.62\sqrt{D^3/\lambda}, \quad R_2 = 2D^2/\lambda$

Definition 9 (Reactive Near-Field Region $R < 0.62\sqrt{D^3/\lambda}$)**.** The portion of the near-field region immediately surrounding the antenna wherein the reactive field (non-radiating field) predominates.

Definition 10 (Radiating Near-Field (Fresnel) Region $0.62\sqrt{D^3/\lambda} \le R < 2D^2/\lambda$)**.** The region of the field of an antenna between the reactive near-field region and the far-field region wherein radiation fields predominate and wherein the angular field distribution is dependent upon the distance from the antenna. If the antenna has a maximum dimension that is not large compared to the wavelength, this region may not exist.

Definition 11 (Far-Field (Fraunhofer) Region $2D^2/\lambda \ge R$)**.** The region of the field of an antenna where the angular field distribution is essentially independent of the distance from the antenna.

3 Radiation Power Density



Poynting Vector

- The quantity used to describe the power associated with an electromagnetic wave is the instantaneous Poynting vector defined as

$$\mathscr{W} = \mathscr{E} \times \mathscr{H}, \tag{1}$$

where

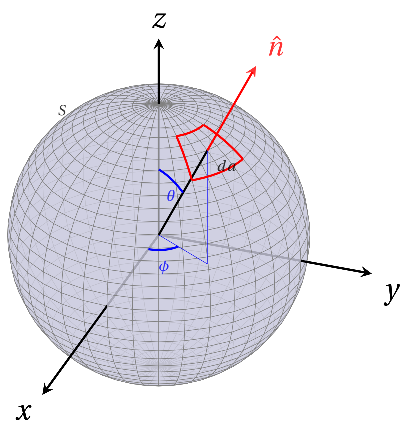
- $\mathscr{W}$ = instantaneous Poynting vector (W/m^2), a power density.
- $\mathscr{E}$ = instantaneous electric-field intensity (V/m).
- $\mathscr{H}$ = instantaneous magnetic-field intensity (A/m).

- The total power crossing a closed surface

$$\mathscr{P} = \oiint_s \mathscr{W} \cdot d\boldsymbol{s} = \oiint_s \mathscr{W} \cdot \hat{\boldsymbol{n}} da.$$

where

- $\mathscr{P}$ = instantaneous total power (W).
- $\hat{\boldsymbol{n}}$ = unit vector normal to the surface.
- a = infinitesimal area of the closed surface (m^2).



Average Power Density

- For applications of time-varying fields, it is desirable to find the average power density.
- The average power density is obtained by integrating the instantaneous Poynting vector over one period and dividing by the period.
- For time-harmonic variations of the form $e^{j\omega t}$, we define the complex fields $\boldsymbol{E}$ and $\boldsymbol{H}$ which are related to their instantaneous counterparts $\mathscr{E}$ and $\mathscr{H}$ by

$$\mathscr{E}(x, y, z; t) = \text{Re}[\boldsymbol{E}(x, y, z)e^{j\omega t}],$$
$$\mathscr{H}(x, y, z; t) = \text{Re}[\boldsymbol{H}(x, y, z)e^{j\omega t}].$$

Example 12. Show that

$$\boldsymbol{W}_{\rm av} = \frac{1}{2}\,\mathrm{Re}[\boldsymbol{E} \times \boldsymbol{H}^*] \quad (\mathrm{W/m^2}).$$

Hint: Use the identity $\mathrm{Re}\,[\boldsymbol{E}] = \frac{1}{2}[\boldsymbol{E}e^{j\omega t} + \boldsymbol{E}^* e^{-j\omega t}]$.

