Impact of Partial Slip and Heat Source on MHD Mixed Convection Flow of Nanofluid in a Double Lid-Driven Cavity Containing Insulated Obstacle

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The current investigation analyzes the effects of partial slip and heat generation on the mixed convection flow with heat transfer in an inclined double lid-driven square cavity containing centered square adiabatic obstacle in the presence of magnetic field. The used cavity is subjected to constant heat flux and filled with Cu-water nanofluid. The top and bottom horizontal walls are thermally insulated and move with uniform velocity while the right vertical wall is maintained at a constant low temperature. A uniform heat flux is located in a part of the left wall of the cavity while the remaining part of this wall is thermally insulated. Finite volume technique is utilized to solve dimensionless governing equations of the problem. The proposed method is validated with the previous published numerical studies which distinctly offer a good agreement. The obtained results show that changing in the heat source length affects much the flow and thermal fields than the position of heat source. The average Nusselt number decreases when the aspect ratio of the obstacle and heat source length increases. The heat transfer rate behaves nonlinearly with inclination of the cavity.

KEYWORDS: Mixed Convection, Cavity, Slip, MHD (Magnetohydrodynamics), Nanofluid, Adiabatic Block.

1. INTRODUCTION
Nanofluid is a colloidal mixture by adding nanoparticles having typical size less than 100 nm in working liquids. An innovative process to upgrade the efficiency and performance of thermal systems is to suspend the nanoscale sized particles in the working liquids.1, 2 Substantially poor heat transfer abilities have been occupied by the working liquids like water, ethylene glycol and propylene glycol because of their low thermal efficiency. Consequently, addition of nanoparticles in such kind of working liquids is known as nanoliquids. Additionally, nanoliquid plays a fundamental role to meet the cooling rate requirements with high thermal efficiency.3-5 Investigations on nanofluid applications have received more attention in recent times due to their possible application for growth of thermal conductivity in cooling of solar devices. The outcomes benefited in expanding the thermal conductivity of the working fluid in contrast with the conventional base fluid. Improvement in thermal conductivity helped industry particularly in heat transfer applications, Wen et al.6 and Godson et al.7 pointed out that improvement in thermal conductivity was helped industry particularly in heat transfer applications, when utilize the nanofluid, the heat exchanger pumping power reduces drastically. Another application in the biomedical engineering and medicine was given by Saidur et al.8 Bhuvaneswari et al.9 and Sivasankaran and Pan10 studied convection analysis in heat transfer improvement of a nanofluid filled cavity with different wall conditions. Selective investigations of lid-driven enclosures filled with pure fluids can be alluded to Sivasankaran et al.,11 Sivasankaran and Pan12 or in nanofluid filled enclosures as in Abu-Nada and Chamkha,13 Talebi et al.,14 Tiwari and Das,15 Chamkha and Abu-Nada,16 and Sheremet and Pop.17 Many authors
discussed the effects of nanoparticles for different fluid models, see for example,\textsuperscript{18–26}.

Lately, applying of external magnetic field on the convection heat transfer within cavities has been the interest of various investigations due to its wide applications such as in the polymer industry, coolers of nuclear reactors, purification of molten metals and various other important applications which can be utilized to control the convection inside cavities.\textsuperscript{27–28} However, we will limit our literature review on the effect of the magnetic field on the mixed convection heat transfer in cavities. Al-Salem et al.\textsuperscript{29} studied the effects of moving lid-direction on MHD mixed convection flow in a linearly heated cavity. They found that heat transfer is decreased with increasing the magnetic field. Öztop et al.\textsuperscript{30} investigated the MHD mixed convection flow in a top-sides lid-driven cavity heated by a corner heater. They found that heat transfer decreases with increasing the Hartmann number and magnetic field is important role to control heat transfer and fluid flow. Rahman et al.\textsuperscript{31} investigated the magneto-hydrodynamics mixed convection flow and the conjugate influence of Joule heating in an obstructed lid-driven square cavity saturated with an electrically conducting fluid. They found that the Joule heating parameter and the Hartmann number have expressible effect on fluid flow and heat transfer. Some other studies on mixed convection flow of magneto-nanofluid in enclosures are given in Refs. [32–44].

