Applying the LMS and RLS Beamforming Algorithms on Actual Linear and Planar Antenna Array

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Abstract

After revolution in wireless communication system, it is observed that the number of users and the requirements of wireless services increase at exponential rate. The wireless services need wider convergence area and higher transmission quality rises. The smart antenna system is used to achieve wireless services demands by using direction of arrival (DOA) and adaptive beamforming algorithms. Most of the famous adaptive antenna algorithms are applied using linear antenna array and the antenna elements are considered to be isotropic point sources. In this paper, the performance of the least mean square (LMS) and recursive least squares (RLS) algorithms using planar array are applied instead of using linear antenna array. Also, Finite size antenna elements are considered to study the effect of mutual coupling between antenna elements. Those algorithms are used to minimize the error, which occur due to high data rate and multiusers. The simulation results for two algorithms using both antenna arrays, linear and planar, in the absence and presence of mutual coupling are carried out to compare between LMS and RLS algorithms. The array factor, error squared, weights adaptation, desired and the array output plots are carried out. As expected, they show that the performance for both algorithms using planar array are better than using linear array.

Keywords— Adaptive antenna, Smart antenna, LMS and RLS algorithms.

Introduction

The least mean square (LMS) algorithm has been widely used in wireless
communication systems [1] for functions such as multi-user detection algorithms, adaptive antenna arrays, narrowband interference cancellation, and equalization algorithms. The LMS algorithm is important due to its low computational requirements and ease of implementation. The standard LMS algorithm does not use any specific structural information about the parameters being estimated. However, if such structural information on the parameters is known, it may be possible to improve the performance of the LMS algorithm by modifying it according to this additional knowledge. There has been some work done to improve the LMS algorithm by exploiting the sparsity of systems and signals [2,3]. The way that the problem is approached in these works is to add a penalty term to the traditional LMS cost function. This penalty term is chosen so to force the resulting signal to be sparse. Sparsity aware modifications of other estimation algorithms such as recursive least squares (RLS) and Kalman filter have also been proposed in recent years [4,5].

The RLS algorithm plays the major role in estimation theory for signal processing [6]. The adaptive algorithm takes the fixed beamforming process which allows for calculation of continuously update weights. The RLS algorithm solves this optimization problem adaptively using only a signal input and output instance at a time [7].

To arrange the structure (number of elements, distance between elements, pattern) of antenna array properly can improve the performance of the adaptive array. The frequently used array structures include a uniform linear array and a uniform circular array [1]. This paper analyzes the two structures $1 \times 4$ linear array and $4 \times 4$ rectangular array.

**Antenna Array Model**

**Linear Array**

The structure of linear array composed of N-elements placed on one dimension (1-D) as shown in Figure 1. The distance between the signal source and the antenna array is bigger than the size of antenna array.

![Figure 1: Array along Y-axis.](image-url)
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The received signal vector $\mathbf{x}(k)$ is given by [3]

$$\mathbf{x}(k) = \mathbf{S}(k)\mathbf{a}(\theta_0) + \sum_{i=1}^{N_u} u_i(k)\mathbf{a}(\theta_i) + \mathbf{n}(k)$$ (1)

$N_u$ is the number of interferers, $\mathbf{S}(k)$ and $u_i(k)$ denotes the desired signal arriving at angle $\theta_0$ and the $i^{th}$ interfering signal arriving at angle of incidence $\theta_i$ respectively. $\mathbf{n}(k)$ is the additive zero mean white Gaussian noise vector with variance $\sigma_n^2$. $\mathbf{a}(\theta)$ and $\mathbf{a}(\theta_i)$ represent the steering vectors for the desired signal and $i^{th}$ interfering signal respectively. The steering vector of a linear array for any arbitrary direction is given by

$$\mathbf{a}(\theta) = \begin{bmatrix} 1 & e^{j \frac{2\pi}{\lambda} \sin \theta} & e^{j \frac{4\pi}{\lambda} \sin \theta} & \cdots & e^{j \frac{2\pi}{\lambda} (N-1) \sin \theta} \end{bmatrix}^T$$ (2)

where $\lambda$ is the transmission wavelength.

The goal of applying adaptive algorithm in adaptive antenna array is to reconstruct the desired signal from the received signal and avoid the interfering signal and the additional white Gaussian noise [8].

The outputs of the individual sensors are linearly combined after being scaled using corresponding weights such that the antenna array pattern is optimized to have maximum possible gain in the direction of the desired signal and nulls in the direction of the interferers. Therefore, the output signal of the linear array is given by

$$\mathbf{y}(k) = \mathbf{w}^H(k)\mathbf{x}(k)$$ (3)

where $\mathbf{w}(k) = [w_1 \ w_2 \ \cdots \ w_N]$ the weight vector and $H$ denotes the hermitian. The weights here will be computed using LMS or RLS algorithms based on minimum squared error (MSE) criterion.