Average Radiated Power

- The power density associated with the electromagnetic fields of an antenna in its far-field region is predominately real and will be referred to as *radiation density*.
- The average power radiated by an antenna (radiated power) can be written as

$$\begin{aligned} P_{\rm rad} = P_{\rm av} &= \oiint_s \mathscr{W}_{\rm rad} \cdot d\boldsymbol{s} = \oiint_s \mathscr{W}_{\rm av} \cdot \hat{\boldsymbol{n}} da, \\ &= \frac{1}{2} \oiint_s \mathrm{Re}\left[\boldsymbol{E} \times \boldsymbol{H}^*\right] \cdot d\boldsymbol{s}. \end{aligned}$$

Power Pattern Versus Average Radiated Power

- The power pattern of the antenna is the average power density radiated by the antenna as a function of the direction.
- The observations are usually made on a large sphere of constant radius extending into the far field.

- In practice, absolute power patterns are usually not desired, but the performance of the antenna is measured in terms of relative power patterns.
- Three-dimensional patterns cannot be measured, but they can be constructed with a number of two-dimensional cuts.

Example 13. Determine the total radiated power, if the radial component of the radiated power density of an antenna is given by

$$\boldsymbol{W}_{\text{rad}} = \hat{\boldsymbol{a}}_r W_r = \hat{\boldsymbol{a}}_r A_0 \frac{\sin\theta}{r^2} \quad (\text{W/m}^2),$$

where A_0 is the peak value of the power density.

4 Radiation Intensity

Steradian

- One steradian is defined as the solid angle with its vertex at the center of a sphere of radius r that is subtended by a spherical surface area equal to that of a square with each side of length r.

- Since the area of a sphere of radius r is $A = 4\pi r^2$, there are 4π sr in a closed sphere.

Definition 14 (Radiation Intensity)**.** Radiation intensity in a given direction is defined as the power radiated from an antenna per unit solid angle.

- The radiation intensity is a far-field parameter.
- It can be obtained by simply multiplying the radiation density by the square of the distance.

$$U = r^2 W_{\text{rad}}.$$

Total Radiated Power Using Radiation Intensity

The total power is obtained by integrating the radiation intensity over the entire solid angle of 4π. Thus

$$P_{\text{rad}} = \oiint_{\Omega} U d\Omega = \int_0^{\pi} \int_0^{2\pi} U \sin\theta \, d\theta \, d\phi.$$

where $d\Omega$ is the element of solid angle = $\sin\theta \, d\theta \, d\phi$.

Example 15. Using the concept of radiation intensity, determine the total radiated power, if the radial component of the radiated power density of an antenna is given by

$$\boldsymbol{W}_{\text{rad}} = \hat{\boldsymbol{a}}_r W_r = \hat{\boldsymbol{a}}_r A_0 \frac{\sin\theta}{r^2} \quad (\text{W/m}^2),$$

where A_0 is the peak value of the power density.

For anisotropic source the radiation intensity U will be independent of the angles θ and ϕ, as was the case for W_{rad}.

Example 16. What is the radiation intensity due to an isotropic source with a total radiated power of P_{rad}?

5 Directivity

Definition 17 (Directivity)**.** The ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions.

- The average radiation intensity: total power radiated by the antenna divided by 4π.
- Stated more simply, the directivity of a nonisotropic source is equal to the ratio of its radiation intensity in a given direction over that of an isotropic source.

$$D = D(\theta,\phi) = \frac{U(\theta,\phi)}{U_0} = \frac{4\pi U(\theta,\phi)}{P_{\text{rad}}}.$$

If the direction is not specified, the direction of maximum radiation intensity is implied.

$$D_{\max} = D_0 = \frac{U}{U_0} = \frac{U|_{\max}}{U_0} = \frac{U_{\max}}{U_0} = \frac{4\pi U_{\max}}{P_{\text{rad}}}.$$

- D = directivity (dimensionless)
- D_0 = maximum directivity (dimensionless)
- $U = U(\theta,\phi)$ = radiation intensity (W/sr)
- $U_{\max}$ = maximum radiation intensity (W/sr)
- U_0 = radiation intensity of isotropic source (W/sr)
- P_{rad} = total radiated power (W)

Example 18. Determine the directivity and the maximum directivity, if the radial component of the radiated power density of an antenna is given by

$$\boldsymbol{W}_{\text{rad}} = \hat{\boldsymbol{a}}_r W_r = \hat{\boldsymbol{a}}_r A_0 \frac{\sin\theta}{r^2} \quad (\text{W/m}^2),$$

where A_0 is the peak value of the power density.