According to aforementioned investigations and to the authors’ best knowledge, there has no investigation studied the MHD mixed convection heat transfer in a double lid-driven cavity using the two-phase nanofluid model. Thus, authors of the present study believe that this work is valuable. Therefore, the aim of this comprehensive numerical study is to investigate the effects of partial slip, heat source position and length, solid obstacle, inclination angle on mixed convection flow of magneto-nanofluid in a double lid-driven square cavity in the presence of a magnetic field. We believe that this work is a good contribution for improving the thermal performance and the heat transfer enhancement in some engineering instruments.

2. MATHEMATICAL MODELING

Figure 1 exhibits the schematic of 2D inclined square cavity of length ($H$) filled with Cu-water nanofluid with internal heat generation. The angle of inclination of the enclosure from horizontal in the counterclockwise direction is denoted by $\Phi$. Both the upper and bottom walls of the cavity are moving with constant speed $U_0$ in its own plane, where the impact of partial slip is imposed on these walls. The cavity is containing an adiabatic square obstacle in the center of the cavity. A magnetic field with strength $B_0$ is utilized on the left side of the enclosure in the horizontal direction. Heat source with a constant volumetric rate ($q'$) is located on a part of the left wall with length b, and the other remaining parts of this wall are thermally insulated. The right wall of the enclosure is maintained at cooled temperature $T_c$, while, both the upper and bottom walls of the enclosure are considered to be adiabatic. The nanoparticles (Cu) and base fluid (water) are assumed to be in thermal equilibrium. The incompressible, laminar and generates heat at a uniform rate $Q_0$ assumptions are used for modeling the nanofluid. The thermo-physical properties of the base fluid and the nanoparticles are given in Table I. The thermo-physical properties of the nanofluid are assumed constant except for the density variation, which is determined, based on the Boussinesq approximation.

The single phase approach is used for modeling the nanofluid heat transfer and therefore the governing equations can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(1)

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} + v_{nf} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

$$+ \frac{(\rho \beta)_{nf}}{\rho_{nf}} g(T-T_c) \sin \Phi$$

(2)

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho_0} \frac{\partial p}{\partial y} + v_{nf} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)$$

$$+ \frac{(\rho \beta)_{nf}}{\rho_{nf}} g(T-T_c) \cos \Phi - \frac{\sigma_{nf} B_0^2}{\rho_{nf}} v$$

(3)

Table I. Thermophysical properties of water and copper.

<table>
<thead>
<tr>
<th>Property</th>
<th>Water</th>
<th>Copper(Cu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>997.1</td>
<td>8933</td>
</tr>
<tr>
<td>$C_p$</td>
<td>4179</td>
<td>385</td>
</tr>
<tr>
<td>$k$</td>
<td>0.613</td>
<td>401</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$21 \times 10^{-5}$</td>
<td>$1.67 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.05</td>
<td>5.96 x 10^2</td>
</tr>
</tbody>
</table>
Table II. Comparisons of the mean Nusselt number at the top wall for $Pr = 0.71$, $Gr = 10^2$.

<table>
<thead>
<tr>
<th>$Re$</th>
<th>Iwatsu et al. [45]</th>
<th>Khanafar and Chamkha [46]</th>
<th>Present data</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>1.94</td>
<td>2.01</td>
<td>1.93</td>
</tr>
<tr>
<td>400</td>
<td>3.84</td>
<td>3.91</td>
<td>3.91</td>
</tr>
<tr>
<td>1000</td>
<td>6.33</td>
<td>6.33</td>
<td>6.31</td>
</tr>
</tbody>
</table>

\[
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\alpha_{sf}}{\mu_{sf}} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \frac{Q_0}{\rho C_p} (T - T_c) + \frac{Q_0}{\rho C_p} (T - T_c) + \frac{Q_0}{\rho C_p} (T - T_c)
\]

(4)

where $u$ and $v$ are the velocity components along the $x$- and $y$- axes respectively, $T$ is the fluid temperature, $p$ is the fluid pressure, $g$ is the gravity acceleration, $\mu_{sf}$ is the dynamic viscosity, $\nu_{sf}$ is the kinematic viscosity.

The boundary conditions are

On the bottom wall

\[
u = 0, \quad 0 \leq x \leq H, \quad \nu = \frac{\partial T}{\partial y} = 0
\]

On the top wall

\[
u = \lambda_i U_0 + \nu_{sf} \frac{\partial u}{\partial y}
\]

$u = 0, \quad v = 0 = \frac{\partial T}{\partial y} = 0$

\[
u = 0, \quad u = v = 0 = 0
\]

Fig. 2. Comparison of the present study with $Re = 1000$, $Pr = 0.71$, $Gr = 10^2$, $Ha = \phi = 0$.