**Planar Array**

The elements are arranged on a two dimensional plane (2-D) and by using the origin of coordinates as the setting point, there are two angles to represent the direction of one signal, the elevation angle, $\theta$ and azimuth angle, $\phi$. All the distances between the elements on the directions of x-axis and y-axis are considered to be $\lambda/2$. The received signal matrix of the planar array composed of $M \times N$ elements with uniformly spaced elements in each direction as shown in Fig. 2 can be written by:

$$\mathbf{\tilde{x}}(k) = \mathbf{S}(k)\mathbf{\tilde{a}}(\theta, \phi)\mathbf{a}(\theta, \phi)^T + \mathbf{V}(k)$$ (4)

$$\mathbf{\tilde{a}}_x(\theta, \phi) = \begin{bmatrix} 1 & e^{j \frac{2\pi}{\lambda} \sin \theta \cos \phi} & e^{j \frac{4\pi}{\lambda} \sin \theta \cos \phi} & \cdots & e^{j \frac{2\pi}{\lambda} (M-1) \sin \theta \cos \phi} \end{bmatrix}^T$$ (5)

$$\mathbf{\tilde{a}}_y(\theta, \phi) = \begin{bmatrix} 1 & e^{j \frac{2\pi}{\lambda} \sin \theta \sin \phi} & e^{j \frac{4\pi}{\lambda} \sin \theta \sin \phi} & \cdots & e^{j \frac{2\pi}{\lambda} (N-1) \sin \theta \sin \phi} \end{bmatrix}^T$$ (6)
$V(k)$ is the additive zero mean white Gaussian noise $M \times N$ Matrix with variance $\sigma_n^2$. The output signal of planar array then is given by

$$y(k) = \sum_{m=1}^{M} \sum_{n=1}^{N} w_{mn} x_{mn}(k)(7)$$

where $x_{mn}(k)$ is the $m^{th}$ row and $n^{th}$ column value of the received signal matrix, $\bar{x}(k)$ and $w_{mn}$ is the associated weight of this received element.

**Least Mean Square (LMS)**

Consider a Uniform Linear Array (ULA) with $N$ isotropic elements, which forms the integral part of the adaptive beam forming system as shown in Figure 3.

![Figure 3: LMS adaptive beam forming network](image-url)
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The LMS algorithm is commonly used to adapt the weights. It uses steepest descent algorithm to update the weight vector and produces weight vector that converge to optimum by Wiener Hopf solution, the updating weight vector is [4]

$$\mathbf{w}(k + 1) = \mathbf{w}(k) + 2\mu \mathbf{e}^*(k)\mathbf{x}(k)(8)$$

Where $\mu$ is the step size and $\mathbf{e}(k)$ is the error between desired signal and the array output signal and is given by

$$\mathbf{e}(k) = \mathbf{S}(k) - \mathbf{y}(k)$$

The array output, $\mathbf{y}(k)$ depends on adaptation of weight, therefore the optimum weights which are obtained by using LMS algorithm is required. The convergent speed depends on the step size value.

**Recursive Least Square (RLS)**

The RLS algorithm is widely used in signal processing. It is dependent on weight vector updates. The formula in RLS adaptive filtering algorithm is given by [3]

$$\mathbf{w}(k) = \mathbf{w}(k - 1) + \mathbf{g}(k)[\mathbf{S}^*(k) - \mathbf{x}^H\mathbf{w}(k - 1)](10)$$

Where $\mathbf{g}(k)$ is again vector calculate by

$$\mathbf{g}(k) = \frac{\mathbf{\alpha}^{-1}\mathbf{R}_{\mathbf{x}\mathbf{x}}(k-1)\mathbf{x}(k)}{1 + \mathbf{\alpha}^{-2}\mathbf{\bar{R}}_{\mathbf{x}\mathbf{x}}(k-1)\mathbf{x}(k)}(11)$$

where $\mathbf{\alpha}$ is the forgetting factor ($\mathbf{\alpha} \approx 1$)

Forgetting factor is referred to exponential weighting factor. $\mathbf{\alpha}$ is positive constant $0 \leq \mathbf{\alpha} \leq 1$ where $\mathbf{\alpha} = 1$ restore the ordinary least square algorithm also, indicate infant memory[3].

To obtain weight vector at iteration $k$ [3], $\mathbf{\bar{w}}(k)$, recursively it is required to determine the received signal correlation matrix and correlation vector between the received and desired signals, the estimate of correlation matrix and correlation vector was taken as the sum of terms[3]

$$\mathbf{\bar{R}}_{\mathbf{x}\mathbf{x}} = \sum_{i=1}^{k} \mathbf{x}(i)\mathbf{x}^H(i)(12)$$

$$\mathbf{\bar{r}}(k) = \sum_{i=1}^{k} \mathbf{d}^T(i)\mathbf{x}(i)(13)$$

**Simulation Results**

In this section, the performance of LMS and RLS algorithms for 1-D and 2-D beamforming using linear and planar arrays respectively are determined. The two algorithms are applied on signals impinging on an array of isotropic point sources and the actual elements. In order to take the mutual coupling between actual elements into consideration, each element is considered to be a half wavelength dipole antenna loaded by 50Ω at the center and has radius of $\frac{\mathbf{a}}{200}$. Linear array of $\mathbf{N} = 4$ and planar array of $\mathbf{M} \times \mathbf{N} = 4 \times 4$ elements are used. For linear array, the desired signal is assumed to be arrived from $\theta_0 = 40^\circ$ and the interference signal arrived from $\theta_1 = -60^\circ$ with SNR = 20 dB and the spacing between elements is considered to be $\frac{\mathbf{\lambda}}{2}$. 