-

1. What is the directivity of an isotropic source?
2. What can you say about the directivity of any other source?

1. 1.
2. $D_0 \geq 1$, $0 < D \leq D_0$.

General Expression of Directivity

Radiation in tensity of an antenna:

$$U = B_0 F(\theta,\phi) \simeq \frac{1}{2\eta}\left[\left|E_\theta^0(\theta,\phi)\right|^2 + \left|E_\phi^0(\theta,\phi)\right|^2\right].$$

where

$$\begin{aligned} B_0 &= \text{a constant}, \\ E_\theta^0, E_\phi^0 &= \text{far-zone electric field components}, \\ \eta &= \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 120\pi\Omega. \end{aligned}$$

$$P_{\text{rad}} = B_0 \int_0^{2\pi}\int_0^{\pi} F(\theta,\phi)\sin\theta\, d\theta\, d\phi.$$

Directivity:

$$D(\theta,\phi) = \frac{U(\theta,\phi)}{U_0} = \frac{4\pi U(\theta,\phi)}{P_{\text{rad}}} = \frac{4\pi F(\theta,\phi)}{\int_0^{2\pi}\int_0^{\pi} F(\theta,\phi)\sin\theta\, d\theta}.$$

Maximum directivity:

$$D_0 == \frac{4\pi \left|F(\theta,\phi)\right|_{\max}}{\int_0^{2\pi}\int_0^{\pi} F(\theta,\phi)\sin\theta\, d\theta}.$$

Beam Solid Angle

Definition 19 (Beam Solid Angle)**.** The beam solid angle Ω_A is defined as the solid angle through which all the power of the antenna would flow if its radiation intensity is constant (and equal to the maximum value of U) for all angles within Ω_A.

$$
\begin{aligned}
P_{\text{rad}} &= \oiint_{\Omega} U_0 \, d\Omega, \\
&= U_{\max} \Omega_A, \\
&= 4\pi U_0. \\
\Omega_A &= \frac{P_{\text{rad}}}{U_{\max}}, \\
&= \frac{\int_0^{2\pi} \int_0^{\pi} F(\theta, \phi) \sin\theta \, d\theta \, d\phi}{F_{\max}}.
\end{aligned}
$$

$$
\boxed{D = \frac{4\pi}{\Omega_A}.}
$$

Approximate Calculation of Directivity

Instead of using the exact expression $\frac{4\pi}{\Omega_A}$ to compute the directivity, it is often convenient to use simpler but approximate expressions.

	Kraus	Tai and Pereira
Use when	$\Theta_{1d}, \Theta_{2d} > 39.77°$	$\Theta_{1d}, \Theta_{2d} < 39.77°$
In radians	$D_0 \simeq \dfrac{4\pi}{\Theta_{1r}\Theta_{2r}}$	$D_0 \simeq \dfrac{4\pi}{\Theta_{1r}^2 + \Theta_{2r}^2}$
In degrees	$D_0 \simeq \dfrac{41253}{\Theta_{1d}\Theta_{2d}}$	$D_0 \simeq \dfrac{72815}{\Theta_{1d}^2 + \Theta_{2d}^2}$

Example 20. The radiation intensity of the major lobe of many antennas can be adequately represented by

$$
U = B_0 \cos\theta,
$$

where B_0 is the maximum radiation intensity. The radiation intensity exists only in the upper hemisphere ($0 \le \theta \le \pi/2, 0 \le \phi \le 2\pi$). Find the beam solid angle: exact and approximate, maximum directivity: exact and approximate.

6 Antenna Efficiency and Gain

- The total antenna efficiency e_0 is used to take into account losses at the input terminals and within the structure of the antenna.