Fig. 3. Streamlines (up), and isothermal (down) for Cu-water at $\phi = 0.05$, $Ha = 10$, $D = 0.5$, $B = 0.5$, $Q = 1$, $\Phi = 30^\circ$, $S_c = 1$. 

\[
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{sf} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \frac{Q_0}{\rho C_p} (T - T_c) + \frac{Q_0}{\rho C_p} (T - T_c)
\]

(4)
On the left wall
\[
\frac{\partial T}{\partial x} = -\frac{q''}{k_{nf}}, \quad d + 0.5b \leq y \leq d + 0.5b
\]
\[
\frac{\partial T}{\partial x} = 0 \quad \text{otherwise}
\]

On the right wall
\[
x = 1, \quad u = v = 0 = 0,
\]
\[
T = T_e
\]

Various models for the thermo-physical properties of nanofluids are suggested in the literature. In the current
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InVESTIGATION, we are adopting the relations which depend on the nanoparticles volume fraction only.

The effective density of the nanofluid is given as:

$$\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_p$$

(6)

Where $\phi$ is the solid volume fraction of the nanofluid, $\rho_f$ and $\rho_p$ are the densities of the fluid and of the solid fractions respectively, and the heat capacitance of the nanofluid is given as:

$$\rho c_p_{nf} = (1 - \phi)\rho c_p_f + \phi\rho c_p_p$$

(7)

The thermal expansion coefficient of the nanofluid can be determined by:

$$\rho\beta_{nf} = (1 - \phi)\rho\beta_f + \phi\rho\beta_p$$

(8)

where $\beta_f$ and $\beta_p$ are the coefficients of thermal expansion of the fluid and of the solid fractions respectively.

Thermal diffusivity, $\alpha_{nf}$ of the nanofluid is defined as:

$$\alpha_{nf} = \frac{k_{nf}}{\rho_{nf} c_p_{nf}}$$

(9)

In Eq. (9), $k_{nf}$ is the thermal conductivity of the nanofluid which for spherical nanoparticles, according to the Maxwell-Garnetts model and it is defined as:

$$k_{nf} = \frac{k_f (k_f + 2k_p) - 2\phi (k_f - k_p)}{(k_f + 2k_p) + \phi (k_f - k_p)}$$

(10)

The effective dynamic viscosity of the nanofluid based on the Brinkman model is given by

$$\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{\frac{5}{3}}}$$

(11)

where $\mu_f$ is the viscosity of the fluid fraction and the effective electrical conductivity of nanofluid was presented by Maxwell as:

$$\frac{\sigma_{nf}}{\sigma_f} = 1 + \frac{\gamma - 1 - \phi}{\gamma + 2 - (\gamma - 1)\phi}$$

(12)

where $\gamma = (\sigma_p)/\sigma_f$

Fig. 6. Local Nusselt number with different $\text{AR}$ at $\phi = 0.05$, $Ha = 10$, $D = 0.5$, $B = 0.5$, $Q = 1$, $\Phi = 30^\circ$ (a) $S_5 = 0$, (b) $S_5 = 1$.

Fig. 7. Local Nusselt number with different $B$ at $\phi = 0.05$, $Ha = 10$, $D = 0.5$, $AR = 0.3$, $Q = 1$, $\Phi = 30^\circ$, (a) $S_5 = 0$, (b) $S_5 = 1$. 
Introducing the following dimensionless set:

\[
X = \frac{x}{H}, \quad Y = \frac{y}{H}, \quad U = \frac{u}{U_0}, \quad V = \frac{v}{U_0},
\]
\[
P = \frac{p}{\rho_n U_0^2}, \quad \theta = \frac{(T - T_0)}{\Delta T}, \quad Ri = \frac{Gr}{Re^2},
\]
\[
S_0 = \frac{\mu_{nf}}{H}, \quad \Delta T = (T_n - T_0), \quad Q = \frac{Q_n H^2}{(\rho c_p)_{nf}},
\]

into Eqs. (1)–(4) yields the following dimensionless equations:

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0
\]  
(14)

\[
U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re} \left( \frac{\nu_{nf}}{\nu_f} \right) \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right)
\]
\[
+ Ri \frac{(\rho \beta)_{nf}}{\rho_n \beta_f} \cos \Phi \cdot \theta
\]

\[
U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{Re} \left( \frac{\nu_{nf}}{\nu_f} \right) \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} - \frac{V}{Da} \right)
\]  
(15)

where \( Pr = \nu_f/\alpha_f \), \( Re = (U_0 H)/\nu_f \), \( Gr = (g \beta_f q' H^2)/\nu_f^2 \), \( Ha = B_0 H \sqrt{\sigma_f/\mu_f} \), \( Ri = Gr/Re^2 \) are respectively the Prandtl number, Reynolds number, Grashof number, Hartmann number and Richardson number.