Same givens are considered in the planar array but the azimuth angle of both signals is assumed to be $90^\circ$.

Figure 4, 5 and 6, illustrate the comparison between the performances of both LMS and RLS algorithms for linear array composed of point sources. The Array factor in RLS and LMS approximately give the main lobe direction towards the desired signal and deep null in the direction of interference signal. Figure 5 shows the square error against iterations number and Figure 6 shows the desired and output signals of LMS and RLS beamformers and it can be seen that the convergence speed of RLS algorithm is faster than LMS. The RLS reaches to the minimum squared error after 27 iterations where squared error $|e|^2 = 0.0008498$, but the LMS algorithm the squared error $|e|^2 = 0.0009402$ reaches to that squared error after 61 iterations, approximately, so the convergence speed of RLS is faster than LMS.

![Figure 4: Array factor of LMS and RLS beamformers for linear array composed of isotropic point sources](image)

Figure 7, 8 and 9, illustrate the comparison performance of both LMS and RLS algorithms for linear array composed of actual elements. By applying the two algorithms on the received voltages contaminated with effect of mutual coupling, the error between the desired signal and received signal is more than in isotropic point source case. This in turn requires more iteration to reach to optimum solution.
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Figure 5: The square error of LMS and RLS beamformers for linear array composed of isotropic point sources.

However, the RLS algorithm converges faster than LMS algorithm where the squared error $|e|^2 = 0.04183$ in LMS and in the RLS $|e|^2 = 0.0003387$ after 27 iterations, as shown in Figure 8 and 9. According to the Figure 7, the LMS and RLS both algorithms give the same beam pattern approximately. The squared error in RLS is minimum than LMS as shown in Figure 8.

Figure 6: The desired and output signals of LMS and RLS beamformers for linear array composed of isotropic point sources.

Figure 7: Array factor of LMS and RLS beamformers for linear array composed of actual elements.
To apply LMS and RLS algorithms on signals received by a planar array, it is required to rearrange the matrix of received signals of $4 \times 4$ elements that are distributed on $x$-$y$ plane to a vector composed of $1 \times 16$ with same values.

Figure 10, 11 and 12, illustrate the comparison performance of both LMS and RLS algorithms for planar array composed of point sources. And Figure 13, 14 and 15, illustrate the comparison performance of both LMS and RLS algorithms for planar array composed of actual elements. It can be noticed that by using planar antenna array, the convergence speed is faster than linear one and the RLS algorithm still converges faster than LMS algorithm. Again, all array factors of different proposed cases success to give maximum radiation at the desired direction ($40^\circ$) and deep null at the interference direction ($-60^\circ$).
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Figure 10: Array factor of LMS and RLS beamformers for planar array composed of isotropic point sources.

Figure 11: The square error of LMS and RLS beamformers for planar array composed of isotropic point sources.

Figure 12: The desired and output signals of LMS and RLS beamformers for planar array composed of isotropic point sources.

Figure 11, the planar array point source with SNR 20db the squared error in RLS is decay faster than LMS. The RLS reach to minimum error after 20 iterations the \(|e|^2=0.0001386\) but LMS reach to \(|e|^2=0.0003793\) after 50 iteration, so the convergence speed in RLS is faster than LMS. In linear element array point source in
RLS the squared error reach to minimum after 27 iterations and in LMS after 61 iterations, so the convergence speed in planar array point source is faster than linear element point source.

Figure 13: Array factor of LMS and RLS beamformers for planar array composed of actual elements.

In actual element planar antenna array, the RLS reach to minimum error after 15 iteration where $|e|^2=0.0008517$ but in $|e|^2=0.0002423$ after 22 iterations So that the RLS is give minimum error and the convergence speed is faster than LMS. Figure 11 and 14, show the convergence speed in actual element planar array is better than in linear array point source and actual element array and planar array point source, Inplanar antenna array point source element reach to minimum squared error in RLS and LMS after 20 and 50 iterations, respectively which give the best result is planar antenna array actual element.

Figure 14: The square error of LMS and RLS beamformers for planar array composed of actual elements.
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Conclusion
This paper illustrates the comparison performance of both LMS and RLS algorithms for linear array and Planar composed of Isotropic point source and actual elements. The RLS algorithm gives minimum squared error, good array factor performance in both dimensions "1-D and 2-D". Fig. 8, 11 are show the performance of planar array actual element is give minimum error than linear array actual element which give best array factor and Fig. 5, 11 shows the performance of planar array point source is better than linear array point source. From all previous figures, show the planar array actual element is the best performance of array factor which give the maximum in the desired signal, deep null direction and minimum squared error after 15 iterations in RLS algorithm and after 25 iterations in LMS, So RLS planar array actual element is the best performance in convergence speed and array factor.

References