- e_0 is due to the combination of number of efficiencies:

$$\boxed{e_0 = e_r e_c e_d}.$$

$$\begin{aligned} e_o &= \text{total efficiency}, \\ e_r &= \text{reflection(mismatch) eff.}, \\ &= (1 - |\Gamma|^2), \\ e_c &= \text{conduction efficiency}, \\ e_d &= \text{dielectric efficiency}, \\ \Gamma &= \frac{Z_{\text{in}} - Z_0}{Z_{\text{in}} + Z_0}, \\ \text{VSWR} &= \frac{1 + |\Gamma|}{1 - |\Gamma|}. \end{aligned}$$

Γ = voltage reflection coefficient at the input terminals of the antenna Z_{in} = antenna input impedance, Z_0 = characteristic impedance of the transmission line. VSWR = voltage standing wave ratio

- Usually e_c and e_d are very difficult to compute, but they can be determined experimentally.

- It is usually more convenient to write e_0 as

$$e_o = e_r e_{cd} = e_{cd}(1 - |\Gamma|^2).$$

  where $e_{cd} = e_c e_d$ = antenna radiation efficiency, which is used to relate the gain and directivity.

Gain

- The gain of the antenna is closely related to the directivity.

- In addition to the directional capabilities it accounts for the efficiency of the antenna.

- Gain does not account for losses arising from impedance mismatches (reflection losses) and polarization mismatches (losses).

Definition 21. Gain The ratio of the intensity, in a given direction, to the radiation intensity that would be obtained if the power accepted by the antenna were radiated isotropically.

$$\text{Gain} = 4\pi \frac{\text{radiation intensity}}{\text{total input accepted power}} = 4\pi \frac{U(\theta,\phi)}{P_{\text{in}}} \text{ (dimensionless)}.$$

- We can write that the total radiated power (Prad) is related to the total input power (Pin) by

$$P_{\text{rad}} = e_{cd} P_{\text{in}}.$$
$$G(\theta,\phi) = e_{cd} \left[4\pi \frac{U(\theta,\phi)}{P_{\text{rad}}} \right]$$
$$\boxed{G(\theta,\phi) = e_{cd} D(\theta,\phi).}$$

- The maximum value of the gain is related to the maximum directivity

$$\boxed{G_0 = e_{cd} D_0.}$$

Absolute Gain

- We can introduce an absolute gain G_{abs} that takes into account the reflection or mismatch losses (due to the connection of the antenna element to the transmission line)

$$\boxed{G_{\text{abs}} = e_r G(\theta,\phi) = (1-|\Gamma|^2) G(\theta,\phi) = e_r e_{cd} D(\theta,\phi) = e_o D(\theta,\phi).}$$

where

$$e_r = (1-|\Gamma|^2), \text{ reflection (mismatch) efficiency,}$$
$$e_o = \text{overall efficiency.}$$

- If the antenna is matched to the transmission line, that is, the antenna input impedance Z_{in} is equal to the characteristic impedance Z_c of the line ($|\Gamma| = 0$), then the two gains are equal ($G_{\text{abs}} = G$).

- For the maximum values

$$\boxed{G_{0\text{abs}} = e_o D_0.}$$

Example 22. A lossless resonant half-wavelength dipole antenna, with input impedance of 73 ohms, is connected to a transmission line whose characteristic impedance is 50 ohms. Assuming that the pattern of the antenna is given approximately by

$$U = B_0 \sin^3\theta.$$

find the maximum absolute gain of this antenna.