The dimensionless boundary condition for Eqs. (14)–(17) are as follows:

\[
Y = 0, \quad 0 \leq X \leq 1, \quad V = \frac{\partial \theta}{\partial Y} = 0
\]

\[\frac{\partial \theta}{\partial X} = \frac{\partial \theta}{\partial Y} = 0\]

\[
0.0 \leq Y \leq 1.0
\]

Fig. 8. Local Nusselt number with different \( D \) at \( \phi = 0.05, \ Ha = 10, \ B = 0.5, \ AR = 0.3, \ Q = 1, \ \Phi = 30^\circ \). (a) \( S_0 = 0 \), (b) \( S_0 = 1 \).

Fig. 9. Local Nusselt number with different \( Ha \) at \( \phi = 0.05, \ D = 0.5, \ B = 0.5, \ AR = 0.3, \ Q = 1, \ (a) S_0 = 0 \), (b) \( S_0 = 1 \).
On the bottom wall
\[ U = \lambda_b + S_b \frac{\mu_{nf}}{\mu_f} \frac{\partial U}{\partial Y} \]
\[ Y = 1, \ 0 \leq X \leq 1, \ V = \frac{\partial \theta}{\partial Y} = 0 \]

On the top wall
\[ U = \lambda_t + S_t \frac{\mu_{nf}}{\mu_f} \frac{\partial U}{\partial Y} \]
\[ X = 0, \ U = V = 0, \]

On the right wall
\[ \frac{\partial \theta}{\partial X} = -\frac{k_f}{k_{nf}}, \ D - 0.5B \leq Y \leq D + 0.5B, \]
\[ \frac{\partial \theta}{\partial X} = 0 \ \text{otherwise} \]

On the left wall
\[ X = 1, \ U = V = 0, \ m \]
\[ \theta = 0 \] (18)

The quantities of physical interest are defined below. The local Nusselt number is defined as:
\[ Nu_s = \frac{1}{(\theta)_{\text{heat source}}} \] (19)

The average Nusselt number is defined as:
\[ Nu_m = \frac{1}{B} \int_{D-0.5B}^{D+0.5B} Nu_s dY \] (20)

3. NUMERICAL METHOD AND VALIDATION

The modeled Eqs. (14)–(17) with the boundary conditions (18) have been solved numerically using the collocated finite volume method. The first upwind and central difference approaches have been used to approximate the convective and diffusive terms, respectively. The resulting discretized equations have been solved iteratively, through alternate direction implicit (ADI) method using the SIMPLE algorithm. The velocity correction has been made using the Rhie and Chow interpolation. For convergence, under-relaxation technique has been employed. To check the convergence, the mass residue of each control volume
has been calculated and the maximum value has been used to check the convergence. The convergence criterion was set as $10^{-5}$, and a uniform grid resolution of $81 \times 81$ is found to be suitable.

In order to verify the accuracy of present method, the obtained results in special cases are compared with the results obtained by Iwatsu et al. and Khanafer and Chamkha in terms of the mean Nusselt number at the top wall, for different values of $Re$. As we can see form Table II, the results are found in a good agreement with these results. These favorable comparisons lend confidence in the numerical results to be reported subsequently.

Figure 2 displays a comparison between the temperature contour presented in this work with those of Khanafer and Chamkha and Iwatsu et al. The result shows a very good agreement between this work and the previously published work.

4. RESULTS AND DISCUSSION

The effects of partial slip, inclination angle of the geometry, heat generation and heat source size and position on the magneto-mixed convection flow and heat transfer of Cu-water nanofluid in an inclined double lid-driven square cavity with centrally placed square adiabatic obstacle subjected to constant heat flux are examined. The pertinent parameters involved here are heat source length, heat source position, nanoparticle volume fraction, Hartmann number, heat generation parameter, aspect ratio of solid block, inclination angle of the cavity, and partial slip parameter. The values of Grashof number ($Gr = 10^4$), Reynolds number ($Re = 10$), direction of moving walls are fixed throughout the study. The results are analyzed for various pertinent parameters involved here with the fixed values of the $Gr = 10^4$, $Re = 10$, $\lambda_s = -\lambda_r = 1$, $S_x = S_y$.

4.1. Flow and Thermal Fields

Figure 3 demonstrated the influence of the aspect ratio of the centrally placed square adiabatic solid block on flow and thermal fields with $Ha = 10$, $D = 0.5$, $B = 0.5$, $Q = 1$, $S_y = 1$, $\varphi = 0.05$, and $\Phi = 30^\circ$. The flow speed is reduced when the aspect ratio of the adiabatic solid block is increased from AR=0.1 to 0.5. Two inner circulating
cells are formed above and below the obstacle and the above cell gets weakened when increasing the values of aspect ratio of the adiabatic solid block. The corresponding thermal distributions shows the convective heat transfer inside the enclosure. Figure 4 shows the effect of iso-flux heat source length on streamlines and isotherms for various values $B$ with $Ha = 10$, $D = 0.5$, $AR = 0.3$, $Q = 1$, $S_b = 1$, $\varphi = 0.05$, and $\Phi = 30^\circ$. The flow field is not strong in the upper portion of the cavity for smaller heat source length ($B = 0.2$). When increasing the heat source length, the strength of the flow field is increased and the cell in the upper portion of the cavity is strengthened. The thermal distribution is weak inside the cavity for smaller heat source length. The effects of the position of the heater on flow and thermal fields are displayed in the Figure 5. The isotherms are clearly indicated the effect of location of the heater inside the cavity. The flow is not affected much while changing the position of the heater as like the isotherms. It is found that the thermal boundary layer is formed at the upper-right corner of the cavity in most of the situations examined.

### 4.2. Heat Transfer Rate

In order to find the local heat transfer rate on various parameters, the local Nusselt number is plotted along the heat source and they are displayed in the Figures 6–9. The effect of aspect ratio of the adiabatic solid body on local heat transfer rate is portrayed in the Figure 6. The Nusselt number profiles clearly show that the heat transfer rate decreases when increasing the aspect ratio of the solid block. The local heat transfer attains its maximum at the leading edge of the heater and it decreases gradually and then increases when approaching the vertical length to trailing edge for the case $S_b = 0$. However, the local heat transfer is found to be high at the leading edge of the heater and steadily decreases thereafter for the case $S_b = 1$.

Figure 7 shows the effect of length of the heat source on local heat transfer rate with $Ha = 10$, $D = 0.5$, $AR = 0.3$, $Q = 1$, $\varphi = 0.05$, and $\Phi = 30^\circ$ for two values of $S_b = 0$ (a) and $S_b = 1$ (b). The local heat transfer is found to be higher for a small value of heater length ($B$). It can be seen that the local Nusselt number decreases with increasing heater length. Along the heater, the local heat transfer rate decreases starting from its maximum value at the lower or leading edge to a minimum value and then increases again.
a little toward the upper or trailing edge of the heater. When \( S_b = 0 \), the heat transfer is increased near the trailing edge of the heater. However, the opposite trend is found on local heat transfer for the case \( S_b = 1 \), which is clearly seen from Figure 7(b). Figure 8 displays the heater position effect on local Nusselt number with \( Ha = 10, D = 0.5, AR = 0.3, Q = 1, \phi = 0.05 \), and \( \Phi = 30^\circ \) for two values of \( S_b = 0 \) (a) and \( S_b = 1 \) (b). The trend of local energy transport is similar for all values of \( D \). Figure 9 shows the magnetic field effect on local Nusselt number with \( B = 0.5, D = 0.5, AR = 0.3, Q = 1, \phi = 0.05 \), and \( \Phi = 30^\circ \) for two values of \( S_b = 0 \) (a) and \( S_b = 1 \) (b). The strengthening the values of the Hartmann number reduces the heat transfer rate.

The averaged energy transport across the cavity is explored with various combinations of the pertinent parameters involved here through the Figures 10–17. Figure 10 also demonstrated the heat transfer deteriorates with the aspect ratio of the solid body and Hartmann number. The deviation is very high in the absence of the magnetic field. The effect of inclination angle of the cavity is displayed in the Figure 10. The heat transfer rate acts nonlinearly with the inclination angle of the cavity. It is also observed that the heat transfer rate is suppressed when the size of the solid block is increased. The highest averaged heat transfer is attained at \( \Phi = 90^\circ \) for all sizes of the solid block. However, the lowest heat transfer is attained at \( \Phi = 270^\circ \) for both cases.