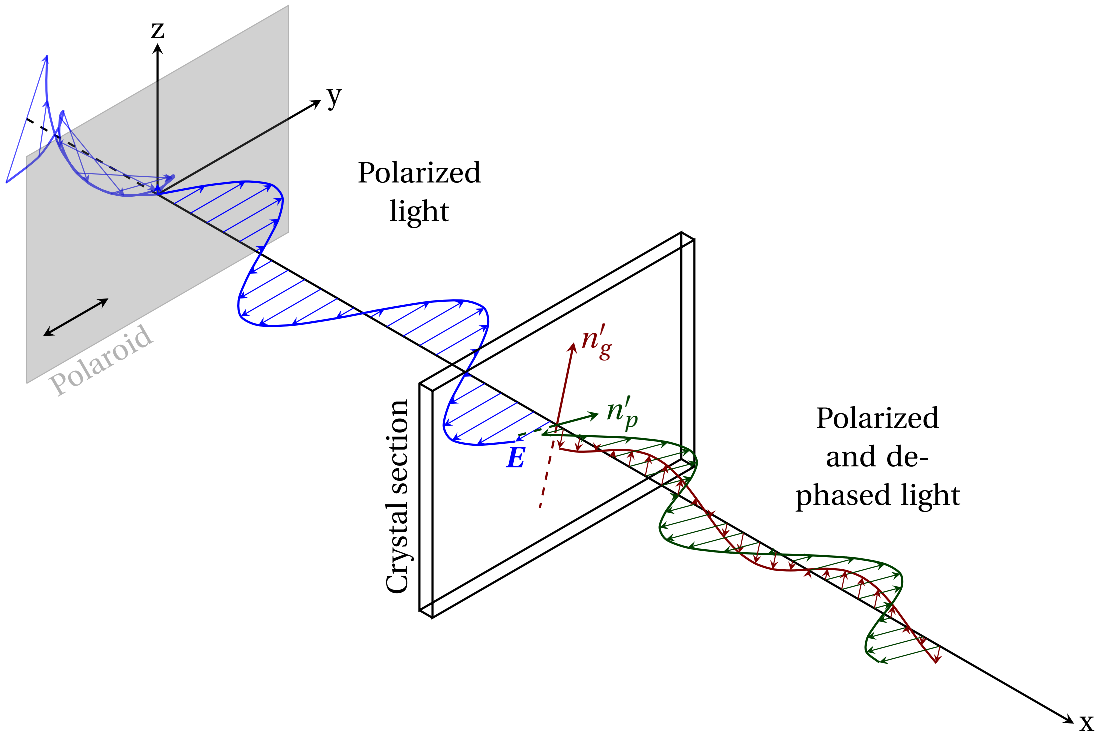
Bandwidth

- For broadband antennas, the bandwidth is usually expressed as the ratio of the upper-to-lower frequencies of acceptable operation. For example, a 10:1 bandwidth indicates that the upper frequency is 10 times greater than the lower.
- For narrowband antennas, the bandwidth is expressed as a percentage of the frequency difference (upper minus lower) over the center frequency of the bandwidth. For example, a 5% bandwidth indicates that the frequency difference of acceptable operation is 5% of the center frequency of the bandwidth.

7 Polarization

Definition 23 (Polarization)**.** Polarization is the curve traced by the end point of the arrow (vector) representing the instantaneous electric field. The field must be observed along the direction of propagation.

- Polarization is classified as linear, circular, or elliptical.
- If the vector that describes the electric field at a point in space as a function of time is always directed along a line, the field is said to be linearly polarized.
- In general, the figure that the electric field traces is an ellipse, and the field is said to be elliptically polarized.



Polarization Types

- Linear polarization and circular polarization are special cases of elliptic polarization.
- Polarization can be clockwise (CW, right-hand polarization), or counter clockwise (CCW, left-hand polarization).

Linear, Circular and Elliptic Polarization

- The instantaneous electric field of a plane wave, traveling in the negative z direction, can be written as

$$\mathscr{E}(z;t) = \hat{\boldsymbol{a}}_x \mathscr{E}_x(z;t) + \hat{\boldsymbol{a}}_y \mathscr{E}_y(z;t).$$

- By considering the complex counterpart of these instantaneous components, we can write

$$\mathscr{E}_x(z;t) = E_{xo} \cos(\omega t + kz + \phi_x),$$
$$\mathscr{E}_y(z;t) = E_{yo} \cos(\omega t + kz + \phi_y).$$

  where E_{xo} and E_{yo} are the maximum magnitudes of the x- and y-components.

- By defining $\Delta\phi = \phi_y - \phi_x$, we can state these as

$$\mathscr{E}_x(z;t) = E_{xo} \cos(\omega t + kz),$$
$$\mathscr{E}_y(z;t) = E_{yo} \cos(\omega t + kz + \Delta\phi).$$

- Linear polarization

$$\Delta\phi = n\pi, \quad n = 1, 2, \ldots$$