Figure 11 displays the influence of average Nusselt number versus inclination angle for different values of aspect ratio of solid block. It is observed that the heat transfer rate behaves nonlinearly with inclination angle. There is no uniform trend on energy transfer with inclination. The higher values of heat transfer is found at \( \Phi = 90^\circ \) for all sizes of the solid block when \( S_b = 0 \). But, the higher amount of heat transfer is found at \( \Phi = 60^\circ \) for all sizes of the solid block when \( S_b = 1 \). It is also observed that the minimum value of heat transfer is found at \( \Phi = 270^\circ \) for all sizes of the solid block and both cases of \( S_b (= 0 \text{ and } 1) \). Figure 12 displays the impact of averaged Nusselt number versus Hartmann number for different values of heat source length. It is observed that the heat transfer rate decreases when increasing the heat source length. It is also witnessed that the mean heat transfer rate declines gradually with Hartmann number. Figure 13 shows the effect of...
averaged Nusselt number versus inclination angle for different values of heat source length. The heat transfer rate acts non-linearly with inclination of the cavity. The higher values of heat transfer is established at $\Phi = 90^\circ$ when $S_h = 0$ and at $\Phi = 60^\circ$ when $S_h = 1$ for all heat source length. It is detected that the lowest value of heat transfer is found at $\Phi = 270^\circ$ for all heat source length and both cases of $S_h$ ($= 0$ and $1$).

Figure 14 exhibits the influence of averaged Nusselt number versus Hartmann number for different places of heat source. It is found that the mean heat transfer rate decreases when increasing the position of heat source from bottom. It is concluded that the bottom position provides higher heat transfer rate than other positions considered. It is also seen that the mean Nusselt number decays gradually when increasing the values of the Hartmann number. Figure 15 displays the influence of mean Nusselt number versus inclination angle for different values of heat source length. It is observed that the heat transfer rate behaves non-linearly with inclination angle. Figure 16 displays the influence of average Nusselt number versus inclination angle for different values of heat generation parameter. It is observed that the averaged heat transfer rate decreases when increasing the international heat generation parameter. Comparing these figures, it is witnessed that the mean heat transfer rate is low in the presence of slip parameter. The effect of nanoparticle volume fraction on averaged heat transfer rate is clearly seen from the Figure 17. The mean Nusselt number reduces when increasing the volume fraction of nanoparticle. It is observed that there is no difference on rate of mean heat transfer when inclination angle is $150^\circ$ for all values of the nanoparticle volume fraction.

4.3. Correlation for Heat Transfer Rate

Finally, the least square method is used to derive a correlation between average Nusselt number and Richardson number ($Ri$), Hartmann number ($Ha$) and the inclination angle of the cavity ($\Phi$). A correlation based on the computational data has been obtained as follows:

$$Nu_{av} = 1.0471 Ri^{0.01071} Ha^{-0.0026} \Phi^{-0.0023}$$  \hspace{1cm} (21)

5. CONCLUSIONS

The numerical study on the magneto-mixed convection flow and heat transfer in an inclined double lid-driven square cavity with centered square adiabatic obstacle is examined here. The effects of partial slip, heat source length, heat source position, inclination angle of the cavity and internal heat generation are examined thoroughly under various combinations of pertinent parameters involved here. The collocated finite volume method is used to solve the governing equations. The following observations are concluded from the study. Changing the length of heat source affects much the flow and thermal fields than the position of heat source. The heat transfer rate decreases when increasing the aspect ratio of the solid block and heat source length. The heat transfer rate behaves non-linearly with inclination of the cavity. The heat source at bottom provides higher heat transfer rate than other positions along the vertical left wall. The average heat transfer rate declines when increasing the values of the Hartmann number and heat generation parameter.

Conflicts of Interest

The authors declare no conflicts of interest.

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References and Notes

cita Iranica* 26, 2817 (2019).
4. K. Narrein, S. Sivasankaran, and P. Ganesan, *Numerical Heat Trans-
5. K. Narrein, S. Sivasankaran, and P. Ganesan, *Numerical Heat Trans-
8. R. Saidur, K. Y. Leong, and H. A. Mohammed, *Renewable and Sus-
Applications* 65, 247 (2014).
B/Fluids* 29, 472 (2010).
B/Fluids* 36, 82 (2012).